Bayesianism and Wishful Thinking are Compatible

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ABSTRACT

Bayesian principles show up across many domains of human cognition, but wishful thinking—where beliefs are updated in the direction of desired outcomes rather than what the evidence implies—seems to threaten the universality of Bayesian approaches to the mind. In this paper, we show that Bayesian optimality and wishful thinking are, despite first appearances, compatible. The setting of opposing goals can cause two groups of people with identical prior beliefs to reach opposite conclusions about the same evidence through fully Bayesian calculations. We show that this is possible because, when people set goals, they receive privileged information in the form of affective experiences, and this information systematically supports goal-consistent conclusions. We ground this idea in a formal, Bayesian model in which affective prediction errors drive wishful thinking. We obtain empirical support for our model across five studies.

Among the most influential ideas in the cognitive sciences is that the mind emerges from approximations to Bayesian calculations[1–4]. This idea, commonly called the "Bayesian brain hypothesis," has been applied successfully to a wide range of psychological processes, from low-level perception to categorization, reasoning, and emotion[5–9]. However, some phenomena appear deeply incompatible with Bayesian principles, raising the question of how fundamental these principles really are to mental processing.

Arguably the greatest threat to the Bayesian brain hypothesis is the phenomenon of *belief polarization*[10–12], which occurs when two people update their beliefs in opposing directions on the basis of the same information[13]. For example, someone may conclude that their preferred political candidate has a greater chance of winning after seeing the latest polls, whereas a detractor may conclude from the same data that the candidate's odds have declined[14]. Because Bayesian norms demand that beliefs be updated in the direction of the evidence, belief polarization seemingly cannot result from Bayesian inference. Instead, belief polarization appears to emerge from "wishful thinking," defined as belief updating driven by the directional motivation to reach a particular conclusion (as opposed to the non-directional motivation to be accurate)[15–17]. On this account, when people who encounter identical data shift their beliefs in opposing directions, they do so because they want to reach opposing conclusions, not because of any rational Bayesian calculus.

Defenders of the Bayesian brain hypothesis are acutely aware of the existential threat posed by belief polarization. In response, they have sought to establish that belief polarization is not driven by wishful thinking, and that sufficiently sophisticated Bayesian models can explain it without appealing to directional motivation. They have taken an *eliminative* stance toward wishful thinking, pitting it against Bayesian accounts of belief polarization in a zero-sum game in which the latter must emerge victorious for the Bayesian brain hypothesis to survive.

The eliminative approach has had some success. Bayesian models have proven capable of reproducing many examples of belief polarization in the absence of wishful thinking. But as we will see, the eliminative approach cannot save Bayesianism on its own.

To counter claims that wishful thinking drives belief polarization, eliminativists have appealed to the fact that belief polarization can be Bayes-rational, and not directionally motivated, when it occurs among people with different priors that bear on the meaning of the evidence[11,18–21]. Consider again the supporter and detractor whose beliefs about a candidate's prospects diverged on the basis of the same poll. These partisans were not randomly assigned to their positions. Before encountering the poll they likely had different priors. The detractor may have believed that polls systematically overestimate support for the candidate in question, and the supporter may have believed the opposite. In this scenario, a neutral polling result would give the detractor reason for pessimism about the candidate's prospects, and the supporter reason for optimism. Thus, Bayesian inference could lead the supporter and detractor to draw opposing conclusions from the same poll without directional motivation. This principle is applicable to the majority of demonstrations of belief polarization, especially within the motivated reasoning literature, where the typical study design involves presenting evidence to groups of people with different priors[11,13,15].

The problem for the eliminativist project is that group differences in priors, though perhaps a common source of belief polarization, is not the only source. Belief polarization occurs even when priors are held constant. In one set of studies, participants were randomly assigned to prosecute or defend a person in a mock trial in exchange for money[22]. Participants did not advocate, nor did they agree to advocate—they merely learned that they could earn money by doing so. Despite receiving identical evidence, participants assigned to defend came to regard the defendant's guilt as less likely, and participants assigned to prosecute came to regard the defendant's guilt as more likely.

This result does not depend on people possessing different priors—priors were held constant through random assignment. Rather, this is a form of belief polarization driven by the motivation to defend opposing positions—a clear case of *wishful belief polarization*, distinct from belief polarization that is attributable to group differences in prior beliefs.

All existing models of wishful belief polarization either omit Bayesian mechanisms entirely[12,13,15,23] or supplement Bayesian mechanisms with non-Bayesian processes[16,17,24]. Wishful belief polarization therefore represents a huge and unignorable challenge to claims that the mind is fully pervaded by Bayesian reasoning. We take up this challenge in the current paper.

Our approach is a novel one. Unlike previous, eliminativist defenses of Bayesianism, our solution is compatibilist: Rather than pitting Bayesian inference and wishful thinking against each other, we contend that the two can coexist. We develop and validate a fully Bayesian model of wishful belief polarization—one in which Bayesian calculations implement, rather than replace, a causal path from directional motivation to diverging beliefs. In so doing, we reconcile one of the most influential accounts of human mental function with one of its greatest empirical threats.

Reconciling Bayesianism with Wishful Belief Polarization

In the aforementioned study, in which goal attainment required defending one of two randomly assigned positions, the experimenters provided all participants with identical information about the trial. But maybe the information provided by experimenters was not the only information participants used. Maybe participants had access to some less obvious, "hidden" information.

Past research indicates that the brain takes a rather ecumenical approach to what counts as information. From the brain's perspective, information is not limited to the sort of things that would be presented in a court of law—DNA from a crime scene, say—but includes internal evidence, such as emotions, mood, and other features of affective processing. Affect is frequently used to build out a full model of a situation, and is used to shape judgments, decisions, and other cognitive processes[9,25–28]. If I feel nostalgia while recalling an ambiguous childhood memory, I may infer this was a good experience; if I feel uneasy about a business deal, I may infer something about it must be amiss. The major point to take away from this literature is that people treat affect as a form of information. Affective information, therefore, is potentially used along with other information to perform Bayesian belief updating[9,29–32].

The novel contribution of this paper is the idea that affective information could be the key to reconciling the Bayesian brain hypothesis with wishful belief polarization. We formalize this idea in a fully Bayesian model of wishful belief polarization (see Methods), which we interrogate across five empirical studies.

From Affective Prediction Errors to Wishful Belief Polarization

How, exactly, is affect used to update beliefs? The Bayesian brain hypothesis makes specific and unique predictions. One prediction is that people should not update their beliefs based on affect per se. Instead, people should update their beliefs based on *affective prediction errors* (APEs): how much better or worse they feel relative to expectations[28,33]. The logic behind this is straightforward. When expectations are confirmed, this implies that the beliefs underlying those expectations are correct. Conversely, when expectations are violated, this suggests that the beliefs underlying those expectations are wrong. A large body of work shows that brains predict upcoming sensory events, including affective experiences, and use deviations from these predictions to update their models of the external world[4,7,9,29]. This process unfolds automatically, regardless of explicit intentions.

Now, suppose a person feels better than expected about defending a position. This positive APE would lead a Bayesian brain to update its beliefs. Our model (see Methods) formalizes this process, and predicts that the update would involve attributing greater validity to the to-be-defended position. Informally, our model implements the following line of reasoning: "The more valid a position is, the better I feel about defending it[†]; I feel better than expected about the prospect of defending this position; therefore, this position is more valid than I initially thought." Now suppose that someone feels worse than expected about defending a position—they encode a negative APE. The same Bayesian inference would lead this person to assign less validity to the to-be-defended position. Should a person feel exactly as they expected to feel about defending a position, the absence of an APE would, under our model, induce no belief updating.

To see how our model explains wishful belief polarization, imagine someone is given an incentive, such as money, to defend a position. This person would come to feel better about the prospect of defending the position; by making the act of defending a position instrumental to a goal, the valence of the act would become more positive[36–38]. But—and this is critical—the person would not merely feel better about defending the position: they would likely feel better than expected. People systematically underestimate how much their feelings will shift as a result of changing incentives due to a general tendency to overestimate the degree to which future affective states will resemble current affective states ("projection bias")[39,40]. For example, people who have just eaten tend to underestimate how much they will enjoy food when they are hungry again[41]. This suggests that people will underestimate the degree to which incentives to defend a position will increase the subjective pleasantness of doing so, resulting in positive APEs. Such APEs should, for reasons described above, increase the subjective validity of the to-be-defended position. This process could explain how the directional motivation to defend opposing positions could lead two people to update their beliefs in opposing directions despite receiving the same external evidence and holding identical priors.

For example, consider two people, one incentivized to defend, and the other incentivized to oppose, capital punishment. Both people wind up feeling better-than-predicted about the prospect of defending their respective positions. Based on these APEs, Bayesian calculations lead the pro-capital punishment individual to more strongly endorse capital punishment, and the anti-capital punishment individual to more strongly oppose capital punishment. This pattern of polarization is Bayesian and, at the same time, directionally motivated: the APEs that drove it reflect desires to defend specific positions.

To summarize, we contend that when people are incentivized to defend a position, they encode positive APEs, a kind of "hidden" information that supports the position's validity. This information is used to update beliefs in a Bayesian fashion, and can account for a phenomenon—wishful belief polarization—long touted as a threat to the Bayesian brain hypothesis. Three hypotheses follow from our model, which we describe below. We then test these hypothesis across five studies while ruling out alternative models, including models derived from cognitive dissonance theory[34] and non-Bayesian models that rely exclusively on observed affect.

Hypotheses

One hypothesis of our model is the *underestimation hypothesis*: Incentives to defend positions systematically induce positive APEs.

The second hypothesis is the *error-based updating hypothesis*: People update their beliefs in proportion to the size of the APE. This is important for distinguishing our model from non-Bayesian models. For example, a non-Bayesian model may predict that, regardless of how people expect to feel about the prospect of defending a position, when they feel good they assign greater validity to the position, and when they feel bad they assign less validity to that position.

The third hypothesis is also important for distinguishing Bayesian from non-Bayesian inference. Bayesian inference stipulates that belief updating is inversely proportional to subjective observation noise—a person's belief about the inherent randomness, or unpredictability, of the observations they use to update their beliefs[33]. Suppose that someone thinks their own affective experiences are very noisy and therefore inherently unpredictable. According to Bayes' rule, this person should not update their beliefs much in response to APEs; if affect is inherently unpredictable, then APEs may stem not from erroneous beliefs, but from random noise. Conversely, suppose that someone thinks their own affective experiences are not noisy at all. This person, according to Bayes' rule, should update their beliefs substantially in response to APEs. If affect is highly predictable, then APEs are unlikely to result from random noise—they probably result from erroneous beliefs that ought to be changed. In short, Bayesian principles dictate that the noisier a person thinks their affective experiences are, the less they should update their beliefs in response to APEs (see Methods). This is the *noise hypothesis*. Evidence for the noise hypothesis would distinguish our model from alternatives in which belief updating is error-based but non-Bayesian, such as Rescorla-Wagner[42] and other delta-rule models[28,43–45].

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†Our model assumes, uncontroversially, that people expected to feel better about defending valid positions than invalid positions[34,35].

Results

Study 1

Study 1 provides support for the underestimation hypothesis. To begin, participants were informed that, later in the experiment, they would be incentivized to defend either the plaintiff or defendant in a mock trial. Participants did not learn *which* side they would be incentivized to defend—only that, if they secured a victory, they would win cash. Participants then read one of six randomly selected court briefs based on actual cases from the United States legal system. The cases covered a diverse range of issues, from a neo-Nazi group's right to demonstrate to the constitutionality of the Affordable Care Act (for full briefs, see Supporting Information). Participants predicted how good they would feel about defending both the plaintiff's side and the defendant's side. Next, we revealed to participants which side they needed to defend to win the cash prize. Participants then reported how they actually felt about the prospect of defending their assigned position.

The only relevant change that occurred between the reporting of predicted and actual affect was the degree of incentive to defend the assigned positions. This feature of the study design is crucial; it ensures that any APEs reflect a change in motivation to defend the assigned position, and are therefore are capable, in principle, of driving directionally motivated belief change.

The underestimation hypothesis says that regardless of which position they are incentivized to defend, participants should feel better about doing so than predicted. This is what we found (Figure 1). We regressed affect on three variables: a binary variable denoting whether the affect was predicted or observed, legal case, and assigned position. For every combination of legal case and assigned position, observed affect was significantly more positive than predicted affect. This finding represents the first piece of our Bayesian account of wishful belief polarization, which says that the affective prediction errors observed here are used, in a Bayesian manner, to update beliefs.

Study 2

Study 2 serves as an initial test of the error-based updating and noise hypotheses. Participants read a vignette about a fictional woman, Harriet, described as a corporate executive on trial for financial fraud and embezzlement. After reading the vignette, participants learned that, in the next few minutes, they would be assigned to one of two roles in Harriet's trial: prosecuting attorney (which entails defending the position that Harriet is guilty) or defense attorney (which entails defending the position that Harriet is innocent). Participants also learned that if they succeeded in defending their assigned position, they would win cash. After providing this information, but before assigning participants to defend a particular position, we collected self-report measures of the subjective probability of Harriet's guilt (Pr(guilty)), predicted affect conditional on prosecuting Harriet, predicted affect conditional on defending Harriet, and subjective observation noise (i.e., the subjective noisiness of affect).

Next, all participants were assigned to the role of prosecuting attorney, introducing an incentive to defend the position that Harriet is guilty. Immediately after, we measured how participants felt about the prospect of prosecuting Harriet. As in Study 1, the introduction of the incentive was the only relevant change that occurred between the reporting of predicted and actual affect. Accordingly, any APEs reflect a change in motivation to defend the assigned position, and can therefore drive directionally motivated belief change. Consistent with the underestimation hypothesis, APEs were systematically positive ($b = .53$, $SE = .07$, $t(568) = 8.05$, $p < .001$; Figure 2A).

After measuring actual affect, we gave participants a description of what Harriet's trial would entail: during the trial, evidence would appear on their screen, and their job would be to present incriminating evidence to the jury by pressing their spacebar whenever the evidence made Harriet seem guilty. This task was never performed. The purpose of describing it was to obscure the true purpose of the study by bolstering our cover story, according to which the study was designed to examine memory for information presented during criminal trials.

After learning what Harriet's trial would involve, participants reported, for the second time, Pr(guilty). Wishful belief updating was operationalized as the change in Pr(guilty) from time 1 (before prosecuting was incentivized) to time 2 (after prosecuting was incentivized). Participants exhibited a significant amount of wishful belief updating: Harriet was judged more likely to be guilty once participants were incentivized to prosecute her ($b = .04$, $SE = .006$, $t(568) = 6.97$, $p < .001$).

We found support for the error-based updating hypothesis in the form of a significant effect of APE on wishful belief updating ($b = .02$, $SE = .004$, $t(567) = 6.29$, $p < .001$). Larger APEs were associated with greater increases in Pr(guilty). To confirm that wishful belief updating varied with the difference between predicted and observed affect, and not just one of these factors on its own, we regressed it on predicted and observed affect simultaneously (Figure 2B). Observed affect had a positive effect ($b = .03$, $SE = .004$, $t(566) = 7.92$, $p < .001$) and predicted affect

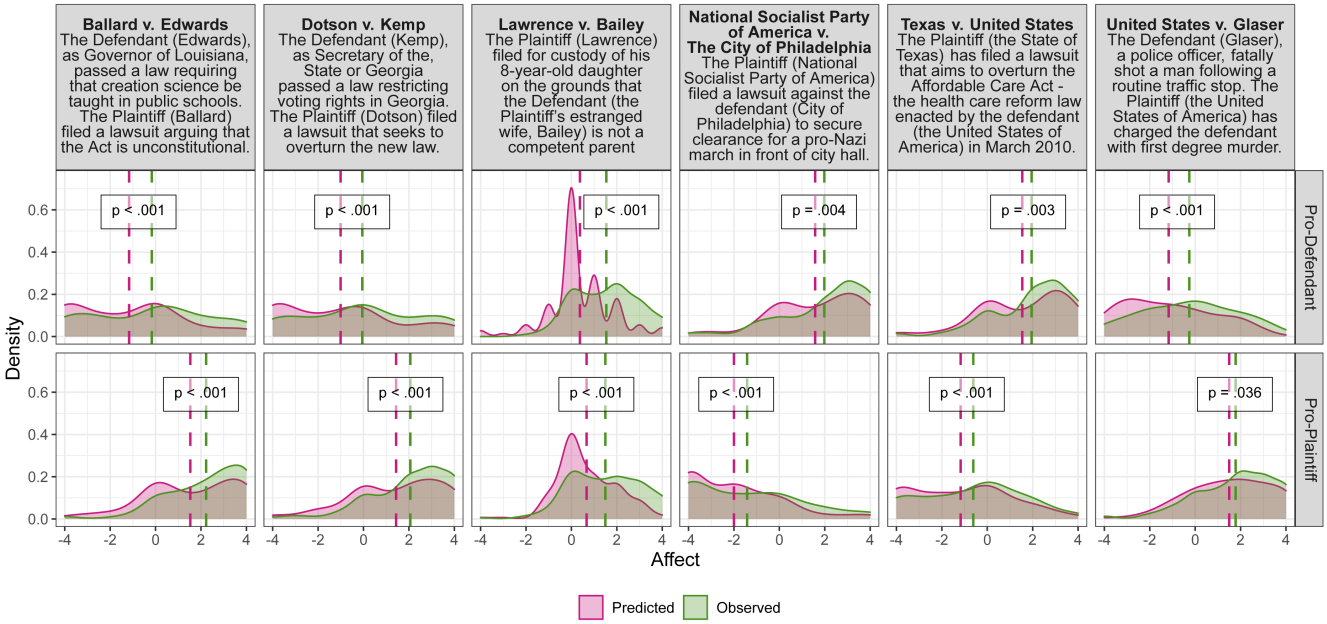

Figure 1. Predicted versus observed affect as a function of case and whether participants were incentivized to defend the plaintiff's position (Pro-Plaintiff) or the defendant's position (Pro-Defendant) in Study 1. Distributions of predicted affect are in pink, and distributions of observed affect are in green. Pink and green dashed lines represent average predicted and observed affect, respectively. P-values correspond to the contrasts between predicted and observed affect for each combination of case and position. These are real cases, but names were changed to prevent identification. The real cases, left to right, are: Edwards v. Aguillard; Sixth District of the African Methodist Episcopal Church v. Kemp; Bailey v. Sarina; National Socialist Party of America v. Village of Skokie ; California v. Texas ; United States v. Slager. Descriptions of each case are abbreviated. For full descriptions and case citations, see Supporting Information. For an equivalent plot that displays distributions of APEs, see Supplementary Figure 1.

had a negative effect ($b = -.01$, $SE = .004$, $t(566) = 3.1$, $p = .002$), suggesting that it is the difference between these variables, rather than either variable on its own, that drives wishful belief updating.

The main effect of APE was qualified by an interaction with our continuous measure of subjective observation noise ($b = .01$, $SE = .004$, $t(565) = 2.56$, $p = .01$; Figure 2C). In line with the noise hypothesis, greater levels of subjective observation noise were associated with weaker effects of APE on wishful belief updating.

An alternative account of these findings is that APEs are outcomes, rather than causes, of wishful belief updating. On this account, participants shifted their beliefs, which led them to feel better than predicted about the prospect of defending their assigned position. This account can be ruled out however, as it does not explain why the subjective noisiness of affect attenuates the relationship between APEs and wishful belief updating. Unless APEs are used as information to update beliefs, there is no reason why their subjective noisiness (which determines how diagnostic they are about the state of the world) should weaken their relationship with belief updating.

One may wonder if the results of Study 2 can be explained in terms of dissonance reduction. Cognitive dissonance theory[46] says that when someone agrees to defend a position that they believe is invalid, they experience negative affect[47], which they strive to reduce by aligning their beliefs with their behavior. In Study 2, however, negative affect was not the driver of belief change. On the contrary, participants updated their beliefs to the extent that they experienced more *positive* affect than anticipated. This result is antithetical to the predictions of dissonance theory.

It should come as no surprise that wishful belief updating was unrelated to cognitive dissonance, as dissonance is theorized to occur only when the choice to defend a subjectively invalid position is made freely[34]. Support for this idea comes from decades of research using the induced compliance paradigm, in which participants are asked to defend a position, and either explicitly informed that they can opt out of doing so (high choice), or not (low choice). Dissonance effects emerge in high choice settings, and not low choice settings. Our paradigm is a low choice setting, as participants cannot opt out of defending their assigned position without foregoing payment.

For these reasons it is doubtful that cognitive dissonance can account for our findings. Nonetheless, we provide even stronger evidence against dissonance in the next study. In Study 3, we measure perceptions of free choice, allowing us to test our claims that perceptions of free choice in our paradigm are low to non-existent, and unrelated to wishful belief updating.

Study 3

Study 3 was identical to Study 2 with two exceptions. Instead of prosecuting Harriet, participants were incentivized to advocate for the position that Harriet is innocent as her defense attorney. Also, at the end of the study, we asked participants to rate how free they were to opt out of defending their assigned position.

Consistent with the underestimation hypothesis, APEs were positive overall (b = .82, SE = .09, $t(310)$ = 8.76, $p < .001$; Figure 2D). We also observed a significant amount of wishful belief updating. After being incentivized to defend Harriet, Pr(guilty) was reduced (b = -.08, SE = .01, $t(310)$ = 6.9, $p < .001$).

We obtained support for the error-based updating hypothesis in the form of a significant effect of APE on wishful belief updating (b = .03, SE = .006, $t(309)$ = 3.78, $p < .001$); larger APEs predicted greater decreases in Pr(guilty). This effect was driven by APEs per se (Figure 2E). When predicted and observed affect were used simultaneously to predict wishful belief updating, we found a positive effect of observed affect (b = .03, SE = .007, $t(307)$ = 4.93, $p < .001$) and a negative effect of predicted affect (b = -.02, SE = .008, $t(307)$ = 2.01, p = .046).

The effect APE on wishful belief updating was qualified by a significant interaction with our continuous measure of subjective observation noise (b = .01, SE = .004, $t(565)$ = 2.56, p = .01; Figure 2F). In line with the noise hypothesis, greater levels of subjective observation noise were associated with weaker effects of APE on wishful belief updating.

Consistent with our claim that our paradigm induces low perceptions of free choice, the modal response on our free choice scale was zero (Mdn = 1, M = 1.74, SD = 1.95), corresponding to the perception of no freedom of choice. In addition, perceptions of free choice had no effect on wishful belief updating ($b = -.003$, $SE = -.006$, $t(304)$ = .57, p = .569). These results confirm that cognitive dissonance—which occurs only under conditions of high perceived choice—cannot account for wishful belief updating in our paradigm.

Studies 4 and 5

If APEs drive wishful belief polarization, then wishful belief polarization should be reduced by manipulations that attenuate APEs. We tested this hypothesis Study 4 and our pre-registered (`aspredicted.org/NL3_RJW`) Study 5.

In Study 4, participants learned that they would prosecute or defend two defendants across separate trials, and would win cash for each trial they won. To manipulate APEs, we randomly assigned participants to one of two conditions. One was the "learning condition." Here, participants read about the first defendant, then were incentivized to prosecute or defend the first defendant, then read about the second defendant, and finally were incentivized to prosecute or defend the second defendant. The same action was always incentivized at both time points (participants did not know this would be the case). This procedure provides a learning opportunity: By observing their affective response to the first incentive, participants could better predict their affective response to the second incentive. This learning opportunity was omitted in the other, "no learning condition." Here, participants read about both defendants, and then, at a single time point, were incentivized to prosecute or defend both. Therefore, in the no learning condition, affective responses to one incentive could not be used to better predict affective responses to the other incentive.

We measured predicted affect, observed affect, and change in Pr(guilty) regarding the second defendant only. Across all conditions, APEs were significantly greater than zero and predicted wishful belief updating, supporting the underestimation and error-based updating hypotheses (see Supplementary Results). Critically, APEs were attenuated in the learning condition (b = .23, SE = .08, $t(567)$ = 2.75, p = .006; Figure 3A), and this effect was unmoderated by incentivized position (prosecute vs. defend; b = .04, SE = .17, $t(566)$ = .23, p = .821). Our model therefore predicts attenuated wishful belief polarization in the learning condition.

This prediction was confirmed (Figure 3B). In the no learning condition, Pr(guilty) diverged as a function of incentivized position (b = .12, SE = .02, $t(566)$ = 7.47, $p < .001$), increasing among participants incentivized to prosecute (b = .06, SE = .01, $t(566)$ = 5.02, $p < .001$) and decreasing among participants incentivized to defend (b = -.07, SE = .01, $t(566)$ = 5.55, $p < .001$). The degree of divergence was reduced in the learning condition (b = .06, SE = .02, $t(566)$ = 2.71, p = .007), but still significant (b = .06, SE = .02, $t(566)$ = 3.96, $p < .001$), such that incentives to

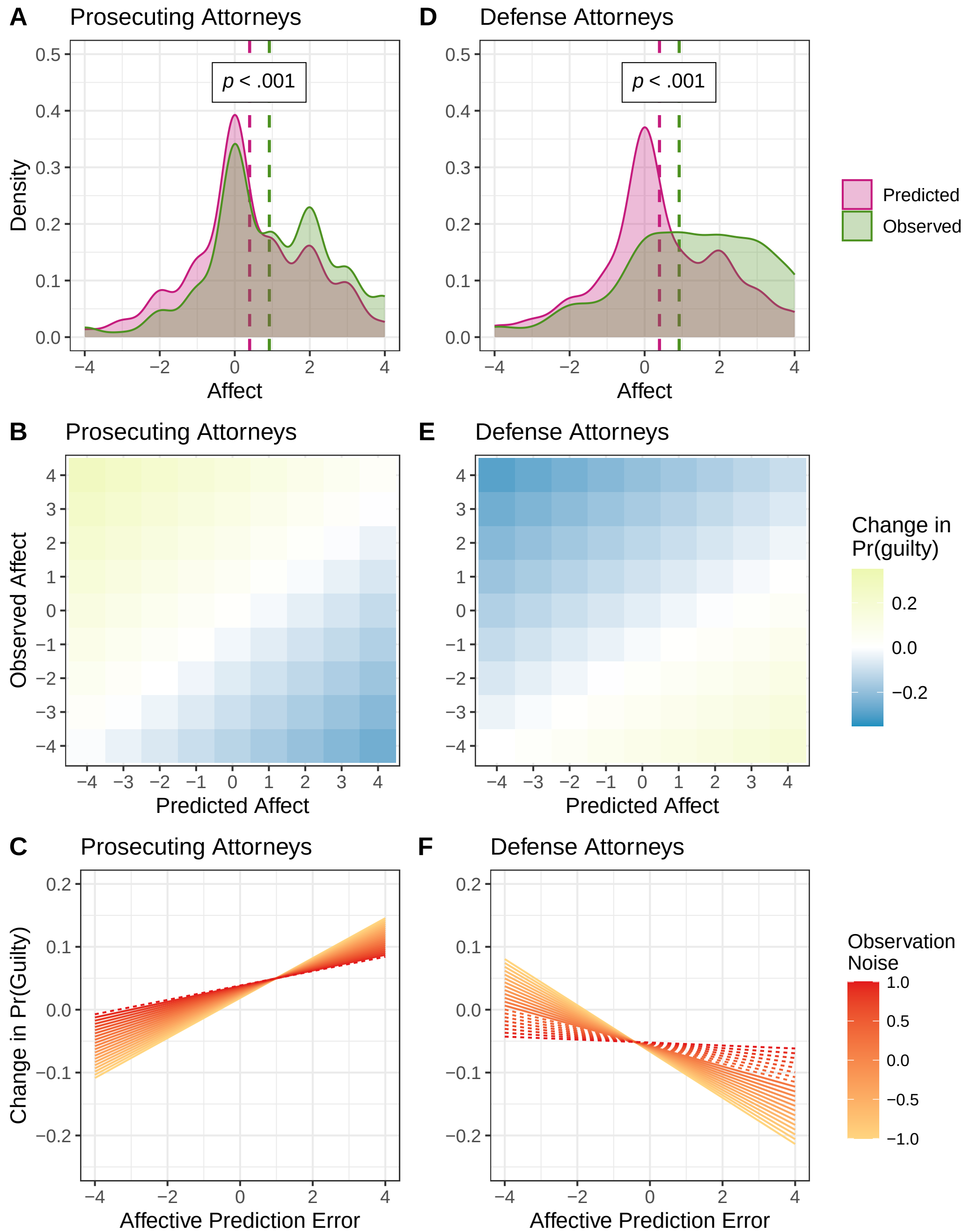

Figure 2. The results of Study 2 (**A–C**) and Study 3 (**D–F**). **A.** Predicted versus observed affect about defending the position that Harriet is guilty. For distributions of affective prediction errors, see Supplementary Figure 2A. **B.** Independent effects of predicted and observed affect on change in Pr(guilty). **C.** Change in Pr(guilty) as a function of APEs (observed minus predicted affect) and subjective observation noise among prosecuting attorneys. Observation noise is standardized such that values denote SDs from the mean. Solid lines denote slopes significantly different from zero at $p < .05$; dashed lines denote slopes non-significantly different from zero. **D.** Predicted versus actual affect about defending the position that Harriet is innocent. For distributions of affective prediction errors, see Supplementary Figure 2B. **E.** Independent effects of predicted and observed values of affect on change in Pr(guilty). **F.** Change in Pr(guilty) as a function of APEs and subjective observation noise among defense attorneys. Observation noise is standardized such that values denote SDs from the mean. Solid lines denote slopes significantly different from zero at $p < .05$; dashed lines denote slopes non-significantly different from zero.

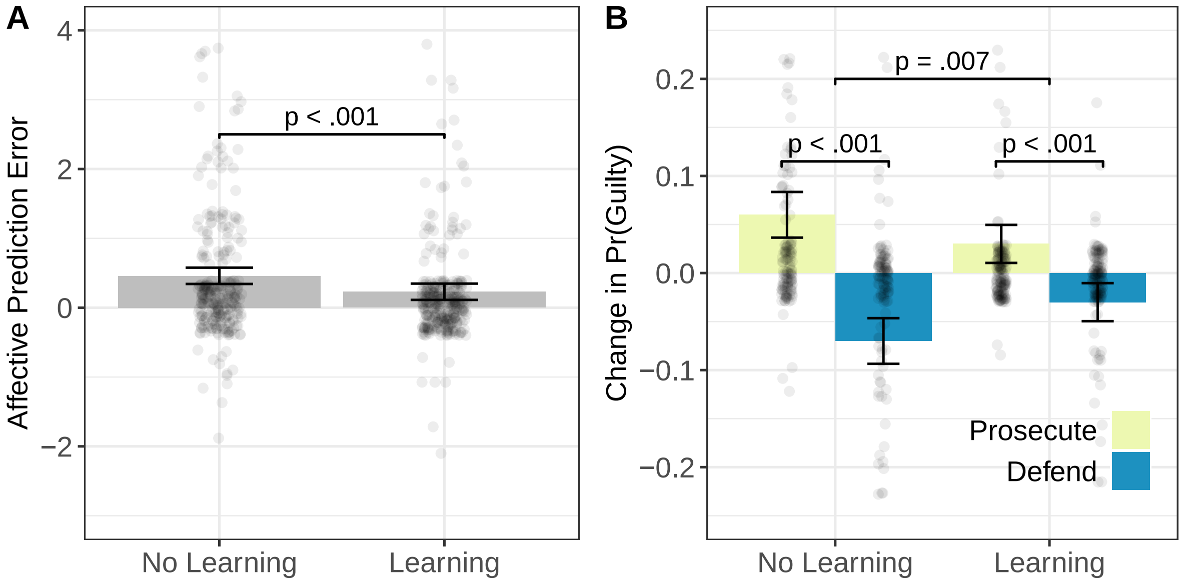

Figure 3. The results of Study 4. **A.** APEs in the Learning and No Learning conditions. Error bars denote 95% CIs. **B.** Belief updating as a function of goal-conducive action (Prosecute vs. Defend) in the Learning and No Learning conditions. Error bars denote 95% CIs.

prosecute increased Pr(guilty) ($b = .03$, $SE = .01$, $t(566) = 2.82$, $p = .005$) and incentives to defend decreased Pr(guilty) ($b = -.03$, $SE = .01$, $t(566) = 2.78$, $p = .008$). The effect of learning on wishful belief polarization was mediated by attenuated APEs (see Supplementary Results).

In Study 4, our key manipulation involved nothing more than delivering incentives simultaneously (preventing learning) or one after the other (permitting learning). By reducing APEs, this simple manipulation reduced wishful belief polarization—a result uniquely predicted by our model.

We conceptually replicated these findings in Study 5 using a novel manipulation of APEs. Here, we delivered incentives one at a time, as in the learning condition of Study 4, and measured wishful belief polarization at either time 1 or time 2. APEs should be reduced at time 2 due to learning, so wishful belief polarization should be reduced at time 2 as well. We confirmed this prediction, and found that the effect of time on wishful belief polarization was mediated by APEs (see Supplementary Methods and Results).

Discussion

On the face of it, wishful thinking seems incompatible with the Bayesian brain hypothesis. This is why defenses of Bayesianism have taken an eliminative stance toward wishful thinking, showing that many apparent instances of wishful thinking are not wishful after all[11,18–20]. This strategy has succeeded in defeating many challenges to the Bayesian brain hypothesis, with at least one major exception: the phenomenon of wishful belief polarization.

Instead of recasting wishful belief polarization in non-wishful terms, we have established that wishful belief polarization and Bayesianism are compatible. Our data support a fully Bayesian account of wishful belief polarization. In our model, when defending a position is incentivized, the motivation to obtain the incentive leads to better-than-predicted feelings about defending the position. The resulting APE is used to shift beliefs into alignment with the to-be-defended position—a process that is at once directionally motivated and Bayes-rational.

In addition to reconciling the Bayesian brain hypothesis with wishful thinking, our model advances theories of how people use their feelings as information about the external world. That people use their feelings as information is well-established[9,25–27], but accounts of the computational mechanisms underlying this process are in their infancy. Recent work suggests that feelings are used to update beliefs according to Bayesian principles[29,48,49]. Our work supports this idea by providing evidence that affect-based belief updating adheres to two predictions of Bayesian optimality: (i) it is a function not of affect per se, but of APEs, and (ii) the noisier people think their affect is, the weaker the relationship between APEs and belief updating.

Our results highlight the fact that Bayesian brains, though rational in one sense of the word, are perfectly capable of producing irrational output. This is because the rationality of a Bayesian brain is limited to the structure of its learning process: It updates its beliefs rationally given the data it encounters and its current internal model. When fed false data, or when using a flawed internal model, Bayesian logic can produce irrational outcomes[18]. Here, the irrational outcome is wishful belief polarization, and the modeling flaw that seems to produce it is an overly pessimistic belief about how good it will feel to defend a position in the service of a desired outcome.

Where does this overly pessimistic belief come from, and why does it persist in the face of disconfirming evidence? Why has a lifetime of learning not produced a more accurate internal model? One possibility is that evolution has endowed people with an extremely strong prior that affect is more autocorrelated than it really is, and therefore less responsive than it really is to situational factors like incentives to advocate. This hypothesis is supported by decades of research on the projection bias[40], and fits comfortably in a Bayesian framework. It also raises a new question: Why would evolution select for an extremely strong and spuriously high prior on the autocorrelation of affect? We suggest that such a prior is advantageous precisely because it drives wishful thinking, which often promotes adaptive behavior[21,50,51]. People are more persuasive, for instance, when they actually believe what it is they are arguing[50,51]. Bayesian brains therefore stand to gain from believing in the positions they are incentivized to defend even if those positions defy the available evidence. Future work could explore this possibility by examining whether APEs in response to incentives to advocate lead to more successful advocacy, especially among people with low subjective observation noise.

Another reason why affective predictions are so frequently wrong may be a failure to generalize appropriately. If a person feels better-than-expected about defending a position in one specific context, that person may infer that their feelings are less autocorrelated than previously thought *in that particular context*, but not in general. This would lead to more accurate affective predictions in the context where the initial prediction error was encoded (as we show in Studies 4 and 5) but not in other contexts.

There may be additional paths, besides the one uncovered here, from Bayesian inference to wishful belief polarization. One possibility that has been raised is that wishful belief polarization emerges from a strong prior that nature is "benevolent," or "rigged" in one's favor[52]. Simulation studies have shown that if an agent holds this belief and observes that they would benefit from a particular event, the agent will conclude that the event will occur[52]. Future work should look for evidence of this mechanism, along with other Bayesian mechanisms of wishful belief polarization, in human belief updating.

Perhaps the deepest question for future work is the one we started with: Is the brain Bayesian? The current investigation does not settle the issue, but it does lend support to proponents of the Bayesian approach by defending it against one of its greatest empirical challenges.

Methods

All studies were approved by the Institutional Review Board of the University of Pennsylvania. All participants gave informed consent and were compensated for their time. Data and analysis scripts are available on the OSF repository at: `https://osf.io/59dmr/?view_only=b8ea1a66b5e84d1e8d67391662b60d82`. Surveys were administered, and data collected, with Qualtrics. Using RStudio version 1.3 and R version 3.6, all between-subjects analyses were conducted using linear regression and fit using the 'stats' package, and all within-subjects and mixed analyses were conducted using linear mixed models with subject-level random intercepts and fit using the 'lmer' package. All tests of statistical significance are two-sided.

In each study, participants had a goal to prosecute or defend someone in a criminal trial, but never actually participated in a trial. Instead, all participants learned that that survey would end early and that they would receive the financial bonus they had expected to compete for during the trial. Throughout each study, participants were asked to summarize key information to promote attention and comprehension. In addition, we included several questions in each study to confirm that participants had read and understood pertinent facts about the case and their task (see Supplementary Information).

Power

Data for establishing empirical estimates of effect sizes were unavailable, so we adopted a conservative approach of assuming, in all studies, that all effect of interest would be small-to-medium in size (d = .3, f^2 = .03). Under this assumption we computed the sample size necessary to achieve 80% power ($p < .05$, two-sided). In Study 1, the effects of interest were within-subject differences between predicted and observed affect across 12 between-subjects conditions, so N = 1070 was required to achieve sufficient power. In Studies 2 and 3, the effect of interest was a two-way interaction between APEs and observation noise, so N = 262 was required to achieve sufficient power. In Study 4, the effect of interest was a between-subject effect of learning, so N = 351 was required to achieve sufficient power.

Participants

We recruited participants using Prolific for Studies 1, 2, 4, and 5, and CloudResearch for Study 3. We note that the exclusion rate (see below) was substantially higher in Study 3 relative to the other studies, which is likely due to the CloudResearch subject pool being less reliable than the Prolific subject pool, as all other relevant features of the studies were held constant. Our initial sample sizes were N = 1502 in Study 1, N = 603 in Study 2, N = 571 in Study 3, N = 625 in Study 4, and N = 451 in Study 5. Our final samples after exclusions were N = 1370 in Study 1 (59% female; Mdn_{age} = 32), N = 569 in Study 2 (47% female; Mdn_{age} = 33), N = 311 in Study 3 (46% female; Mdn_{age} = 35), N = 570 in Study 4 (49% female; Mdn_{age} = 36), and N = 375 in Study 5 (52% female; Mdn_{age} = 30).

Attrition did not vary significantly by condition in any study. For instance, in Study 1, where statistical power to detect such an effect was greatest, only 1.8% of participants who were assigned to defend a particular position failed to complete the survey, and the likelihood of abandoning the study prematurely was unrelated to (i) the predicted affect of defending the assigned position (b = .54, SE = .45, $t(1528)$ = 1.19, p = .232), and (ii) whether participants were assigned to their preferred position (b = .54, SE = .44, z = 1.22, p = .224).

Model

Let a_K be the act of defending, or otherwise acting in favor of, some position K, and let $V(a_K)$ be the subjective valence of this action—that is, how positive or negative a person feels about the prospect of defending K. We assume that people regard $V(a_K)$ as a function of N different variables including, but not limited to, the validity of K, denoted as x_1, and the goal-conduciveness of a_K, denoted as x_2. The value of x_1 is based all information about position K made available to the reasoner prior to the observation of $V(a_K)$.

Let all N variables that people use to predict $V(a_K)$ be elements of the vector $\mathbf{x} = [x_1, \ldots, x_N]$. Though $\mathbf{x}$ is predictive of $V(a_K)$, it is not perfectly so; $V(a_K)$ is inherently noisy. We assume that people represent this uncertainty. Formally, we assume that people represent $V(a_K)$ as a random draw from a normal distribution:

$$V(a_K) \sim \mathcal{N}\left(\boldsymbol{\beta}\mathbf{x}^{\mathrm{T}},\, \sigma_v^2\right) \tag{1}$$

$\boldsymbol{\beta} = [\beta_1, \ldots, \beta_N]$ is a vector of weights corresponding to beliefs about the magnitude and direction of the relationship between each element of $\mathbf{x}$ and $V(a_K)$. β_1 denotes a person's belief about the relationship between x_1 (the validity of K) and $V(a_K)$, and β_2 denotes a person's belief about the relationship between x_2 (the goal-conduciveness of a_K) and $V(a_K)$. We take β_1 and β_2 to be positive. The variance, σ_v^2, denotes a person's belief about observation noise, with greater values of σ_v^2 corresponding to greater noise.

$\mathbf{x}$ comprises variables whose values cannot be known with certainty. For instance, a person cannot know for sure how valid a position is. People can only form *beliefs* about the elements of $\mathbf{x}$—beliefs that may be wrong. Let a person's beliefs about the elements of $\mathbf{x}$ be elements of the vector $\hat{\mathbf{x}} = [\hat{x}_1, \ldots, \hat{x}_N]$. $\hat{x}_1$ is a person's belief about the validity of K (a person's belief about the value of x_1) and $\hat{x}_2$ is a person's belief about the goal-conduciveness of a_K (a person's belief about the value of x_2). We assume that people represent $\mathbf{x}$ as normally distributed around $\hat{\mathbf{x}}$:

$$\mathbf{x} \sim \mathcal{N}(\hat{\mathbf{x}}, \mathbf{Q}) \tag{2}$$

where $\mathbf{Q} = \mathrm{diag}(q_1, \ldots, q_N)$ is a covariance matrix.

Given the generative model defined by equations 1 and 2, Bayes' rule stipulates the optimal way to update $\hat{x}_1$ (the subjective validity of K) upon observing $V(a_K)$. The update involves three steps. In the first step, $\hat{\mathbf{x}}$ is used to predict the value of $V(a_K)$. This prediction, denoted a $\hat{V}(a_K)$, quantifies how a person expects to feel about the prospect of defending K. It is computed as follows:

$$\hat{V}(a_K) = \boldsymbol{\beta}\hat{\mathbf{x}}^{\mathrm{T}} \tag{3}$$

The next step involves computing the APE, δ_K, which quantifies how much better or worse a person feels about the prospect of defending K than expected:

$$\delta_K = V(a_K) - \hat{V}(a_K) \tag{4}$$

Note that δ_K is goal-dependent when it results from an overly small value of β_2 (i.e., an underestimation of the degree to which $V(a_K)$ increases with the goal-conduciveness of a_K). Finally, δ_K is used to update $\hat{x}_1$:

$$\hat{x}_1' = \hat{x}_1 + \delta_K \beta_1 \alpha \tag{5}$$

$\hat{x}_1'$ is the new belief about the validity of K, and α is a non-negative weighting factor that controls the amount of updating:

$$\alpha = \frac{q_1}{\boldsymbol{\beta} \mathbf{Q} \boldsymbol{\beta}^{\mathrm{T}} + \sigma_v} \tag{6}$$

where q_1 is the element of covariance matrix $\mathbf{Q}$ corresponding to the variance of x_1. Notice that α is a decreasing function of observation noise, σ_v^2. Accordingly, equations 5 and 6 say that as observation noise increases, the amount of updating in response to δ_K decreases, in line with the noise hypothesis. Also notice that, according to equation 5, if $\beta_1 > 0$ (i.e., if people expect $V(a_K)$ to increase with the validity of K), then the subjective validity of K should increase as a positive function of δ_K, in line with the error-based updating hypothesis.

Goal-Dependent Prediction Errors

We quantified goal-dependent prediction errors as the difference between the predicted and observed values of $V(a_K)$. We measured predictions and observations of $V(a_K)$ by asking participants how positive or negative they thought they would feel—or how positive or negative they did feel—about (i) representing the plaintiff or defendant (Study 1), or (ii) prosecuting or defending the defendant (Studies 2–4). Participants responded on a 9-point scale from 1 (extremely negative) to 9 (extremely positive), with 5 (neutral) in the middle.

Observation Noise

In Study 2, we measured affective noisiness in two steps. First, we asked participants four questions of the following form: "If you knew for sure that Harriet was [innocent / guilty], how do you think you would feel about the prospect of [defending / prosecuting] her?" Next, we asked participants to rate their confidence in each answer. Participants responded to four questions of the following form: "You predicted that if you knew for sure that Harriet was [innocent / guilty], you'd feel X about [defending / prosecuting] her. How certain are you that this is how you'd feel?" where X is the relevant affective prediction. Participants responded on 9-point scales from 0 ("Not at all certain") to 9 ("Completely certain"). For participants assigned to defend a particular position (i.e., participants assigned to prosecute or defend Harriet), observation noise was computed as the average degree of uncertainty about the subjective valence of defending that position (i) if Harriet was known to be guilty and (ii) if Harriet was known to be innocent. The logic underlying this approach is the following: If someone is highly uncertain of how they would feel about prosecuting or defending someone they know to be guilty or innocent, this uncertainty must reflect the belief that observation noise is high (i.e., the belief that the subjective valence of prosecuting or defending is noisy). Conversely, if someone is highly certain of how they would feel about prosecuting or defending someone they know they to guilty or innocent, this certainty must reflect the belief that observation noise is low (i.e., the belief that the subjective valence of prosecuting or defending is predictable).

In Study 3, we adopted a simpler, more straightforward approach to measuring the subjective noisiness of affect. After participants reported their initial affective predictions, we asked participants to rate their confidence in their predictions. We used the following item: "You predicted that you'd feel X about [defending / prosecuting] Harriet. How certain are you that this is how you'd feel?" where X is the relevant affective prediction. Participants responded on 9-point scales from 0 ("Not at all certain") to 9 ("Completely certain").

Freedom of Choice

In Study 3, we measured perceived freedom of choice using two self-report items asking participants to rate how much they agree or disagree with the following statements: "Had I wanted to, I could have declined to perform the role of [prosecuting attorney / defense attorney]" and "I could not have declined to perform the role of [prosecuting attorney / defense attorney] even if I had wanted to" (reverse coded). Participants responded on 7-point scales from 0 ("Not at all accurate") to 6 ("Completely accurate").

Data availability

The data that support the findings of this study are available at https://osf.io/59dmr/.

Code availability

The custom code that supports the findings of this study is available at https://osf.io/59dmr/.

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Author contributions statement

D.E.M. and N.S. jointly designed the studies, analyzed the data, and wrote the paper. D.E.M. is responsible for the formal model and, in an equally weighty intellectual achievement, N.S. is responsible for typesetting that model in LaTeX.

Additional information

The authors declare no conflict of interest.