Imperfect Automata:
Effects of “mutation” on the evolution of
automaton 01101110 (i.e., rule 110)

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Outline

Background. Cellular automata (CA) are mathematical models. They consist of cells, which can exist in a limited number of finite states. The state of each cell at time t is determined by simple rules, based on the states of its neighboring cells. CAs have been studied systematically since the early 40s and were shown to be relevant to many disciplines (Schiff, 2011). Their notoriety increased significantly with the publication of Wolfram’s *A New Kind of Science* (2002), a monumental endeavor that entailed the examination of trillions of CAs. In it, Wolfram shows that some CAs, despite their conceptual simplicity, exhibit fascinating behaviors, and proposes that the Universe itself may undergo an automaton-like evolutionary process, with structures and phenomena of mind-bending complexity emerging from simple rules.

Motivation. While exploring their relevance to behavioral sciences (McDowell & Popa, 2009), one aspect caught my attention: it seemed that all emerging structures and patterns could be traced back to the first few generations; patterns were evolving, colliding, changing, and disappearing, but no new patterns or structures were emerging from non-patterns (i.e., from “noise”). In biological systems, *novelty* is made possible by spontaneous variability – i.e., *mutation* (Thomas, 1974), a concept central to my computational work on learning (Popa & McDowell, 2016) and on the emergence of psychological objects and phenomena from changes in neuronal activation states (Popa, 2019). Because CAs are sets of precise rules, the state of each new cell is *determined* by the states of its neighbors. The question examined here was: *what if every time a cell is created, there’s a small probability to make a mistake, and write 0 instead of 1 and vice-versa?*

Results. I explored this idea in the context of Rule 110 (01101110), an elementary CA notorious for its fascinating properties (Wolfram, 2002). A small amount of *mutation* – e.g., 0.00005 probability to make mistakes – facilitated the emergence of new patterns and structures, disconnected from the initial conditions. Mutation rates of 0.0001 – 0.0005 produced an abundance of irregular, interacting structures. Mutation rates higher than 1% prevented the emergence of discernable patterns, producing instead an amalgam of ill-defined, organic-looking structures. These results suggested that imperfect automata may provide useful insight on the evolution of non-deterministic systems and on the emergence and quantification of novelty – two key topics in machine learning and artificial intelligence.

Cellular Automata (CA)

Cellular automata (CA) are discrete mathematical models, discovered in the 1940s by Stanislaw Ulam and John von Neumann (Schiff, 2011). They consist of cells. Each cell can exist in a limited number of mutually exclusive states, like 0 or 1. The state of each cell at any given time is determined by simple rules, applied to the states of a pre-defined neighborhood. Figure 1 shows automata in which the state of each cell is defined by a neighborhood of ten and three cells, respectively.

Figure 1 shows examples of neighborhoods; time flows downwards.

and before that	1	1	0	0	0	1
before	0	1	1	1	0	0
now	1	1	0	0	0	1
next			?			

and before that	0	1	0	0	0	1
before	1	1	0	0	1	1
now	0	0	1	1	0	0
next			?			

Despite their apparent simplicity, some CAs were shown to exhibit complicated properties and behaviors, difficult (if not impossible) to predict by examining the rules that generated them (Figure 2, **red ellipses**). This property made them relevant to scholars from many disciplines, from computer science to philosophy (Schiff, 2011). Stephen Wolfram, for example, proposes that studying automata will shed light on notoriously difficult research questions, like the origins of the universe and the emergence of consciousness (Wolfram, 2002).

Figure 2. Patterns evolve, collide (red circles), change, and disappear, but no new ones appear from uniformity (i.e., non-patterns, or “noise”). Pattern X, for example, can be traced back (red arrows) all the way to two areas of origin. Keep in mind that the automaton wraps upon itself, like a tube.

Emergence, determinism, and novelty

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In biological systems, *novelty* is made possible by spontaneous variability – i.e., *mutation* (Thomas, 1974), a concept central to my computational work on learning (Popa & McDowell, 2016) and on the emergence of psychological objects and phenomena from changes in neuronal activation states (Popa, 2019). Because CAs are sets of precise rules, the state of each new cell is *determined* by the states of its neighbors. The question examined here was: *what if every time a cell is created, there's a small probability to make a mistake, and write 0 instead of 1 and vice-versa, as shown in Figure 3?*

Figure 3. The figure illustrates the idea of mutation applied to cellular automata.

Method

The program, available on GitHub, was written by the author in **Python 3**, using *tkinter*¹ for the interface (Figure 4) and *Pandas* for data handling. Some of its features are listed in Table 1.

Table 1. Features included in imperfect_automata.py.

run any elementary automata (Rules 0 to 255);
run and display two automata at the same time, each with its own “mutation” rate (i.e., probability to break the rule)
specify the number of different cells in initial conditions, i.e., “butterflies”;
compute and display the Hamming Distance between the two automata in real time;
save data for future analyses: cells, mutants, and Hamming Distances;
save the patterns as Postscript files;

The Hamming Distance between two strings of equal length is defined as the number of bits that must be flipped in order to obtain one string from the other (Hamming, 1950). For example, the Hamming Distance between 0011 and 0000 is two, and between 1111 and 0001 – three.

Hamming Distances are relevant to many fields, including behavior of human and digital agents, as shown in Popa (2010; 2013, pp. 13, 19-21, 56-60).

When running two automata at the same time, the program computes the Hamming Distance between them at each generation; for ease of interpretation, this value is expressed in percentages. Imagine, for example, two automata with 330 cells each, like the ones discussed in this paper. Each generation, or column, is essentially a string with 330 bits. If the two string are identical, the Hamming Distance between them is 0. If all bits are the same except for one, the Hamming Distance is one, or 0.03% (1/330, Figure 6). A 50% Hamming Distance at generation n means that ~165 of the 330 cells of the top automaton are in the same state as in the bottom automaton, whereas the other 165 are in opposite state. Transforming the n^{th} “top” generation into the n^{th} “bottom” generation would require “flipping” 165 bits from 0 to 1 or vice-versa.

¹ The tooltip class was retrieved from <https://code.sololearn.com/c5dk37BDJEeK/#py>.

Figure 4. Features implemented in `imperfect_automata.py`. The `cell_size` parameter is useful for illustrative purposes. In this example, each cell was 50 pixels wide, so each automaton had six cells. The first column is filled at random; this column defines the automaton's initial conditions. In this example, the initial conditions of the two automata were identical, with the exception of one cell, in red. The Hamming distance between their initial conditions was one, or 16.6% (1/6). At generation 2 (second column), the Hamming Distance increased to 50%, because the first three bits were identical and the last three bits were different.

Note: all automata discussed here are wrapped upon themselves, like cylinders that grow in time, to ensure that the first and last cell in each column has three neighbors to define its state. The cell size was 1 pixel in all cases. The Rule investigated was 110, in all cases.

Results overview

Small amounts of spontaneous variability, or *mutation* (Figure 6) facilitated the emergence of new patterns and structures, disconnected from the initial conditions. Mutation rates of 0.0001 – 0.0005 (Figure 7) produced an abundance of irregular, interacting structures; in behavioral terms, one might characterize their behavior as “effervescent”, or “impulsive”. Increasing the rate of mutation even further prevented the emergence of discernable patterns, producing instead amalgams of ill-defined, organic-looking structures (Figures 8 and 9).

These results suggested that imperfect automata may provide useful insight on the evolution of non-deterministic systems and on the emergence and quantification of *novelty* – two key topics in machine learning and artificial intelligence.

Coming up next: *how many mutants, where?*

Figures 5 – 9

Figure 5. Sensitivity to initial conditions. Both automata started from identical initial conditions (*first column*), except for the **224th** cell (out of 330 cells); the probability to break the rules was zero, in both cases. Over generations (columns), the difference increased more and more, as illustrated by the gradual increased in Hamming Distance between the two automata. After a few hundred generations, the number of bits that were different between the two automata increased from 1/330 to 165/330, or from 0.3% to 50%, after which it remained relatively constant.

Figure 6. Mutation rate 0.0 and 0.00005; identical initial conditions; times flows towards the right.

Figure 7. Probability to break the rules was 0.0001 (top) and 0.0005 (bottom). Their behavior could be described as “*effervescent*”, “*disorganized*”, or “*impulsive*”.

Figure 8. Increasing the probability to break the rules even further, produced Mutation rates higher than 1% prevented the emergence of discernable patterns, producing instead an amalgam of ill-defined, organic-looking structures.

Figure 8.1. The first half of patterns shown in Figure 8, zoomed in for clarity.

Figure 9. Mutation rates higher than 1% prevented the emergence of discernable patterns, producing instead an amalgam of ill-defined, organic-looking structures. Both canvases show the same automaton, with and without identifying the mutants (yellow).

Figure 9.1. The first half of patterns shown in Figure 9, zoomed in for clarity.

References

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Links

Code: https://github.com/andreipopa515/imperfect_automata

Tooltip class for tkinter: <https://code.sololearn.com/c5dk37BDJEeK/#py>

Naming convention: <http://mathworld.wolfram.com/ElementaryCellularAutomaton.html>

Rule 110: <http://www.wolframalpha.com/input/?i=rule+110>)

A New Kind of Science (full text): <https://www.wolframscience.com/nks/>