Leaping across the mental canyon: Higher-order long-distance analogical retrieval

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Abstract

Previous experiments have shown that a comparison of two written narratives highlights their shared relational structure, which in turn facilitates the retrieval of analogous narratives from the past (e.g., Gentner, Loewenstein, Thompson, & Forbus, 2009). However, analogical retrieval occurs across domains that appear more conceptually distant than merely different narratives, and the deepest analogies use matches in higher-order relational structure. The present study investigated whether comparison can facilitate analogical retrieval of higher-order relations across written narratives and abstract symbolic problems. Participants read stories which became retrieval targets after a delay, cued by either analogous stories or letter-strings. In Experiment 1 we replicated Gentner et al. who used narrative retrieval cues, and also found preliminary evidence for retrieval between narrative and symbolic domains. In Experiment 2 we found clear evidence that a comparison of analogous letter-string problems facilitated the retrieval of source stories with analogous higher-order relations. Experiment 3 replicated the retrieval results of Experiment 2 but with a longer delay between encoding and recall, and a greater number of distractor source stories. These experiments offer support for the schema induction account of analogical retrieval (Gentner et al., 2009) and show that the schemas abstracted from comparison of narratives can be transferred to non-semantic symbolic domains.

Keywords: analogy; memory; analogical retrieval; letter-string analogies; cross-domain retrieval; long-distance analogy.

Word count: 8,302

1. Introduction

Analogical retrieval has led to many important insights in science and design. These insights often emerge as a result of the application of prior knowledge from vastly different domains to one's present situation. Prominent examples include the development of Velcro by analogy to burdock burrs, the development of the Rutherford–Bohr model by analogy to the solar system, and Claude Shannon's development of the computer chip by drawing an analogy between Boolean logic and electric circuits. Despite this, most studies of analogical retrieval have primarily focused on mere narrative differences between written text, whereas rarer is a focus on retrieval that requires a connection between written narratives and an abstract symbolic domain. Gick and Holyoak (1983) found that comparing two narratives that share a common conceptual structure allows people to better use that structure in a subsequent novel situation. Similarly, case comparison has been shown to facilitate learning (Alfieri, Nokes-Malach, & Schunn, 2013). The *schema induction account* of analogical retrieval (Gentner, Loewenstein, Thompson, & Forbus, 2009) explains this effect by suggesting that a comparison between two cases promotes an alignment of common conceptual structures (relations), resulting in an abstraction of the two cases into a more general principle, or *schema*. This schema is then available as a memory probe for future analogical retrieval. Inherent to the schema induction account is the assumption that analogical comparison highlights relational structure regardless of surface features. That is, it allows a connection from the initial case that is to be retrieved (the source analogue) and the subsequent case that cues the retrieval of the source after a delay (the target analogue). Despite this focus on abstraction, the existing research is based on demonstrating transfer from one semantically-rich narrative to another semantically-rich narrative. Narrative has structure, conventions and common themes, and their use introduces the risk of inadvertent semantic overlap, raising the possibility that these schemas are less abstract than assumed. It is at least an open question whether the same transfer phenomena apply between narrative and non-narrative materials, and a major goal of the current research is to address this question.

Gentner et al. (2009) provided strong evidence for the schema induction account by showing that comparing written passages that are analogous helps people remember other analogous passages. Undergraduates read seven passages about contract disputes, and how they were resolved with negotiation. Critically, only one of the scenarios was resolved via the target negotiation principle (the negotiation principle of contingent contracts, whose retrieval the

researchers were measuring).¹ After a 30-minute delay (with a filler task), half of the participants were given two example cases that were also resolved with the target negotiation principle, and were explicitly asked to compare them, while noting the key parallels. The other half were given the two cases to read but given no instruction to compare them. All participants were subsequently asked to recall a source case that best matched the two target comparison cases. Participants that explicitly compared target cases were significantly more likely to retrieve the relevant source case than participants that did not compare them.² It appeared that the explicit comparison assisted the creation of an abstract schema that then served as a retrieval cue to the original story. Gentner et al. called this effect of facilitating retrieval through the comparison of target cases the *late analogical abstraction effect*. This name was given in order to distinguish it from experiments that facilitate retrieval through a comparison of source cases at the time of initial encoding. This effect has been demonstrated in both studies of cued-retrieval (Gentner, Loewenstein, & Thompson, 2004; Gentner et al.) and problem solving (Kurtz & Loewenstein, 2007).

The studies reported here addressed a major limitation of this research: prior studies mainly measured domain distance of retrieval only as changes in similarity within a certain type of task or materials, for example, written narratives (see Barnett & Ceci, 2002). We examined transfer between written narratives and abstract symbolic stimuli. It is important to investigate methods that facilitate this more conceptually distant analogical retrieval because connections between semantics and formal symbolic domains can facilitate innovation in science and design. Further, these connections can increase learning outcomes in non-expert populations, as frequently mapping between symbolic and semantic structures is a challenge to learners (e.g., Reed, 1987). A second limitation in prior work is that there is often a focus on the retrieval of

¹ *Contingent contracts* are agreements based on terms that depend on some future outcome, e.g., a pay rise if a certain performance level is met. They are particularly useful when the two parties in dispute have different predictions about the future. For example, if an author and publisher predict differing numbers of book sales, then a contingent contract would specify different terms based on what the real sales end up being.

² Some of these studies use the spontaneous application of prior information as the key measure, while others directly ask people what they remember. We group this work together because retrieval seems to be the primary bottleneck of successful application, as often people struggle to draw spontaneous connections to analogous cues that they have been presented with, but have no problem seeing the connections once they are told that some prior information is relevant (e.g., Gick and Holyoak, 1980). Further, research has explicitly shown how retrieval and problem solving are intertwined processes, with the same factors affecting both (e.g., Bernardo, 2001; Mandler & Orlich, 1993; Ross & Kennedy, 1990).

simple first-order relational structures (relations among surface features) but no explicit consideration of more complex higher-order relations (relations among relations), which seem to characterise most expert analogical retrieval (Ahmed & Christensen, 2009; Ball, Ormerod, & Morley, 2004).

We aim to address both limitations by investigating analogical retrieval across domains that share virtually no surface similarity, but do share higher-order relational structure. That is, story narratives and abstract symbolic problems in the form of Hofstadter's (1995) letter-string analogies. We elaborate on these letter-string analogies further below, but it is worth noting two important characteristics. First, their underlying relational structure can be matched to specifically-designed written narratives. That is, while a written, semantically-rich paragraph and a seemingly meaningless string of letters may appear to be very different on the surface, they can be made to share a relational structure (the specific way that we do this will be explained further below). Second, they offer a semantic-free symbolic structure that participants with little math or science expertise can reason with, which further allows us to control for potential variation in prior experience if using a symbolic domain that is used in real-world problem solving. Table 1 presents three examples from Hofstadter (1995), and *some* potential solutions that depend on different representations of the problem. We will later further explain these letter strings' use in the current experiments.

Table 1

Example letter string problems from Hofstadter (1995) and potential solutions

Letter string problem	Short hand	Potential solutions
Suppose the letter-string abc were changed to abd ; how would you change the letter-string ijk in "the same way"?	abc:abd::ijk:?	ijl
Suppose the letter-string abc were changed to abd ; how would you change the letter-string kji in "the same way"?	abc:abd::kji:?	kjj, kjh, lji
Suppose the letter-string abc were changed to abd ; how would you change the letter-string mrrjjj in "the same way"?	abc:abd::mrrjjj:?	mrrjjk, mrrkkk, mrrjjjj

1.1. The limits of schema induction

Similar to the schema induction account, Jung and Hummel (2015) noted that any underlying relation is learnable through comparison as long as the relation is invariant across the examples. This suggests that *cross-domain* connections can be made, that is, connections across instances that differ in surface features and context. For instance, people should be able to connect a written narrative and a letter-string so long as there is a shared relational structure between them. A story about a succession to the throne and the letter string change of *abc* to *abd* both share the concept of succession. However, the distance across which comparisons are made can vary. Dunbar (1995) classified three types of analogies in his study of analogy-use in a set of molecular biology laboratories. First, *local analogies* are analogies that connect cases from the same domain, for example, using knowledge from previous experiments to make inferences about a very similar current experiment. Second, *regional analogies* are analogies that connect cases from similar domains, for example, using knowledge about a well-known virus to inform knowledge about a lesser-known virus. Third, *long-distance analogies* are analogies that connect cases from very different domains, for example, explaining a biological structure that allows molecules to enter but not exit by referencing the song Hotel California.

Similarly, Christensen and Schunn (2007) studied within-domain (equivalent to local analogies) and between-domain analogies (equivalent to long-distance analogies). While *domain* can mean different things across the literature, they define analogical distance as the extent of superficial similarity between the source and target analogues. This is why explaining a biological structure using a similar kind of biological structure is considered a local analogy and explaining it using the song Hotel California is considered a long-distance analogy. As such, since most studies of retrieval (or transfer) involve transfer between the same kind of tasks (e.g., narrative to narrative), they can be considered regional analogies. Transfer between different kinds of tasks (e.g., narrative to letter-string) can therefore be considered as a long-distance analogy.

While usually analogical transfer and retrieval in the literature is studied between narratives (e.g., Gentner et al. 2009), sometimes other task types are used. For instance, between net-logo simulation (Goldstone & Sakamoto, 2003; Goldstone & Son, 2005), math problems (Novick, 1990; Novick & Hmelo, 1994), and letter-string proportional analogies (Burns, 1996). Research on long-distance analogies is sparser. Most work of this kind was done on transfer

between narratives and visual stimuli (e.g., Chen, 1995; Pedone, Hummel, & Holyoak, 2001; Beveridge & Parkins, 1987). This was shown to be useful for design problem solving (Casakin & Goldschmidt, 1999). In fact, the frequency of long-distance analogy use in real-life depends on the setting (Christensen & Schunn, 2007; Dunbar & Blanchette, 2001), but they appear to be harder to make than local analogies. As such, it is important to understand when and how long-distance retrieval can occur.

1.2. Levels of abstraction

We also investigated whether long-distance retrieval is limited to specific levels of abstraction. A *surface feature* is an object or object-attribute (Gentner, 1983). For example, the Sun and Earth are round in the same way that an asteroid and a comet are also round. The object-attribute is ROUND. In Structure Mapping Theory (SMT; Gentner), all aspects of an analogy should be able to be expressed in predicate structure. In predicate structure this object-attribute would be represented as ROUND(Sun) and ROUND(Earth) mapping to ROUND(asteroid) and ROUND(comet). A *first-order relation* is a connection between surface features. For example, the mass of the Sun is greater than the mass of a planet in the same way that the mass of a boulder is greater than the mass of a pebble. The first-order relation is GREATER MASS. In predicate structure this would be represented as GREATER MASS(Sun, Earth) mapping to GREATER MASS(boulder, pebble). A *higher-order relation* is a connection between other relations (Gentner; Goldstone, Gentner, & Medin, 1989). For example, the greater mass of the Sun causes a planet to revolve around it in the same way that the greater mass of the atom nucleus causes an electron to revolve around it.³ The higher-order relation is CAUSE. In predicate structure this would be represented as CAUSE(GREATER MASS(sun, planet), REVOLVE AROUND(planet, sun)) mapping to CAUSE(GREATER MASS(nucleus, electron), REVOLVE AROUND(electron, nucleus)). It is important to note that there are many ways of representing knowledge, so each example is only one possible representation and is used merely to illustrate the basic concepts that underly SMT.

The present study focused on retrieval rooted in higher-order relational structures as they are particularly powerful for driving analogical inference. This has been shown both with naïve

³ We are aware that this is not how this process works at the subatomic level, but Gentner's original SMT paper uses this explanation, so we decided to use it for simple illustrative purposes.

laboratory participants, and in studies examining real-world insight and discovery in, for example, professional design and engineering teams (Ahmed & Christensen, 2009; Ball, Ormerod, & Morley, 2004; Christensen & Schunn, 2007; Markman, 1995). The distinction between higher- and lower-order levels of abstraction is also of central importance to theories of relational cognition; the relational reinterpretation hypothesis (Penn, Holyoak, & Povinelli, 2008) suggests that the qualitative distinction between human animals and nonhuman animals lies largely in that humans alone can reinterpret merely surface (perceptual) connections as higher-order relations, and that other animals are limited to first-order relational interpretations. Although our focus is on higher-order relational retrieval, we include both surface and first-order relational matches between encoding and retrieval stimuli as the controls. Doing this allows us to test whether any benefits of the methods to foster schema induction are specific to levels of abstraction.

1.3. Letter-string analogies

The task we chose for investigating long-distance retrieval of higher-order relations from narrative problems was Hofstadter's (1995) letter-string proportional analogies. These have a number of properties that make them particularly appropriate. First, they involve relations between abstract symbols with no inherent semantics. As such, stories that are written to match the relational structure that underlies a letter-string analogy only match according to that relational structure. There is no possibility for experimental participants to use a more superficial semantic structure to connect the disparate domains, ensuring that we are examining long-distance analogical retrieval when an individual is reminded by a semantic narrative. Second, they are structured in a way that responses to them express different levels of abstraction. An example of a letter-string proportional analogy is: "Suppose the letter-string *abc* were changed to *abd*; how would you change the letter-string *mrrjjj* in 'the same way'?" (Hofstadter, p. 238). Due to the abstract domain, there are many ways to solve these types of analogies that indicate different levels of representing the analogy (that is, through a surface, first-order, or higher-order representation; Holyoak & Thagard, 1995). For instance, the most common representation is considering the change of *abc* to *abd* as being one of succession. That is, the last instance, *c*, is replaced by its successor, *d*. In the same vein, the last instance of *mrrjjj* is *jjj*, and its successor is *kkk*. As such, as shown in the upper section of Figure 1, the solution would be *mrrkkk*. In predicate structure, this change would be written as *SUCCESSOR(d, c)*, which maps to

SUCCESSOR(kkk, jjj). This example illustrates the potential of letter-strings to embody the same concepts as those in more semantically rich and more realistic situations.

Figure 1. A representation of two possible responses to the analogy $abc:abd::mrrjjj:?$. The first-order relational response (a) considers the change from c to d as one of ordinal succession, as per their order in the alphabet. Since c is the last term in the string, jjj , as the last string of $mrrjjj$, is also changed to its successor in the alphabet: kkk . The higher-order response (b) represents abc as 1-2-3 and abd as 1-2-4, as per their order in the alphabet. The quantity of different letters in the string $mrrjjj$ can also be represented numerically as 1-2-3. This higher-

order relational mapping leads to the inference that the fourth term in the analogy should be a quantity successor of *mrrjjj* that can be numerically represented as 1-2-4, that is, *mrrjjjj*.

However, both Hofstadter (1995) and Boden (1994) noted that the *mrrkkk* answer for the *mrrjjj* string only uses the last letter of the string and effectively ignores the alphabetic successor relations inherent in *abc*. They suggest that an answer such as *mrrjjjj* captures the higher-order structure of numerical representation. This answer connects two first-order relations: ordinal succession and quantity succession. The higher-order relational structure is shown in the lower section of Figure 1. In predicate structure, this change would be written as $\text{NUMERICAL}(\text{ORDINAL}, \text{LETTER GROUPS}(a, b, c))$, which maps to $\text{NUMERICAL}(\text{QUANTITY}, \text{LETTER GROUPS}(m, rr, jjj))$. The NUMERICAL higher-order relation is critical to the analogy because without it there is no connection between *abd* and *mrrjjjj*. In fact, people usually rate this solution quite poorly, compared to the *mrrkkk* solution (Burns, 1996). Most other letter-string analogies detailed by Hofstadter strongly suggest one representation, but the *mrrjjj* based analogy can be conceivably represented in distinctly different ways. Thus, we use this characteristic in order to probe people's representation (detailed further below). It is also worth noting that it is possible that mathematically adept reasoners could potentially find these abstract schemas easier to grasp. However, these kinds of individual differences were beyond the scope of the current study. In sum, the advantages of letter-string analogies are that they are a novel symbolic domain that naïve participants can reason with, controlling for variation in background knowledge; they provide a test of long-distance analogical retrieval with written narratives; and they allow for precise control over the level of relational abstraction between encoding and retrieval stimuli.

1.4 Matching narrative and letter-string structure.

To test how letter-string problems can cue retrieval of narratives, we wrote narratives illustrating the kinds of relations from the above example letter-string problems. As just noted, the first-order relation from the letter-string problem was *succession*, for example *C changing to D* was mapped to *JJJ changing to KKK*. This kind of problem was matched with a story about succession to a throne. The higher-order relation from the letter-string problems mapped an order relation to a quantity, for example *C changing to D* was mapped to *JJJ changing to JJJJ*. This was matched with a story about a visitor's centre whose number of staff is dependent on the day of the week, such that Monday will have one worker, Tuesday will have two workers, etc. In

addition, we created a surface match between a letter-string problem and a narrative in which, for example, *C changing to D* was matched to a story about a typographical error that would replace the letter *C* with the letter *D*.

We note that the specifics of the problems and narratives (such as the specific letters, and how the letter mapped to other kinds of features, such as geometric shapes) differ in our actual materials (see below), but these examples illustrate how narrative relations can be matched to the relations of the non-symbolic semantic domain of letter-string analogies.

1.5 Present studies

Experiment 1 will first replicate Gentner et al.'s (2009) results by showing that a comparison of story narratives facilitates the retrieval of analogous story narratives and provide preliminary results for story retrieval cued by a comparison of letter-strings. Experiment 2 will provide evidence for a high-order long-distance retrieval in a more statistically high-powered investigation of the effect. Finally, Experiment 3 will replicate this effect in a more challenging retrieval environment. There will be a longer filler task between encoding and retrieval, and more source stories at the time of encoding to compete with each other at retrieval. Our analysis of each experiment will test the same three hypotheses: 1) participants given materials likely to invoke a surface-level schema will be more likely to retrieve the surface-level story than participants in the other schema conditions; 2) participants given materials likely to invoke a first-order-level schema will be more likely to retrieve the first-order-level story than participants in the other schema conditions; 3) participants given materials likely to invoke a higher-order-level schema will be more likely to retrieve the higher-order-level story than participants in the other schema conditions. Support of each hypothesis will provide evidence of analogical retrieval, but the level will show how deep that analogical retrieval could be. If the hypotheses are confirmed, then this experiment would provide converging evidence for the schema induction account of analogical retrieval (Gentner et al., 2009). Critically, they will suggest that schema induction can facilitate retrieval across more conceptually distant cases than originally thought, that is, long-distance analogical retrieval between written narratives and abstract symbolic problems.

2. Experiment 1

Experiment 1 replicated Gentner et al.'s (2009) finding that comparison of current cases facilitates retrieval of analogous past cases and explored preliminary results for retrieval of stories from a comparison of analogous letter-string cues (see Figure 2).

Participants were randomly allocated to either the *letter-string* or *story* analogy condition. Within these conditions, participants were further allocated to one of three schema conditions: *surface schema*, *first-order relational schema*, and *higher-order relational schema*. Participants had to retrieve one of three stories cued by either new story narratives if they were in the *story condition*, or letter-string analogies if they were in the *letter-string condition* (detailed further below). We hypothesised correct source story retrieval after the story comparison in the story condition, which would provide evidence for the late analogical abstraction effect (Gentner et al.); and hypothesised correct retrieval after the letter-string comparison in the letter-string condition, which would provide evidence for long-distance analogical retrieval.

Figure 2. Comparison of simplified designs in Gentner et al. (2009) and the present study. While participants in Gentner et al. retrieved source stories from a comparison of *story* cues, participants in the present study retrieved source stories from a comparison of *letter-string* cues.

2.1. Method

2.1.1. Participants. A hundred and twenty-five first-year undergraduates (87 female) from the University of Sydney were recruited with informed consent and given course credit for their participation. The age range was 17-29, with a mean age of 19.4 ($SD = 2.163$). One participant (in the letter-string surface schema condition) did not complete the first filler task due to computer error, so their data was excluded from the analysis.

2.1.2. Apparatus. The experiment was completed online so the computers, operating systems, and browsers varied. However, the experiment included a function that blocks access from a phone or tablet. Further, participants were prohibited by the software from navigating backwards through the pages.

2.1.3. Materials. All materials were webpages coded with HTML and JavaScript.

Source stories. The three source stories, shown in Table 2, were designed to differ in semantic content, but be equivalent in structure and length. Each story presented an initial conflict, and a subsequent resolution. Critically, the resolution of each story also provided the information that made up the target schema for that story, which would subsequently either match or mismatch with the later story or letter-string analogies. This instantiated the following three schema condition manipulations:

For the *surface schema* the story hinged on simply *changing an E to an F*, expressed in the story as the response to a typographical error in a businessman's flyers. It is considered a surface story because its similarity to the later cues is based on a simple attribute mapping. In predicate structure, this could be represented as E(letter) and F(letter).

For the *first-order relational schema* the story hinged on *succession*, expressed in the story as the succession to the throne by one sister, called Sylvia, after her older sister, Valerie, ran away. This resolution is a first-order relation because it is related to the later cues by virtue of mapping one relation (succession) and no surface features. In predicate structure this could be represented as SUCCESSOR(Sylvia, Valerie).

For the *higher-order relational schema* the story hinged on the *correspondence of quantity to an order*, expressed in the story by the correspondence of the sequence of number of staff in a centre to a day's sequential order in the week. This resolution is a higher-order relation because it relies on a connection of first-order relations. That is, numerical representation connects the first-order relational structure of two types of *succession*: ordinal succession, as in

the order of days in the week; and quantity succession, as in the number of staff allocated. In predicate structure this could be represented as NUMERICAL(ORDINAL, WEEKDAYS(Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday), which maps to NUMERICAL(QUANTITY, WORKERS(1, 2, 3, 4, 5, 6, 7)).

Analogy examples. The analogy examples page was designed to give participants training in the structure and function of proportional analogies for the three levels of abstraction, using the same examples that were given in the Introduction. The responses to these incomplete proportional analogies were used to check participant understanding of analogies, and most were able to pass this check.

Letter-string analogue comparison. Participants in the letter-string analogy condition received one of three pairs of proportional letter-string analogies to compare, as shown in Figure 3. In the figure, all three pairs are presented together to facilitate the reader's comparison of the differences between each pair. Each of the three pairs was designed to induce one of the three schemas underlying the source stories. For instance, shapes were used to encourage an abstraction of the schema beyond the domain of letters. Participants in the letter-string condition compared only one of these pairs. The surface comparison (a) makes it apparent that the critical change is that from *E* to *F*, regardless of position in the string or presence of shapes. In the first-order relational schema comparison (b) the increase in the letters' alphabetic order and shape's number of sides, expresses the concept of *succession*. The higher-order relational schema comparison (c) connects the alphabetic succession of the initial strings to the ordinal succession of the latter shape or letter strings. The higher-order relationship *numerical representation* connects these two forms of succession. The same basic structure and symbols (letters and shapes) were used for all three of the comparisons.

Story analogue comparison. Participants in the story analogy condition received one of three story pairs to compare. Each of the three pairs was designed to induce one of the three schemas underlying the source stories.

Table 2

Source Stories and Principle Explanation used in Experiment 1

Schema condition	Source story text	Explanation text for comparison phase
Surface	John is an owner of a small-town computer company and wanted to advertise his company to the town. He printed out some flyers with large font size to put up. However, there was a typographical error in the flyers, with the title printing out as 'Elash Computers' instead of 'Flash Computers', which John knew would confuse potential customers if put up around town. As such, he had to rewrite the company name for the posters, changing the 'E' to an 'F', and printing them again. There was only a typo in the word 'Elash', so only the letter 'E' was changed from the letter 'E', to the letter 'F', correcting the word 'Elash' to 'Flash'.	Both pairs rely on the same rule: Change E to F.
First-order relation	Jerome is an advisor to the King of a large nation and wanted to confirm the successor to the throne. He thought of Valerie, who was the king's eldest daughter. However, the advisor found out that despite being the next in line to the throne, Valerie had run away to a mountain town because she did not want to take on the responsibilities associated with being a Queen. As such, he worked out that Sylvia should be the next in line to the throne as she is the second-oldest sibling. The order of succession in the kingdom is found by birth order, so if the first born child is not able to uphold the throne, then the second born is next in line.	Both pairs rely on the same rule: change the rightmost symbol (letter or shape) to its successor. For example, triangle changes to square because of the number of sides, and G to H because of alphabetic order.
Higher-order relation	Julia is a manager at a local information centre and wanted to staff her centre efficiently. She usually has about three people working every day. However, the number of visitors to the centre increases consistently each day, with almost no visitors on Mondays and peak number of visitors on Sundays, so most days the centre is either overstaffed, or understaffed. As such, she decided that she will roster on an amount of staff that corresponds with the order of that day in the week. The centre will have one staff member on Monday, being the first day of the working week, two on Tuesday, and so on, with seven people working on Sundays.	Both pairs rely on the same rule: change the length of the rightmost group to its successor. For example, E (fifth in the alphabet) changing to F (sixth in the alphabet) = five symbols (letters or shapes) changing to six symbols.

Figure 3. The three pairs of comparison cues for the letter-string analogy condition.

2.1.4. Procedure. The experiment was run as an online study through a series of webpages. Initially participants read an introductory paragraph that explained the simple navigation features of the webpages and completed a short demographics page. The experimental procedure is shown in Figure 4. Participants read three one-paragraph narratives and typed a response to indicate how each story was resolved. Participants were told to memorize the stories and summarize their resolutions after reading. They then completed two minutes of the first filler task and were subsequently presented with the analogy example page.

Participants then completed the comparison task, as per their analogy and schema condition. Underneath this comparison, participants read a short explanation of the target principle, shown in Table 2. Following this, they completed an incomplete proportional analogy to check their understanding of the principle, which most participants were able to do. In the story condition, participants were asked to respond with what they thought were the key parallels between the two stories. The wording of these instructions was almost identical to the instructions used in Gentner et al. (2009). On the subsequent page, participants were asked to retrieve the source story that matched the comparison they just did.

Figure 4. Flowchart of the procedure for Experiment 1.

2.2. Results and Discussion

All analyses were run in IBM SPSS Version 22. The story descriptions generated in the story condition retrieval phase were classified into one of four levels: *null* (did not retrieve any story, or response was not coherent enough), *surface* story, *first-order* story, and *higher-order* story. This classification was determined by which source story the retrieval most resembled and was completed by one of the authors. Table 3 presents the frequency of each classification for each of the three levels of schema condition (surface schema, first-order schema, and higher-

order schema) for the story analogy conditions (upper section) and the letter string analogy condition (lower section).

For the story analogy condition, the overall effect was significant, $\chi^2 (6, N = 64) = 35.714$, $p < .001$, demonstrating an association between participants' schema condition and source story retrieval rates. To test the story analogy hypotheses, the conditions were collapsed into 2 x 2 tables for each of the three hypotheses outlined in the introduction. For each hypothesis, schema condition was recoded into a dichotomous variable of those in the target schema condition and those that are not. The critical comparison is between conditions rather than between stories to account for the possibility that some stories may be easier to retrieve than others. Therefore, for each schema condition we tested the hypothesis that the predicted schema-matched story was retrieved more often than it was in other conditions. Participants in the surface schema condition retrieved the surface schema story (13/21 times, 61.9%) more than those not in the surface schema condition retrieved it (5/43 times, 11.6%), $\chi^2 (1, N = 64) = 17.643$, $p < .001$, $\eta^2 = .525$. Participants in the first-order schema condition retrieved the first-order schema story (13/21 times, 61.9%) more than those not in the first-order schema condition retrieved it (5/43 times, 11.6%), $\chi^2 (1, N = 64) = 17.643$, $p < .001$, $\eta^2 = .525$. Those in the higher-order schema condition retrieved the higher-order schema story (9/22 times, 40.9%) more than those not in the higher-order schema condition retrieved it (5/42 times, 11.9%), $\chi^2 (1, N = 64) = 7.107$, $p < .01$, $\eta^2 = .333$. These results suggest that a comparison of stories can induce a schema that sufficiently cues a previously encoded analogous story, providing converging evidence for the late analogical abstraction effect (Gentner et al., 2009). Further, the effect was replicated in a new knowledge domain, since novel story stimuli were used. These results also showed that the novel source story stimuli used in the present study can be sufficiently encoded by the undergraduate sample to enable later retrieval.

For the letter-string analogy condition, the overall effect was significant, $\chi^2 (6, N = 60) = 16.341$, $p < .05$, suggesting an association between participants' schema condition and source story retrieval rates. The same three hypotheses were investigated as in the story analogy condition. Participants in the surface schema condition retrieved the surface schema story (13/19 times, 68.4%) more than those not in the surface schema condition retrieved it (16/41 times, 39%), $\chi^2 (1, N = 60) = 4.493$, $p < .05$, $\eta^2 = .274$. However, there were no significant differences between the retrieval of the first-order schema story for those in the first-order schema condition

(3/21 times, 14.3%) and those not in the first-order schema condition (14/39 times, 35.9%), $\chi^2 (1, N = 60) = 3.14, p = .076, \eta^2 = .229$. Similarly, there were no significant differences between the retrieval of the higher-order schema story for those in the higher-order schema condition (4/20 times, 20%) and those not in the higher-order schema condition (4/40 times, 10%), $\chi^2 (1, N = 60) = 1.154, p = .283, \eta^2 = .139$. These results suggest that while people seem to be sensitive to *surface* similarity in long-distance analogical retrieval, analogue comparison may not be sufficient to induce a *relational* schema and facilitate retrieval. These results are consistent with the distinction between surface and relational similarity usually found in analogical retrieval (e.g., Gentner et al., 1993).

The hypothesis suggesting that the higher-order relational letter-string comparison cues will facilitate the retrieval of the higher-order relational story was thus not supported. However, the letter-string comparison that highlighted mere surface features facilitated a retrieval of the correct story, which provides preliminary evidence for long-distance analogical retrieval. However, it is possible that the materials did not sufficiently induce the appropriate schema for the first-order and higher-order relational conditions.

To conclude, there were two critical findings in Experiment 1: converging evidence for the late analogical abstraction effect (Gentner et al., 2009) and preliminary evidence for long-distance analogical retrieval. The main unexpected result was the lack of long-distance analogical retrieval effects in the higher-order relational schema condition. It is possible that this unexpected result is a result of low power and issues with the letter-string comparison materials. Given that this was the first attempt to demonstrate retrieval of by letter-string sources of narrative targets, they may need some refinement. Experiment 2 addressed these issues.

Table 3

Frequency of Story Retrievals by Schema Condition and Analogy Condition for Experiment 1

Story retrieval	Schema Condition			Total
	Surface	First-order	Higher-order	
Story analogy condition				
Surface	13	2	3	18
First-order	3	13	2	18
Higher-order	1	4	9	14
Null	4	2	8	14
Total	21	21	22	64
Letter-string analogy condition				
Surface	13	13	3	29
First-order	3	3	11	17
Higher-order	2	2	4	8
Null	1	3	2	6
Total	19	21	20	60

3. Experiment 2

The results of Experiment 1 found clear long-distance retrieval effects only for *surface features* in the letter-string analogy condition. However, the first time a new task is used it may not be optimized, so in Experiment 2 we focused on improving the letter-string condition of Experiment 1, and dropped the story condition because we had successfully replicated the late analogical abstraction effect. We did this by collecting data from a larger sample and we modified the letter-string comparison pages of the first-order and higher-order relational schema conditions to better scaffold the comparison process. This should elicit more robust schemas for retrieval. In the first-order relational comparison, we made changes to the target analogues and the explicit principle. These changes are discussed in more detail below.

Participants were randomly allocated to one of four schema conditions: surface schema, first-order relational schema, original higher-order relational schema, and modified higher-order relational schema. The main hypothesis was that participants who compared higher-order relational letter-string analogues will be more likely to retrieve the higher-order relational source story than those that compared different letter-string analogues.

3.1. Method

3.1.1. Participants. Two hundred and forty-one first-year undergraduates (172 female) from the University of Sydney were recruited using the SONA system with informed consent, and were given course credit for their participation. The age range was 17-68, with a mean age of 20.05 ($SD = 5.408$). One participant did not complete the first filler task, and another did not complete the analogy example page (both due to computer error), so their data was excluded from the analysis of retrieval rates.

3.1.2. Materials. Examination of the results of Experiment 1 in Table 3 suggested that the letter-strings for the first-order schema condition were the most discrepant from predicted. Rather than leading to retrieval of the first-order target, it appeared to most often lead to retrieval of the surface target. This suggested that it may not have invoked the first-order schema strongly enough. As such, we replaced the letter-string materials shown in Figure 3, Panel B, with those in Figure 5. As shown in Figure 5, the modified first-order comparison was now identical to the original higher-order comparison, except that the last symbol group was changed into a set of successor symbols (same symbol as in Figure 3), instead of changing into an increased number of symbols (i.e., the key to the higher-order schema). Otherwise, the letter-string materials were the same as in Experiment 1.

Further, the explicit principle for the first-order condition was changed to be a more general description of the principle. As shown in Table 4, the explicit principle was changed to *succession*. We also created a *modified* higher-order relational comparison that changed its explicit principle to clearly state that *order corresponds to quantity*. The modified principle is also shown in Table 4.

Figure 5. Modified first-order relational letter-string comparison, replacing Figure 3, Panel B.

Table 4

Modified Target Principle Explanations used in Experiment 2

Schema condition	Explanation text
First-order relation	Both pairs rely on the same rule: succession. For example, triangle changes to square because of the number of sides, and G to H because of alphabetic order.
Modified Higher-order relation	Both pairs rely on the same rule: order corresponds to quantity. For example, E (fifth in the alphabet) changing to F (sixth in the alphabet) = five symbols (letters or shapes) changing to six symbols.

3.1.3. Procedure. The procedure was identical to Experiment 1, except that participants were now allocated to one of four analogue comparison conditions.

3.2. Results and Discussion

As in Experiment 1, participants demonstrated a sufficient understanding of the experimental materials when responding to the questions on the example pages. Table 5 shows the cross-tabulation of retrieval by schema condition. As in Experiment 1, all retrieved stories were classified into one of four levels: null retrieval, surface story, first-order story, and higher-order story. Schema condition now had four levels: surface schema, first-order schema condition, original higher-order schema condition, and modified higher-order schema condition.

As Table 5 shows, the overall effect of retrieval by schema condition was significant, $\chi^2(9, N = 239) = 59.497, p < .001$, suggesting an association between people's schema condition and story retrieval rates. To test the specific hypotheses, the conditions were collapsed into 2 x 2 tables for each hypothesis, as in Experiment 1. Participants in the surface schema condition retrieved the surface schema story (51/59 times, 86.4%) significantly more than those not in it retrieved it (64/180, 35.6%), $\chi^2(1, N = 239) = 46.088, p < .001, \eta^2 = .439$. Those in the first-order schema condition retrieved the first-order schema story (19/61 times, 31.1%) more than those not in the first-order schema condition retrieved it (35/178 times, 19.7%), but this difference was not significant, $\chi^2(1, N = 239) = 3.426, p = .064, \eta^2 = .120$. Those in the original higher-order schema condition retrieved the higher-order schema story (20/60 times, 33.3%) more than those not in it retrieved it (35/179 times, 19.6%), $\chi^2(1, N = 239) = 4.817, p < .05, \eta^2 = .142$. Those in the modified higher-order schema condition retrieved the higher-order schema story (37.3%) significantly more than those not in it (18.3%), $\chi^2(1, N = 239) = 9.011, p < .01, \eta^2$

= .194. There were no significant differences in the retrieval of the higher-order schema story between the original and modified higher-order schema conditions, $\chi^2(1, N = 119) = 0.204, p = .652, \eta^2 = .041$. These results replicate the previous experiment's findings of long-distance analogical retrieval of a surface schema but extend it by also demonstrating long-distance analogical retrieval of a higher-order relation.

The main hypothesis was thus mostly supported by these results: most critically, analogical comparison of higher-order relational letter-string analogies facilitated the correct retrieval of a source story that shared only this higher-order relational similarity. However, the first-order schema condition effect was more equivocal, not passing the significance threshold of .05; as in Experiment 1. The effect of the first-order condition was numerically in the same direction and with a similar effect size to the statistically significant effect in the higher-order condition. As such, we cannot draw clear inferences from the first-order condition.

To conclude, the critical finding of Experiment 2 was that there was clear evidence for long-distance analogical retrieval of higher-order relations; participants correctly retrieved a story narrative that only matched the letter-string analogy cue by an underlying higher-order relational structure. This retrieval across task-types is a novel finding, showing how long-distance analogical retrieval can be supported via comparison.

Table 5

Frequency of Story Retrievals by Schema Condition for Experiment 2

Story retrieval	Schema Condition				Total
	Surface	First-order	Original higher-order ⁴	Modified higher-order	
Surface	51	28	16	20	115
First-order	5	19	19	11	54
Higher-order	1	12	20	22	55
Null	2	2	5	6	15
Total	59	61	60	59	239

⁴ The pattern of retrieval frequencies for the original higher-order schema condition appears different from the pattern of retrieval frequencies in Experiment 1. It was unclear why this should be the case, since both manipulations are identical. As such, a chi-square test was run on the cross-tabulation of retrieval by the original higher-order schema condition, comparing across the two Experiments. The differences were not statistically significant, $\chi^2(3, N = 80) = 3.974, p = .264$. It is therefore likely that the pattern found in Experiment 1 was a chance result due to a small sample.

However, aspects of our design may have made this retrieval easier than comparable results from this literature. In fact, it is possible that these apparent *retrieval* effects, might actually be *mapping* effects, if participants were able to hold all stories in short-term memory until the retrieval phase. The experimental design included three source stories, to match the three comparison cues. This is a relatively low number of source stories compared to most retrieval studies. For instance, Gentner et al. (Experiment 4; 2009) used seven source stories, while Gentner et al. (1993) used thirty-two. Further, the delay between source story encoding and the retrieval phase in the present was relatively short. Participants had a minimum delay of two minutes due to the filler task, with an additional few minutes for completing the analogy examples page and the analogue comparison. This varied for each participant. In the late analogical abstraction effect, this delay was 30 minutes (Gentner et al., 2009), while in Gentner et al. (1993) it was six to eight days. Thus, our long-distance analogical effects were shown with fewer competitors for retrieval, and a shorter delay between encoding and test. This combination of a relatively small number of source stories and short delay might mean that participants were mapping the source stories, or at least some part of them, to the target comparison, as opposed to retrieving them. That is, participants could have considered each source story as a potential match to their comparison cue and then actively decided on the best-perceived mapping. In Experiment 3, we attempted to replicate the effects of Experiment 2 without these methodological limitations by increasing the number of source stories and increasing the delay to match Gentner et al. (2009), since their research most closely inspired ours.

4. Experiment 3

Experiment 2 found evidence for cued long-distance retrieval of higher-order relations. However, in both Experiments 1 and 2, the number of source stories was small, and there was a short delay between the source and target stimuli. This means that participants could have chosen a source story at the retrieval stage by considering each source story individually, as opposed to being reminded of a specific source story by the cue. Therefore, in Experiment 3 we included more source stories and increased the length of the filled delay. Specifically, we matched the methodology that Gentner et al. (2009) used, with seven source stories, and a filled delay of 30 minutes such that our long-distance analogical retrieval effects could be compared to their effects

across more similar domains.⁵ We only used the modified, not the original, higher-order schema from Experiment 2, so there were only three schema conditions (i.e., surface, first-order, higher-order).

4.1. Method

4.1.1. Participants. A hundred and twenty-one adults (41 female) were recruited from Amazon's Mechanical Turk with informed consent and were paid \$6 for their participation. The age range was 18-61, with a mean age of 32 ($SD = 8.615$). Participants were randomly allocated to one of three schema conditions: *surface schema* ($N = 40$), *first-order relational schema* ($N = 46$), and *higher-order relational schema* ($N = 35$).

4.1.2. Materials. Four new source stories were written about relatively arbitrary topics and are shown in Table 6. The stories' structure and length were controlled as in the previous experiments. The letter-string cues were the same as in Experiment 2, except that only the modified higher-order schema materials were used for the higher-order schema condition, and the modified first-order schema materials were used for the first-order schema condition.

4.1.3. Procedure. The procedure was identical to the previous experiments, except that participants read seven source stories (the original three plus four new ones) and completed 30 minutes of unrelated insight problems as a filler task.

⁵ We of course recognize that a 30 minute delay is not the most impressive retrieval from long-term memory, but it is certainly long enough that participants are not still actively considering the content of the encoding narratives during the delay task.

Table 6

New Source Stories used in Experiment 3

Story name	Story text
Toy Truck	Andrew was playing in the kindergarten playground when Tom took his truck toy out of his hand. He pushed Tom back onto the sandpit in anger. However, the force made the toy truck fly out of Tom's hand and onto the pavement behind the sandpit, causing it to break into many smaller pieces, and both boys to cry. As such, neither of the boys were able to play with the toy truck because it was so badly broken from the fall. After pushing Tom when he stole his toy truck, Andrew pushed him back for taking his toy, but this ended up making the situation worse for both boys because of the toy truck breaking.
Recycling	Emma reminds her boyfriend to recycle daily, so they can have a more positive impact on the environment. He begrudgingly does the recycling when she tells him. However, after coming back from a holiday, she notices that he did not do any recycling when she was not there because he did not seem to understand why he should be recycling. As such, she explained to him that recycling reduces the total amount of waste that goes to landfill, helping him understand its importance. When Emma forces her boyfriend to recycle, he does it when she is around, but because he did not really understand why it is important, he did not do it independently.
Marine Biologist	Rachael wants to get a job as a marine biologist, but cannot find any in her home town. She decides to move close to the Great Barrier Reef. However, while this would guarantee her a job, the move would mean being far away from her family and friends, and entering a situation that she is not used to. As such, she finds the courage to move to the new location in the Great Barrier Reef and getting the marine biology job that she always wanted. Living in a familiar environment held Rachael back from being in a job that she wanted to be in, but she took a risk by moving away from her home town and ended up getting the job.
Baking a Cake	Samuel wants to prove that he is independent, so decides to bake a cake without his mum's help. His mum gives him some advice about how to bake anyway. However, doesn't listen because he is confident that he can figure out the correct process by himself, but ends up putting too much flour in the mix and over-baking the cake. As such, the cake ends up very dry, and Samuel throws it away to the garbage because no one wanted to eat it. Samuel wants to be independent, but became overconfident in his ability to bake and ends up ruining the cake, despite having him mum there as a source of helpful advice.

4.2. Results and Discussion

All analyses were run in Matlab R2017a (The MathWorks Inc.) and IBM SPSS Version 24. Table 7 shows the cross-tabulation of retrieval by schema condition. The retrieval variable had four levels: surface story, first-order story, higher-order story, or other retrieval (null or of one of the four new stories). Schema condition had three levels: surface schema, first-order schema condition, and higher-order schema condition. The increased delayed and number of source stories greatly increased the number of “other” retrievals compared to the previous two experiments. As a result, the overall effect was not significant, $\chi^2(6, N = 121) = 10.811, p = .094$, suggesting a lack of a general association between people’s schema condition and story retrieval rates. To investigate the specific hypotheses, the conditions were collapsed into 2 x 2 tables for each hypothesis as in the previous experiments.

Participants in the surface schema condition retrieved the surface schema story (13/40 times, 32.5%) at a slightly higher rate than those not in this condition retrieved it (23/81 times, 28.4%), but this difference was not significant, $\chi^2(1, N = 121) = 0.216, p = .642, \eta^2 = .042$. Those in the first-order schema condition retrieved the first-order schema story (5/46 times, 10.9%) at a rate that was not significantly different to the rate that those not in the first-order schema condition retrieved it (8/75 times, 10.7%), $\chi^2(1, N = 121) = 0.001, p = .972, \eta^2 = .003$. Critically though, those in the higher-order schema condition retrieved the higher-order schema story (7/35 times, 20.0%) at a significantly higher rate than those not in the higher-order schema condition retrieved it (4/86 times, 4.7%), $\chi^2(1, N = 121) = 7.091, p < .01, \eta^2 = .242$. These results are shown in Table 7 (see the Appendix for the retrieval frequencies for each source story per condition).

These results replicate the previous experiments’ findings of long-distance analogical retrieval of a higher-order relation, even though retrieval was expected to be much more difficult in Experiment 3. However, the lack of effect for the surface and first-order schema conditions was unexpected. This could be due to unforeseen aspects of the distractor and target stories that made them more memorable in general. For example, the first distractor story (Toy Truck) was retrieved a lot more (27 retrievals) than the other distractors ($M = 3.3$), and the surface story (Flyer Typo) was the most-retrieved story, regardless of schema condition. It is also possible that these effects were not incidental to the semantics of the stories but reflected important psychological differences. For example, schemas rooted in higher-order relations could become

stronger memory probes than first-order relations with a delay because they have more systematic matches with source stories. This is akin to how shared higher-order relations are better drivers of analogical inference (Markman, 1997). Determining whether the analogical retrieval of only higher-order relational matches after a delay (and not superficial or first-order matches) was reflective of a deeper psychological principle or is incidental to the particulars of our materials is a topic for future research.

Table 7

Frequency of Source Story Retrievals by Schema Condition for Experiment 3

Story retrieval	Schema Condition			Total
	Surface	First-order	Higher-order	
Surface	13	14	9	36
First-order	3	5	5	13
Higher-order	0	4	7	11
Other	24	23	14	61
Total	40	46	35	121

5. General Discussion

Successful cross-domain analogical retrieval can be the basis of famous scientific breakthroughs but has been hard to demonstrate experimentally. Despite this, the results of the present study provide evidence that schema induction can facilitate long-distance analogical retrieval (i.e., analogical retrieval across task-types) rooted in shared higher-order relations. The rate of such retrieval was not high, but this is what should be expected given how hard such a process is in general. Experiment 1 had two key findings. First, it replicated the late analogical abstraction effect (Gentner et al., 2009) with novel story stimuli. Second, it found preliminary evidence of long-distance retrieval. Experiment 2 investigated this effect further with a larger sample size, finding that correct source story retrieval was also possible when the letter-string comparison induced a higher-order relational schema. Experiment 3 then extended the key finding of Experiment 2, that analogical comparison supported long-distance analogical retrieval based on higher-order relational matches. Importantly, we made retrieval more difficult by adding more competitor source stories and increasing the delay between encoding and test. This extension was critical because higher-order relational retrievals are of the most theoretical importance. That is, higher-order matches most closely approximate the real-world phenomena

(such as Dyson's bagless vacuum cleaner design-via-analogy to the industrial cyclones used to collect sawdust from the sawmill) that research on analogical retrieval is attempting to illuminate.

5.1. Implications

Previous research has shown that analogical retrieval is facilitated by a comparison of analogues at the retrieval stage (Gentner et al., 2004; Gentner et al., 2009; Kurtz & Loewenstein, 2007). These results support a schema induction account of retrieval: when surface-dissimilar analogues are compared, then their underlying relational structure become aligned and highlighted. However, the main limitation in this literature was that the disparity in analogue domains has been quite limited, with retrieval studies only investigating retrieval within task-types (primarily between different narrative domains). As such, it was unclear whether analogue comparison can facilitate long-distance analogical retrieval (i.e., between written narratives and abstract symbolic problems) as well. This investigation was especially important considering how important this retrieval is for real-life domains. For instance, scientists, engineers, and designers regularly need to make connections between abstract symbolic representations of the underlying principles of their domain, and more concrete semantically rich situations. Such analogies could be criticised as a focus of study because they are rare for most people, so it makes sense that most studies of analogy have used narrative-to-narrative analogies. However, theories of analogy (such as Structure Mapping Theory; Gentner, 1983; or LISA; Hummel, 2003) have no problems being applied to narrative-to-symbols analogies. Letter-string analogies are encountered by very few people and are completely artificial, but if artificiality was a barrier to analogy formation, then theories of analogy would need to be modified to explain why. Our results show that people can form narrative-to-symbols analogies in the way the theories predicts, and thus that the theories can encompass such far-transfer without needing modification. Furthermore, very few studies have investigated higher-order relations other than causation, so investigating other types of higher-order relations was warranted. We addressed these limitations by using letter-string analogies.

The present study extends the existing research by showing that late analogue comparison can facilitate analogical retrieval by using letter-string analogies as cues for source story retrieval with which they share very few surface features. This is a retrieval between tasks that are far more different than the usual narrative-to-narrative retrieval paradigm. Apart for the presence of alphabetic characters, the letter-string analogies share no surface features with the source stories.

Despite this, participants were able to retrieve the relevant source story by virtue of matching underlying higher-order relations, significantly more than those cued by other letter-string analogies. Current theories of analogy, such as SMT and LISA (Gentner, 1983; Hummel, 2003), rest on the assumption that knowledge is constructed as structured representations. The long-distance analogical retrieval effect found in the present study extends this literature further, suggesting that even stimuli as initially meaningless as higher-order letter-string analogies can be abstracted into a meaningful structured representation. As mentioned in the Introduction, it appears that in most studies of retrieval, transfer is only considered across stimuli that differ in narrative content alone rather than across the type of processing required as well. As such, given a sufficient schema induction, it seems that schemas can be extended to domains completely devoid of semantics.

Gentner et al.'s (2009) demonstration of late analogical retrieval through comparing analogues at retrieval time has important implications for the practical use of analogies. Previous demonstrations of comparison as a tool for increasing analogical transfer used comparisons of source analogues made at the time of encoding (e.g., Gick & Holyoak, 1983; Catrambone & Holyoak, 1989). These were effective but to help a person's retrieval the person would have to know ahead of time what analogues might be needed in the future. However, late comparison implies that people could improve their analogical retrieval by making more analogical mappings at the time they need to make retrievals. Such a technique implies there is an effective way to increase analogical problem solving. Our finding that such late comparisons can be from very different domains to the potential source implies even greater scope for this method.

5.2. Limitations & Conclusion

While retrieval based on higher-order matches is most critical to demonstrate, it is worth noting that none of the three experiments found a significant effect for matches based on first-order relations. However, as discussed above, it goes beyond the scope of the current research to determine whether this is due to first-order relations being an insufficient basis for a strong schema, or whether this was incidental to the specific semantic content of the target and distractor stories.

A further limitation of the present studies is that the explicit principle provided underneath the letter-string comparison cues is more semantically coherent than the cues themselves. As such, it is possible that the explicit principle played a role in cuing the source

stories. In general, it seems that explicit principles are not sufficient to induce a schema, but they do seem to facilitate induction. Gick and Holyoak (1983) found that using an explicit principle did not facilitate analogical problem solving independently of analogical comparison. However, the addition of the explicit principle to analogical comparison led to greater transfer rates than for comparison alone. Similarly, Gentner et al. (2004) found that simply presenting an explication of a schema is not sufficient to facilitate analogical retrieval in the same way that comparison does. Further, Goldwater and Gentner (2015) also found that both clear explications of causal structure and comparison of exemplar phenomena that share causal structure enhance participants' sensitivity to causal structure in novel exemplars more than either explications or comparison alone. Across the current and previous findings, a clear pattern emerges that comparing exemplars with each other and with abstract principles work in concert to support long-distance retrieval and transfer. However, it is still unclear how comparing principles and exemplars shapes schema-representation, alters the representations of the exemplars themselves, and how these representational changes affect transfer and retrieval.

A limitation of applying the effects found in our experiment to real-life situations is that people rarely have access to an analogue that can be readily compared with a target problem. It is likely that using comparison as a technique for retrieval would be useful in educational settings. However, in situations in which the correct principle to retrieve is not clear, such as when a novel problem requires a solution, a comparable analogue may not be clear. Previous research found that inventing analogous problems or idealised models facilitates retrieval and problem solving (Minervino, Olguín, & Trench, 2017; Trench, Tavernini, & Goldstone, 2017). As such, future research can test whether these effects replicate in higher-order long-distance retrieval.

To conclude, we demonstrated long-distance retrieval of higher-order relational structures. This finding was replicated with a longer delay and a greater number of source stories. Analogical retrieval, especially through schema induction, may not be as limited as the literature suggests; people with little expertise are capable of leaping across a wider mental canyon than previously demonstrated.

Open Practices Statement

The relevant data, materials, and program code for all experiments are available at <https://osf.io/fmrta/>, and none of the experiments were preregistered.

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Appendix*Frequency of all Source Story Retrievals by Schema Condition for Experiment 3*

Schema Condition	Source Story								Total
	Null	Toy Truck	Flyer Typo	Recycl ing	Next in Line	Marine Biolog ist	Centre Staff	Baking a Cake	
Surface	9	11	13	2	3	1	0	1	40
First- order	8	11	14	2	5	1	4	1	46
Higher- order	7	5	9	0	5	0	7	2	35
Total	24	27	36	4	13	2	11	4	121