Interference in stimuli employed to assess masking by substitution

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ABSTRACT

Enns and Di Lollo (1997, *Psychological Science*, 8, 135-139) described a kind of visual masking termed "masking by substitution" in which a diamond-shaped target stimulus was masked by four small squares. The present study addresses the question of if, or to what extent, interference in the stimuli could reduce their power. Interference was assessed based on (1) the sums of the amplitudes in the Fourier spectra, and (2) the norms of the Fourier spectra. The two methods gave interference effects of 17 % and 28 %, respectively. It is concluded that in the case of the stimuli used by Enns and Di Lollo it cannot be assumed that the target stimulus has the same stimulus power when presented together with the masking stimulus as when presented alone. The interference takes place in the stimuli and does not depend upon the visual system.

Key Words: visual masking; Fourier; norm; stimulus; amplitude; spectrum.

INTRODUCTION

Enns and Di Lollo (1997) described a form of masking in which the presence of four small squares masks a diamond-shaped target stimulus. This was termed "masking by substitution". It has become quite clear that owing to interference the introduction of masking stimuli has the potential to reduce the stimulus power of a target stimulus (Skottun, 2017). This, it needs to be emphasized, is an effect which takes place in the stimuli and does not depend on them being seen or that they activate any visual system. The purpose of the present report is to assess the potential effect of interference between masking and target stimuli in the kinds of stimuli employed by Enns and Di Lollo (1997).

INTERFERENCE

The Fourier Transform transforms a signal into a series of sine functions. Thus, the adding of two stimuli, such as, e.g., the adding of a masking stimulus to a target stimulus, can be understood as the adding of two series of sine functions. That is, to one sine function from the spectrum of one stimulus is added the corresponding sine function (i.e., the sine function of equal frequency) from the spectrum of the other. Adding two sine functions of equal frequency gives a new sine function. The amplitude of this function is determined by the phase relationship between the two summed functions. If these are in-phase the amplitudes will add and if they are out-of-phase they will subtract. Denoting the phases of the two functions by θ_1 and θ_2 , respectively, we have that if $\theta_1 - \theta_2 = n \pi$, where n is an even integer (including 0), the functions will add and if $\theta_1 - \theta_2 = m \pi$, where m is an odd integer (not including 0), they will subtract. In cases where $\theta_1 - \theta_2 \neq n \pi$, with n being an even integer (including 0), the amplitude of the summed signal will be smaller than the sum of the amplitudes of the individual signals. Stated mathematically: $|\sin(x - \theta_1) + \sin(x - \theta_2)| < |\sin(x - \theta_1)| + |\sin(x - \theta_2)|$ when $\theta_1 - \theta_2 \neq 2 n \pi$, with n being an integer (including 0) and where $|\cdot|$ denotes the amplitude operator. This reduction in amplitude is *interference* [FOOTNOTE 1]. In practice two stimuli of some complexity will almost never have identical phase spectra. Thus, interference needs to be considered when adding two stimuli.

Interference is illustrated in Fig. 1A which shows the addition of two sine functions with unit amplitude but different phases. Such functions can be represented by vectors in the plane (i.e., a 2-D vector). The amplitude of the sine function is represented by the length of the vector and the angle (relative to the horizontal) gives its phase, θ . The horizontal and vertical components are given by $\cos(\theta)$ and $\sin(\theta)$, respectively. The adding of two sine functions thus amounts to the adding of two vectors (Fig. 1B). In the case where the two vectors have unit length, which means their functions have unit amplitude, their horizontal and vertical components become $\cos(\theta_1) + \cos(\theta_2)$ and $\sin(\theta_1) + \sin(\theta_2)$, respectively (with θ_1 and θ_2 denoting the angles with the horizontal of the two vectors, respectively). The length of the vector sum, that is to say, the amplitude of the combined function, becomes $\sqrt{(\cos(\theta_1) + \cos(\theta_2))^2 + (\sin(\theta_1) + \sin(\theta_2))^2}$. In the case of the two sine functions in Fig. 1A their phase angles are $\pi/6$ and $3\pi/4$. If we insert these values for θ_1 and θ_2 in the expression for the length we get 1.218 for the length of the vector sum. Thus, the amplitude of two sine functions of unit amplitude one with the phase of $\pi/6$ the other with phase $3\pi/4$ is 1.218. This is clearly smaller than the sum of the amplitudes which is $1 + 1 = 2$. This entails that the full amplitudes of the two sine functions cannot both be contained in the amplitude of the combined function. That is, by combining functions of unequal phase the amplitude of one of the functions or of both is reduced. (To put it colloquially: Something has to give.) (For further discussions of interference in visual stimuli see Skottun, 2017.)

ASSESSING INTERFERENCE

In order to provide a single measure of interference two approaches were adopted. One was based on the sum of the amplitudes. The notion behind this approach is that the total amount of amplitudes in the stimuli provides a measure of stimulation. Thus, if $F(\omega)$ is the Fourier Transform of $f(x)$, with ω denoting spatial frequency, we get that the total amount of stimulation is $\sum_x |F\omega_x|$. For a 2-dimensional stimulus this becomes $\sum_x \sum_y |F(\omega_x, \omega_y)|$, where ω_x and ω_y denote the frequencies along the x and y

Fig. 1

Fig. 1. (A) The combination of two sine functions out of which one has a phase of $\pi/6$ and the other a phase of $3\pi/4$. Each function has an amplitude of 1 as indicated by the double-headed arrow labeled "Each Function". The sum of the two functions is indicated with a dashed line. The amplitude of this sum is indicated by the double-headed arrow marked "Combined". This amplitude is 1.218 which is smaller than the sum of the the amplitudes of the two sine functions (i.e., $1.218 < (1 + 1) = 2$). (B) The vector representation of the addition of the same two sine functions. Again, it is apparent that the length of the vector representing the sum, vector marked $\pi/6 + 3\pi/4$, is shorter than the length of the vector marked $\pi/6$ plus the length of the vector marked $3\pi/4$. These plots illustrate that the amplitude of the sum of two sine functions is smaller than the sum of the amplitudes when their phases differ, which entails that the full amplitudes of both sine functions cannot be contained in the combined function under such conditions.

dimensions, respectively. Interference is then taken to be the difference between the sum of amplitudes in the combined stimulus relative to the sums of amplitudes in the target and masking stimuli determined separately. This is expressed by the *Relative Amplitude Sum* which is calculated as

$\sum_x \sum_y |F_{t+m}(\omega_x, \omega_y)| / (\sum_x \sum_y |F_t(\omega_x, \omega_y)| + \sum_x \sum_y |F_m(\omega_x, \omega_y)|)$, where F_t , F_m and F_{t+m} denote the Fourier Transforms of the target, the masking stimulus and the combined stimulus, respectively. When there is no interference the Relative Amplitude Sum is 1.0 and when interference is present this sum falls below 1.0. The amount of by which the Relative Amplitude Sum falls below 1.0 is a measure of interference.

The second method was based on the norms of the Fourier spectra. The norm of a vector is a measure of its length. In order to calculate the norms the spectra were flattened into 1-D arrays which were treated as vectors with 65536 elements (since the stimuli had 256 x 256 elements, see below, we get a total number of stimulus elements of 256 x 256 = 65536). The norm of a vector f is usually calculated as $\|f\| = \sqrt{\sum_i f_i^2}$, In the case of a Fourier spectrum which has complex-valued components this has to be altered to $\|f\| = \sqrt{\sum_i f_i \overline{f_i}}$ where the overline denotes complex conjugation. The norm of a Fourier spectrum is the square root of the sum of the energies. (It may be asked: Why not use the sum of the energies? The answer is that one needs a measure in which the quantity for the combined stimulus equals the sum of the quantities for the two stimuli determined separately when the two stimuli are identical and have the same position, i.e. identical phase spectra. This is not the case for the sum of the energies [FOOTNOTE 2].) Based on the norms the *Relative Norm* was calculated as $\|F_{t+m}\| / (\|F_t\| + \|F_m\|) = \sqrt{\sum F_{t+m} \overline{F_{t+m}}} / (\sqrt{\sum F_t \overline{F_t}} + \sqrt{\sum F_m \overline{F_m}})$ where F_t , F_m , and F_{t+m} (as above) denote the Fourier spectra of the target stimulus, of the masking stimulus, and of the combined target and mask stimulus, respectively. Thus, the Relative Norm gives the norm of the combined stimulus relative to the sum of the norms of the target and masking stimuli determined separately. A Relative Norm of less than 1.0 denotes interference and the degree to which the Relative Norm falls below 1.0

provides a measure of the magnitude of the interference.

In the present analyses the fundamental component of the Fourier spectra was set equal to zero. This component is simply the sum of the luminance. Thus, by setting this component to zero it is assumed that the stimuli correspond to the deviations in luminance from the background. (For some further remarks on this issue see below.)

STIMULI

Stimuli were generated as 256 x 256 element arrays. The target stimulus was a dark diamond with a height of 104 elements and width of 104 elements centered upon the stimulus array. It is depicted in Fig. 2A. The masking stimulus consisted of four squares each with dimensions of 20 x 20 elements. The locations (of the lower right hand corner of these) in the initial test were (50, 50), (50, 188), (188, 188), and (188, 50). The masking stimulus is shown in Fig. 2B. and the two stimuli together are shown in Fig. 2C. The value for the bright areas was one and that of the background was zero.

RESULTS

The Relative Amplitude Sum for the stimuli shown in Fig. 2 was found to be 0.83. This corresponds to a reduction of 17 %. That is to say, the sums of the amplitudes in the combined stimulus is 17 % smaller than the sum of amplitudes in the target stimulus plus the sum of the amplitudes in the masking stimulus. The value for the Relative Norm was 0.72 which gives a reduction of 28 %. Thus, both measures provide evidence for interference. These reductions, it should be emphasized, are in the stimuli and do not depend upon the visual system.

As already mentioned the dc level was set equal to zero in these analyses. Carrying out the analyses with the dc level in place gave a slightly lower value for the Relative Amplitude Sum, 0.79 versus 0.83, and a much smaller value for the Relative Norm, namely 0.49 versus 0.72. The reason for this is that the dc level, which is simply the sum of the luminance values (multiplied by the scaling factor $1/\sqrt{n}$, where n is the total number of elements in the signal), will dominate the amplitude spectra when the stimuli

Fig. 2

Fig. 2. (A) The target stimulus. (B) The masking stimulus. (C) The target and the masking stimuli presented together.

contain many positive values. Consider the following theoretical example where the amplitude sum with *dc* value set equal to 0 for the two stimuli combined is 1.5 and that of each of the two stimuli measured separately is 1. We then get for the Relative Amplitude Sum: $1.5/(1 + 1) = 1.5/2 = 0.75$. Assume, for the sake of simplicity, that all three stimuli have *dc* values of 1. When adding the *dc* values we then get: $(1.5 + 1)/((1 + 1) + (1 + 1)) = 2.5/4 = 0.625$. Thus, by adding the *dc* levels the Relative Amplitude Sum can become artificially small. To realize that this is artificial consider the effect of adding more white space around the stimuli. This would increase the *dc* level by equal amounts for all three stimuli, so that, for instance, instead of having $(1.5 + 1)/((1 + 1) + (1 + 1)) = 0.625$ we could have $(1.5 + 2)/((1 + 2) + (1 + 2)) = 3.5/6 = 0.58$ for the same stimuli. For stimuli which are modeled as deviations from a background set to zero this is not a major issue but for stimuli which are dominated by positive values (as in the present case) it needs to be taken into account. One way to do so is to set the *dc* values in the Fourier spectra equal to zero. (For the sake of the record the actual *dc* values in the present analyses were 234.5, 249.8, and 228.2 for the target, the mask and the combined stimuli, respectively.)

Enns and Di Lollo (1997) found that the separation between the masking stimuli and the target had little or no effect upon the amount of masking. It was therefore of some interest to find out how the interference effect is affected by this separation. The results are shown in Fig. 3A where the Relative Amplitude Sum is shown with a solid line and the Relative Norm with a dashed line. As can be seen, in agreement with the results of Enns and Di Lollo (1997), changing the separation has little effect upon the magnitude of the interference. This is also consistent with estimates of the effect of separation on the Relative Amplitude Sum for other kinds of stimuli (see Figs. 3a and 3c of Skottun, 2017). These results further document that stimuli need not overlap in order for there to be interference. In fact, it is possible for the largest interference effect to occur when the stimuli do not overlap. This would be because interference is the result of a difference in spatial phase and stimuli which differ in spatial location may be more likely to differ in their spatial phase spectra.

Fig. 3

Fig. 3. (A) The Relative Amplitude Sum as a function of separation between masking and target stimuli is indicated with a solid line and the Relative Norm is given with a dashed line. The term "ratio" refers to the ratio of the value for the target and masking stimuli combined relative to the sum of their separate values. (B) The separation was measured from the edge of the diamond making up the target stimulus and the closest corner of the squares in the masking stimulus as indicated by the two-headed arrow marked "s" for separation. (All four squares were at the same distance from their nearest edge of the target stimulus.)

DISCUSSION

The Relative Amplitude Sum is a measure of the sum of amplitudes in the combined stimulus relative to the sums in the target and mask determined separately. When the Relative Amplitude Sum is less than 1.0 it is not possible for the full amplitudes of both the masking and the target stimuli to be contained in the amplitudes of the combined stimulus. Similarly for the norms: When the Relative Norm is less than 1.0 the full norms of both target and masking stimuli cannot be contained in the norm of the combined stimulus. Thus, when the stimuli are combined either the amplitudes, or norms, in the target are reduced, the amplitudes, or norms, in the masking stimulus are reduced, or both. It is not possible based on the present analyses to determine the amount of amplitude reduction, or norm reduction, linked to each of the two stimuli. In masking experiments the task given the subjects is generally to judge some aspect of the target stimulus. Thus, the critical factor is to what extent are the amplitudes, or norm, linked to this stimulus reduced when it is presented along with a target stimulus. Were we to make the assumption that the two stimuli are equally affected by interference the values of 17 % and 28 % would give the appropriate values for reduction of amplitudes and norm in the target stimulus.

Were we further to assume that the sum of amplitudes or the norm of the Fourier series represent measures of stimulus power we get that the stimulus power of the target and masking stimuli in the masking experiments of Enns and Di Lollo (1997) is reduced when the stimuli are combined. This entails, therefore, that in such experiments it cannot be assumed that the target stimulus presented along with the masking stimulus has the same stimulus power as when it is presented by itself. It should be emphasized, as was pointed out by Skottun, (2017), that this effect takes place in the stimuli and does not depend on the stimuli being seen or being processed by any visual system.

By estimating interference using two methods (i.e., by sums and norms) it is demonstrated that the amount of interference depends, to some degree, on the method of measurement. However, because both methods used here depend on the Fourier spectra it may be thought that interference is a consequence of the Fourier Transform. That this is not the case is indicated by the fact that interference is evident

in the Relative Norms. Because $\|f\| = \|F\|$, where f and F denote the space-domain signal and its Fourier Transform, respectively, interference demonstrated by the norms of the Fourier spectra entails that interference is also present in the norms of the space-domain representations of the stimuli.

The present analyses have involved only the spatial aspects of the stimuli. This may make it seem that the present analyses only deal with simultaneously presented target and masking stimuli. Enns and Di Lollo (1997) assessed the effect of the masking stimulus as a function of Stimulus Onset Asynchrony (SOA). Since there is temporal summation in the visual system it would seem that it is possible for there to be interference between stimuli presented at somewhat different times.

Enns and Di Lollo (1997) found the largest masking effect when the masking stimulus was presented slightly after the target stimulus. If the temporal integration window for the mechanism subserving the target stimulus is skewed in relation time (i.e., if it is not symmetric along the time axis) a skewed SOA function may arise [FOOTNOTE 3]. Most visual stimuli give rise to neuronal responses with an abrupt onset and a shallower decay. If the temporal integration of stimulation followed the response the largest interference effect could occur when the masking stimuli follows some time after the target stimulus.

In order for interference to be present it is required that the two stimuli both fall within the area from which stimulation is integrated. This may correspond to the receptive field of some neurons. Since receptive fields of visual neurons tend to increase in size with eccentricity it is interesting to note that Enns and Di Lollo (1997) found little or no masking for centrally fixated stimuli but considerable masking for stimuli located in the parafovea where the receptive fields, presumably, are larger.

Enns and Di Lollo (1997) discussed "masking by camouflage" as a possible contributor to masking by substitution. How masking by camouflage is related to interference is not clear. Nor is it clear if, or how, masking by camouflage is related to properties of the visual system or how it is to be quantified. In contrast, interference effects can be given precise mathematical expressions and its effects can be given precise measures.

The present analyses do not represent an attempt at explaining masking by substitution. Rather,

they are an attempt at understanding the stimuli involved in such masking. The term "masking" refers to certain responses to visual stimuli. It seems that in order to understand the responses to visual stimuli one must first understand the stimuli. The present study represents an attempt at this.

FOOTNOTES

1. Each component in a Fourier spectrum is given by a complex number of the form: $z = a + b i$, where a and b are real numbers and $i = \sqrt{-1}$. Its amplitude is given as $|z| = |a + b i| = \sqrt{a^2 + b^2}$. The addition of two components then becomes $|z_1 + z_2| = |a_1 + a_2 + (b_1 + b_2) i| = \sqrt{(a_1 + a_2)^2 + (b_1 + b_2)^2}$. Important in this connection is that in general $|z_1 + z_2| \neq |z_1| + |z_2|$. The two complex numbers, z_1 and z_2 , may be represented as vectors in the complex plane. We then get that $|z_1 + z_2| = |z_1| + |z_2|$ only when the two vectors have the same direction. That is, when they have the same phase (i.e., when $b_1/a_1 = b_2/a_2$). (See Fig. 1 and Skottun, 2017, for further details.)

2. Consider the simple case where the target stimulus $\mathbf{t} = \{0, 0, 1, 0\}$ and the masking stimulus $\mathbf{m} = \{1, 0, 0, 0\}$. The Fourier Transforms of these stimuli are $\{0.5, -0.5, 0.5, -0.5\}$ and $\{0.5, 0.5, 0.5, 0.5\}$, respectively, giving energy spectra of $\{0.25, 0.25, 0.25, 0.25\}$ for both stimuli. (The stimuli in these examples were chosen deliberately so as to have real-valued Fourier spectra.) This makes the sum of energies equal to 1 for either stimulus. Were we to add the stimuli we would get $\mathbf{t} + \mathbf{m} = \{1, 0, 1, 0\}$ which has a Fourier spectrum of $\{1, 0, 1, 0\}$, making the energy spectrum also $\{1, 0, 1, 0\}$ with a sum of energies of 2. Thus, in this case the sums of the energies add: $1 + 1 = 2$. Now, consider the case where $\mathbf{t}$ and $\mathbf{m}$ both are $\{0, 0, 1, 0\}$ (when they both have the same phase). Were we to add these we would get $\mathbf{t} + \mathbf{m} = \{0, 0, 2, 0\}$. The Fourier spectrum of this stimulus is $\{1, -1, 1, -1\}$ making the energy spectrum $\{1, 1, 1, 1\}$ which sums to 4. This is clearly not the same as the sum of energies in $\mathbf{t}$ plus the sum of energies in $\mathbf{m}$ which both are 1 (i.e., $1 + 1 \neq 4$). On the other hand, the norms add: $\|\mathbf{t}\| = \sqrt{1} = 1$, $\|\mathbf{m}\| = \sqrt{1} = 1$, and $\|\mathbf{t} + \mathbf{m}\| = \sqrt{4} = 2$, which gives $\|\mathbf{t}\| + \|\mathbf{m}\| = \|\mathbf{t} + \mathbf{m}\| = 2$. (For comparison, when $\mathbf{t} = \{0, 0, 1, 0\}$ and $\mathbf{m} = \{1, 0, 0, 0\}$ we get $\|\mathbf{t}\| = \|\mathbf{m}\| = 1$ making $\|\mathbf{t}\| + \|\mathbf{m}\| = 2$, while $\|\mathbf{t} + \mathbf{m}\| = \sqrt{2}$ which

is smaller than 2, i.e. smaller than $\|\mathbf{t}\| + \|\mathbf{m}\|$, indicating the presence of interference. The interference being $2 - \sqrt{2}$.)

3. A model incorporating spatial and temporal inaccuracy, asymmetric temporal integration and interference in both space and time may cause interference to be largest when the masking stimulus is presented some time after the target. Although this model incorporates features of the visual system (spatial and temporal inaccuracy, and temporal integration) the interference is generated in the stimuli. The properties of visual system simply "sculpts" the interference. The details of this model is beyond the scope of the present report which focuses on the spatial aspects of the stimuli. It is mentioned here only to counter any notion that interference would not be able generate asymmetric SOA functions.

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