From Uncertainty to Insight: An Autocatalytic Framework

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Abstract

We show how uncertainty and insight can be modeled using Reflexively Autocatalytic Foodset-generated (RAF) networks. RAF networks have been used to model the self-organization of adaptive networks associated with the origin and early evolution both biological life, and the kind of cognitive structure necessary for cultural evolution. The RAF approach is applicable in these seemingly disparate cases because it provides a theoretical framework for formally describing systems composed of elements that interact to form new elements, and for studying under what conditions these (initial + new) elements collectively become integrated wholes of various types. Here, the elements are mental representations, and the whole is a conceptual network. The initial components—referred to as *foodset items*—are mental representations that are innate, or were acquired through social learning or individual learning (of *pre-existing* information). The new elements—referred to as *foodset-derived items*—are mental representations that result from creative thought (resulting in *new* information). The demarcation into foodset versus foodset-derived elements provides a natural means of (i) grounding abstract concepts in direct experiences (foodset-derived elements emerge through ‘reactions’ that can be traced back to foodset items), and (ii) precisely describing and tracking how new ideas emerge from earlier ones. Thus, RAFs can model how endogenous conceptual restructuring results in new conduits by which uncertainties can be resolved and needs can be met. A source of uncertainty is modeled as an element that resists integration into the conceptual network, which is described as a *maxRAF* containing the bulk of the individual’s mental representations. This disconnect produces arousal, which catalyzes one or more interactions amongst mental representations. We illustrate the approach using the historical example of Kekulé’s realization that benzene is ring-shaped through a reverie of a snake biting its tail. We show how a single conceptual change can precipitate a cascade of reiterated cognitive ‘reactions’ (self-organized criticality) that affect the network’s global structure, and discuss why this may help explain why cognitive restructuring can be therapeutic. Finally, we discuss educational implications of the RAF approach.

Keywords: analogy, autocatalytic network, conceptual network, creativity, cross-domain transfer, innovation, potentiality, uncertainty

From Uncertainty to Insight: An Autocatalytic Framework

Introduction

Creativity flourishes in situations that involve both constraints and uncertainty (Beghetto, 2019), or what has been referred to as *enabling constraints* (Kauffman, 2016). Ultimately, both the constraints and the uncertainty originate in the dynamics of *concepts*: mental constructs such as DOG or FREEDOM that enable us to interpret new situations in terms of similar previous ones. By the *potentiality* of concepts, we mean their capacity to shift in meaning in ways that are often non-trivial, or even defy classical logic, when they combine or serve as contexts for one another (Aerts, Broekaert, Gabora, & Sozzo, 2016; Aerts, Gabora, & Sozzo, 2013; Hampton, 1988; Osherson & Smith, 1981). For example, though a seemingly defining property of the concept ISLAND is the property ‘surrounded by water’, in the context ‘kitchen,’ as in KITCHEN ISLAND, the property ‘surrounded by water’ is not only unusual, but a sign of trouble!

Accordingly, our group has invested considerable effort into mathematically modeling how creativity derives from such shifts in meaning when concepts combine (Gabora, 2017; Gabora & Carbert, 2015; Gabora & Kitto, 2017), and synthesizing these results with what happens during concept combination at the neural level (Gabora, 2010, 2018; Gabora & Ranjan, 2013). In such models, *constraints arise* because concepts usually appear in predictable contexts; for example, typical contexts for ISLAND are DESERT (as in DESERT ISLAND), or REMOTE (as in REMOTE ISLAND). *Potentiality arises* from the fact that concepts often appear in atypical contexts, and indeed there is no way of circumscribing the possible contexts for any given concept. Moreover, as the KITCHEN ISLAND example shows, atypical contexts can dramatically alter the properties of a given concept. Moreover, the KITCHEN ISLAND example is far from extraordinary. For example, it would seem that all instances of ISLAND have the property ‘larger than a breadbox’ but TOY ISLAND does not; similarly, one might expect that all instances of the concept BUNNY possess the property ‘has a heart,’ but of course CHOCOLATE BUNNY does not. *Uncertainty arises* because to the extent that one cannot completely describe a situation in terms of all relevant concepts and how they mutually serve as contexts for each other (including order effects), one cannot

precisely know how to act in that situation, or accurately predict what will happen.

Although in creativity research one often comes across the term ‘problem domain,’ concepts readily ‘jump’ domains; for example, ISLAND is in the domain but KITCHEN is not, and BUNNY is in the ANIMAL domain, but CHOCOLATE is in the FOOD domain. This is particularly true when concepts are used creatively, as occurs in metaphor (Lakoff, 1993), analogy (Gentner, 1983; Holyoak & Thagard, 1996), or cross-domain transfer, in which a source from one domain (e.g., music) inspires or influences a creative work in another (e.g., painting) (Ranjan, Gabora, & O’Connor, 2013; Scotney, Weissmeyer, Carbert, & Gabora, 2019). For example, George Mestral’s invention of Velcro was inspired by analogy to burdock root seeds (Freeman & Golden, 1997) which, in turn, inspired ‘shoelace-less runners’ (or ‘shoelace-less sneakers’). Ingredients for the creative process can come from just about anywhere. Therefore, understanding what happens when concepts come together in unusual ways is only part of the story of how a creator creates. Another part is: why does the creator bring certain concepts together in the first place? How is it that the creator comes to look at a problem from a particular perspective?

The relationship between uncertainty and creativity, and what it means to approach it at the concept-combination level versus the global conceptual network level, can be understood metaphorically by examining the photograph in Figure 1 of woodcuttings with light shining on them from three different directions, yielding three differently shaped shadows: that of a G, an E, and a B. Without going into the details of why this metaphor is particularly apt,¹ Focusing first on just the upper woodcutting by itself, consider the situation in which you could not see the woodcutting, and had to guess its shape by examining the shadows it cast. Though each shadow is different, they are all projections of this same underlying object. You are in a state of uncertainty regarding its shape, but the shadows provide constraints on the space of viable answers. In the language of the above-mentioned concept combination models (which use a mathematical framework that was first used in quantum mechanics), we can refer to the state of the woodcutting when no light is shining on it as its *ground state*. We would say that the woodcutting has the potentiality to *actualize*

¹ This is done elsewhere (Scotney, Schwartz, Carbert, Saab, & Gabora, 2020).

different ways, and to actualize in one of these ways requires an *observable* or context, in this case, light shining from a particular direction.

Figure 1 sheds light (so to speak) on many aspects of the creative process. Consider first the upper woodcutting. Your uncertainty regarding the the problem solution or creative output is represented by your uncertainty as to the actual shape of the woodcutting. The process of coming to better understand it by looking at it from different perspectives is represented by the act of shining a light on it from different angles. First, this metaphor suggests a way of understanding the underlying form or conception of a creative idea that is consistent with the models of concept combination. Since the different sketches of a painting, or prototypes of an invention, take different forms when expressed in the physical world, it is tempting to assume that that they derive from different underlying conceptions in the mind. Thus, it seems self-evident that a stream of creative thought entails the generation of multiple distinct, separate ideas. However, the above-mentioned concept combination research suggests, much as that the three shadows of each wood-cutting are projections of the same underlying object, there is a single underlying as-yet-poorly articulated vision of the creative project that manifests as different sketch or prototype ideas when looked at from different contexts or perspectives.

Second, the woodcutting metaphor suggests a way of envisioning what is happening as you hone or ‘reflect on’ an idea. Midway through a creative process, you may have an inkling of how to proceed, but not yet know whether, or exactly how, it could work; we say that it is ‘half-baked.’ You entertain wildly different ways of looking at it, and these different perspectives yield wildly different ‘takes’ on the idea, much as shining light on the woodcutting from different directions yields different shadow-letters. However, over time, your reflections on the problem converge. You start looking at it in similar ways, and the images that come to mind overlap more, and seem less like ‘different ideas’ and more like ‘variants of one idea,’ much as shining light on the upper woodcarving from the right at slightly different angles would yield similar shadows. Although some might be wobbly or distorted, they would all be G-shaped.

Third, the woodcutting metaphor provides a nice visual explanation of why we cannot

understand the creative process without investigating the global structure of the creator’s conceptual network. Note that the two wood-cuttings in Figure 1 have different shapes, yet they yield the same three shadows. To distinguish the shape of the woodcutting above from the one below would require that light be shown on them from still more angles, casting shadows that would not look like any particular letters we know. Similarly, the more complex one’s unborn creative idea, the more honing steps required to discern its underlying form. Note also that the situation in creativity may be more complex than the woodcuttings scenario because each time you think about the creative problem from a particular angle, that thought may feed back and alter your conception of the problem. (It’s as if each time you cast a shadow on a woodcarving, that slightly altered the shape of the woodcarving.) In this case, the order of the perspectives from which the problem was looked at alters the solution to the problem. The upshot is that to understand the creative process we need to understand how and why the creator looks at the problem from a particular sequence of perspectives or contexts, in a particular order. In turn, to understand this, we must look beyond models of concept combination to the global structure of how a creator’s concepts are woven together into a conceptual network, and how this network is restructured, fueled by the creator’s needs, goals, desires. For this, creativity research must turn to network science (Kenett, 2018).

This chapter proposes a method for modelling creative insight and its origins in uncertainty using a certain kind of network, referred to as the Reflexively Autocatalytic Foodset Generated (RAF) network. The term *reflexively* is used in its mathematical sense, meaning that each part is related to the whole. The term *autocatalytic* will be defined more precisely shortly, but for now it refers to the fact that the whole can be reconstituted through interactions amongst its parts. The term *foodset* refers to the elements that are initially present, as opposed to those that are the products of interactions amongst them. In the RAF model developed here, these elements are *mental representations* (MRs)² composed of one or more concepts, and the network is a conceptual network: a web of shared properties, contexts, associations, and relationships of logic, causation, and so forth, that bind them together. The

² Although we use the terms ‘mental representation,’ we are sympathetic with the view (common amongst ecological psychologists and in the quantum cognition community) that what we call mental representations do not ‘represent,’ but act as contextually elicited bridges between the mind and the world.

chapter begins with a brief introduction to the literature on analogy and cross-domain thinking to Next, we introduce RAF networks and their application to cognition, and present a RAF network model of insight through cross-domain analogical transfer. We close with some thoughts on educational applications of the model, followed by conclusions, and suggestions for future research.

The Relationship between Uncertainty and Insight

We now discuss the psychological processes that will play a role in the creative process modeled in this chapter: cross-domain transfer, analogy, and self-organized criticality.

Analogy and Cross-domain Transfer

Cross-domain thinking is ubiquitous in the history of innovation (Enkel & Gassmann, 2010; Feinstein, 2006; Kalogerakis, Lu, & Herstatt, 2010). Cross-domain thinking often takes the form of analogy, which involves reducing uncertainty by mapping correspondences between something you don't understand and something else that you do understand which shares its deep structure. Analogy is central to our humanness (Hofstadter & Sander, 2013; Holyoak & Thagard, 1996). A well-known example of analogical transfer is Kekulé's discovery of the structure of benzene by visual analogy to a snake biting its tail (Findlay, 1965). This example reveals an interesting aspect of the phenomenon: the information from the source domain is not always retrieved from memory in the form it was originally encoded. MRs from memory may need to be modified to be useful as a source (Holyoak & Thagard, 1996). (Kekulé had likely never seen a snake bite its tail (though he may have encountered the symbol of the uroboros); however, his daydreaming mind modified the concept SNAKE in a way that elicited this insight.) Alternatively, the cross-domain source material may arise by noticing something in the environment. For example, the concept of wing warping—the key insight in the invention of the airplane—occurred to Wilbur Wright as he idly twisted an inner tube box (Heppenheimer, 2003). He realized that by twisting the trailing edges of the wings in opposite directions it would be possible to control the direction of the aircraft, thereby removing the remaining fundamental obstacle to human flight.

Cross-domain inspiration is also prevalent in the arts; indeed, a film score attempts to translate the emotional tone of an unfolding plot into music. A well-known example is Schubert’s ‘Hymn to Mary’, also known as ‘Ellens dritter Gesang’ (‘Ellen’s Third Song’) or (more famously) ‘Ave Maria.’ This song from the opera *The Lady of the Lake* was inspired by Walter Scott’s epic poem of the same title (Deutsch, 1928). Indeed, the phenomenon of cross-domain creativity may go back much further, as the earliest stringed instrument may have been derived from the bow and arrow (Montagu, 2017).

The cross-domain thought that goes into a creative work is not always evident in the work itself. The prevalence of cross-domain influences on creativity was examined in two studies, one with creative experts, and the other with undergraduate students from diverse academic backgrounds (Scotney et al., 2019). Participants listed both their creative outputs, and the influences (sources of inspiration) associated with each of these outputs. In both studies, cross-domain influences on creativity were found to be widespread, and indeed more frequent than within-domain sources of inspiration. In another study in which painters were instructed to paint what a particular piece of music would ‘look like’ if it were a painting, naïve participants were able to correctly identify at significantly above chance which piece of music inspired which painting (Ranjan et al., 2013). Although the medium of expression was different, something of its essence remained sufficiently intact for people to detect a resemblance between the new creative output and its inspirational source. These studies show that even when the creative output lies squarely in one domain, the process giving rise to it may be rooted in another.

Analogy plays a role not only in the discovery and development of ideas, but in the pedagogical process by which they are transmitted and learned by others (Holyoak & Thagard, 1996). Analogies are often used by parents and educators to explain abstract, or unfamiliar ideas, in terms of concrete or familiar ones. For example, analogy to existing methods of gene editing played a role in the discovery of CRISPR-Cas9 technology (Doudna & Charpentier, 2014), and its discoverers explain how it works through analogies to a weapons defence system and a Swiss army knife (Doudna & Sternberg, 2017; Thagard, 2019). Thus, analogies influence—sometimes misleadingly—how new concepts are understood, and

developed further. Only network models can account for such lines of creative influence.

Self-organized criticality

Innovation may be indicative of a cognitive phase transition arising through self-organized criticality (SOC) (Gabora, 1998, 2017; Schilling, 2005; Stephen, Boncoddo, Magnuson, & Dixon, 2009). SOC is a phenomenon wherein, through simple local interactions, complex systems tend to find a critical state poised at the cusp of a transition between order and chaos, from which a single small perturbation occasionally exerts a disproportionately large effect (Bak, Tang, & Wiesenfeld, 1988). In a sense, self-organized criticality in cognitive enterprises is like kicking the uncertainty down the road; answering one scientific question (or developing a piece of art or technology) leads to another, which leads to another, and so forth.

The signature of SOC is an inverse power law relationship between the magnitude of a critical event, and the frequency of critical events of that magnitude. Perhaps the best-known example of SOC is the theory of *punctuated equilibrium*, which posits that change in biological species is restricted to rare, rapid events interspersed amongst prolonged periods of stasis (Eldridge & Gould, 1972). It has been suggested that insight constitutes a form of punctuated equilibrium (Gabora, 2017). SOC gives rise to structure that exhibits sparse connectivity, short average path lengths, and strong local clustering. Other indications of SOC include long-range correlations in space and time, and rapid reconfiguration in response to external inputs. There is evidence of SOC in the human brain, e.g., with respect to phase synchronisation of large-scale functional networks (Kitzbichler, Smith, Christensen, & Bullmore, 2009). There is also evidence of SOC at the cognitive level; word association studies have shown that concepts are clustered and sparsely connected, with some having many associates and others few (Nelson, Kitto, Galea, McEvoy, & Bruza, 2013; Nelson, McKinney, Gee, & Janczura, 1998). Thus, conceptual networks exhibit the sparse connectivity, short average path lengths, and strong local clustering characteristic of self-organized complexity, and exhibit some aspects of ‘small world’ structure (Bassett & Bullmore, 2006; Steyvers & Tenenbaum, 2005; Zurn & Bassett, 2020). Like other SOC systems, a mind may function within a regime midway between order (systematic progression of thoughts), and chaos

(everything reminds one of everything else). Much as most perturbations in SOC systems have little effect but the occasional perturbation has a dramatic effect, most thoughts also have little effect on one’s worldview, but occasionally, one thought triggers another, which triggers another, and so forth in a chain reaction of conceptual change. This is consistent with findings that large-scale creative conceptual change often follows a series of small conceptual changes (Ward, Smith, & Vaid, 1997), and with evidence that power laws and catastrophe models are applicable to the diffusion of innovations (Jacobsen & Guastello, 2011).

Reflexively Autocatalytic and Foodset Generated (RAF) networks

We now introduce the networks that we will use to model analogical cross-domain transfer, beginning with the rationale and basic idea behind for the approach, followed by the mathematical framework.

Rationale for the Approach

The RAF approach offers a number of benefits over other conceptual network, semantic network, and neural network approaches:

- (1) **Semantic grounding.** The distinction between foodset items and foodset-derived elements provides a natural means of grounding abstract concepts in direct experiences; foodset-derived elements emerge through interactions (which for historical reasons are sometimes referred to as ‘reactions’), that can be traced back to foodset-items.
- (2) **Reactivity of ideas.** The extent to which one MR modifies the meaning of another is referred to here as its *reactivity*. A given MR’s reactivity varies depending on the other MRs present in working memory.³ MRs activate, or ‘react with’ each other, by aligning with needs (e.g., the need to resolve uncertainty or cognitive dissonance, or to solve a problem), and the emotions they elicit. Thus, whereas conventional node-and-edge network models generally require external input to continue processing, RAF networks ‘catalyze’ conceptual change endogenously, resulting in new conduits by which needs

³ For example, the degree to which PYLON qualifies as an instance of HAT increases in the context ‘worn to be funny’ Veloz, Gabora, Eyjolfson, and Aerts (2011).

and goals can be met. A RAF network self-organizes through conceptual restructuring, the effects of which can percolate throughout the network, altering its global structure. The recognition that cognitive networks operate in the service of goals and desires is not new, but here this relationship serves to maintain a reflexively autocatalytic network. By recursively inciting interactions amongst each other (which, again for historical reasons, is sometimes referred to as not inciting but *catalyzing*) MRs self-organize into new structures that can be formally described. MRs are not only activated by stimuli, and participate in pattern learning, but form a network that is self-organizing and entropy-reducing, and conceptual restructuring can percolate throughout the network and affect its global structure. This is consistent with findings that immersion in a creative task can be therapeutic, and accompanied by a sense of release (Barron, 1963; Forgeard, 2013). Treating MRs as not merely passive participants in spreading activation, but active catalysts of conceptual change is central to our strategy for capturing the flexibility of human cognition. It enables RAFs to be used to model cognitive development during childhood of the kind of conceptual structure that actively participates in the generation of the cultural novelty, i.e., the emergence of new ‘hubs’ in the cultural evolution machinery (Gabora & Beckage, submitted).

- (3) **Tracking innovation within and across conceptual networks.** The RAF approach tags mental representations with their source, i.e., whether they were (1) innate, or acquired through social learning or individual learning (of *pre-existing* information), or (2) the result of creative thought (resulting in *new* information). This demarcation makes it possible to trace innovations back to the individuals that generated them, and describe and track how new ideas and creative outputs emerge from previous ones.⁴ Creative thought gets lumped in with individual learning, but there is an important distinction between them. In *individual learning*—obtaining pre-existing

⁴ We note that in the cultural evolution literature, much has been made of ‘Rogers’ paradox’ (that social learning can invade a population without increasing its fitness). However, the apparent ‘paradox’ is an artifact of assumptions built into Rogers’ simulation (Gabora & Tseng, 2017; Kharrazadeh, Montrey, Metz, & Shultz, 2017), including that it incorporates social learning and individual learning (of existing information) but not the creative generation of new information. In the real world, whether or not the environment is temporally varying, we benefit from finding creative new ways of conceptualizing and responding to this world.

information from the environment by nonsocial means through direct perception—the information does not change form once the individual knows it. Noticing for oneself that lightning tends to be followed by thunder is an example of individual learning. In contrast, in *abstract thought*—the processing of internally sourced mental contents—the information is in flux (Barsalou, 2005), and when this incremental honing process results in the generation of new and useful or pleasing ideas, behaviour, or artifacts, it is said to be *creative* (Basadur, 1995; Chan & Schunn, 2015a; Feinstein, 2006; Gabora, 2017).

Distinguishing between individual learning and creative thought enables us to monitor at what point in a cultural lineage each new idea (or idea component) first arose, assess the relative contribution of these different sources on the emerging conceptual networks of individuals and social groups, and track cumulative change step by step.

- (4) **Integrative framework.** The RAF approach offers an established formal framework for integrating creativity research with the explosively growing field of network science, and embedding this synthesis in the study of self-organizing structures and their role in evolutionary processes. RAFs replicate and evolve, as demonstrated (both in theory and in simulation studies), which makes them a viable candidate for explaining how cultural information replicates and evolves (Gabora & Steel, 2020a). Indeed, the fact that autocatalytic networks (such as RAFs) have proven useful for modelling the origins of both biological (Hordijk & Steel, 2016; Steel, Hordijk, & Xavier, 2019; Vasas, Fernando, Santos, Kauffman, & Szathmáry, 2012; Xavier, Hordijk, Kauffman, Steel, & Martin, 2020), and cultural evolution (Gabora, 1998, 2000; Gabora & Steel, 2017, 2020a, 2020b)⁵, suggests that RAF theory may provide a broad conceptual framework that is applicable to the origins and early stages of any evolutionary process. The framework is consistent with the theory that humans possess two levels of complex, adaptive structure: an organismal level, and a psychological level, with the mind playing the role in cultural evolution that the soma plays in biological evolution (Barton, 1994; Maturana & Varela, 1973). This is consistent with the theory that humans possess two

⁵ For related approaches, see (Andersson & Törnberg, 2019; Cabell & Valsiner, 2013; Muthukrishna, Doebeli, Chudek, & Henrich, 2018).

levels of complex, adaptive, self-organizing structure—an organismal level and a psychological level (Barton, 1994; Gabora, 2004; Maturana & Varela, 1973). The self-sustaining, self-protecting nature of a conceptual network is evident in the tendency to reduce cognitive dissonance, resolve inconsistencies, and preserve existing schemas in the face of new information. This is not merely an extension of organismal needs; indeed, these two levels of endogenous control can be at odds (e.g., a scientist immersed in solving a problem may neglect offspring or forget to eat.) Although the contents of a conceptual network change over time, it maintains integrity as a relatively coherent whole. Conceptual structure replicates, in a piecemeal manner, when individuals share ideas and perspectives. We posit that the generation of cumulative cultural novelty reflects the capacity for conceptual networks to evolve.

Because a RAF captures the nature of a structure *as a whole*, a cognitive RAF is, by necessity, abstract. It does not distinguish between semantic memory (memory of words, concepts, propositions, and world knowledge) and episodic memory (personal experiences); indeed, we are sympathetic to the view that these are not as distinct as once thought (Kwantes, 2005). For a detailed comparison between cognitive RAFs and other models used in cognitive science, we refer the reader to papers in which RAFs have been used to model the origin of cultural evolution (Gabora & Steel, 2017, 2020a), and the transition to behavioural and cognitive modernity (Gabora & Steel, 2020b).

Autocatalytic Networks

The theory of autocatalytic networks grew out of studies of the statistical properties of *random graphs* consisting of nodes randomly connected by edges (Erdős & Rényi, 1960). As the ratio of edges to nodes increases, the size of the largest cluster increases, and the probability of a phase transition resulting in a single giant connected cluster also increases. The recognition that connected graphs exhibit phase transitions led to their application to efforts to develop a formal model of the origin of life (OOL), i.e., how abiogenic catalytic molecules crossed the threshold to the kind of collectively self-sustaining, self-replicating, evolving structure we call ‘alive’ Kauffman (1986, 1993). In the application of graph theory to

the OOL, nodes represent catalytic molecules, and edges represent reactions. It is exceedingly improbable that any catalytic molecule present in the primordial soup of Earth’s early atmosphere catalyzed its own formation. However, reactions generate new molecules that catalyze new reactions, and as the variety of molecules increases, the variety of reactions increases faster. As the ratio of reactions to molecules increases, the probability that the system will undergo a phase transition increases. When, for each molecule, there is a catalytic pathway to its formation, the set of molecules is said to be collectively *autocatalytic*, and the process by which this state is achieved has been referred to as *autocatalytic closure* (Kauffman, 1993). Graph theory was also used to model how discrete MRs coalesce into an integrated understanding of the world capable of generating creative novelty (Gabora, 1998). In this application, nodes represent, not catalytic molecules, but MRs, and the edges represent interactions amongst MRs, such as through associative learning or concept combination. The catalyst often takes the form of a problem, desire, or need. It may be a biological need, such as the need for food or shelter, or produced by some earlier ‘reaction’, as when solving one problem leads to another. Very often, a reaction on one MR is catalyzed by another MR that has become aligned with the underlying need. For example, although Wilbur Wright’s invention of wing warping was ultimately motivated by the need to create a flying machine, it was directly ‘catalyzed’ by the notion of a bending box.

Autocatalytic networks have been developed mathematically in the theory of Reflexively Autocatalytic and Foodset-generated (RAF) networks (Hordijk & Steel, 2016; Steel et al., 2019). The term *reflexively* is used in its mathematical sense to mean that every element is related to the whole. The term *foodset* refers to the reactants that are initially present, as opposed to those that are the products of catalytic reactions. RAF theory has been used to model the emergence of a self-sustaining, self-replicating structure (i.e., a living protocell (Hordijk & Steel, 2015)). Thus, RAFs offer a promising avenue for modelling the OOL, and thereby understanding how biological evolution began (Hordijk & Steel, 2016; Steel et al., 2019; Vasas et al., 2012; Xavier et al., 2020). RAFs have also been used to model cognitive transitions underlying cultural evolution (Gabora & Steel, 2017, 2020a, 2020b). Table 1 summarizes how RAF theory terminology applies in biological and cultural/cognitive settings.

The fact that RAFs have proven useful in both these domains suggests that RAF theory may provide a broad conceptual framework that is applicable to the origins and early stages of diverse evolutionary processes.

We now summarize the key concepts of RAF theory. A network of interrelated parts, such as a conceptual network, is referred to as a *catalytic reaction system* (CRS), and is modeled as a tuple $\mathcal{Q} = (X, \mathcal{R}, C, F)$ consisting of a set X of interacting elements, a set $\mathcal{R}$ of reactions, a catalysis set C indicating which types of elements catalyze which reactions, and a subset F of X called the foodset. A *Reflexively Autocatalytic and F-generated* set—i.e., a RAF—is a non-empty subset $\mathcal{R}' \subseteq \mathcal{R}$ of reactions that satisfies the following two properties:

1. *Reflexively autocatalytic*: each reaction $r \in \mathcal{R}'$ is catalyzed by at least one element type that is either produced by $\mathcal{R}'$ or is present in the foodset F ; and
2. *F-generated*: all reactants can be generated from the foodset F through a series of reactions in $\mathcal{R}'$ itself.

A set of reactions that forms a RAF is simultaneously self-sustaining (by the *F-generated* condition) and (collectively) autocatalytic (by the RA condition) because each of its reactions is catalyzed by a molecule associated with the RAF. RAF theory has proven useful for identifying how phase transitions might occur, and at what parameter values. In the OOL context, a RAF emerges in systems of polymers (molecules consisting of repeated units called monomers) when the complexity of these polymers (as measured by maximum length) reaches a certain threshold (Kauffman, 1993; Mossel & Steel, 2005). The phase transition from no RAF to a RAF incorporating most or all of the molecules depends on (1) the probability of any one polymer catalyzing the reaction by which a given other polymer was formed, and (2) the maximum length (number of monomers) of polymers in the system. This transition has been formalized and analyzed (mathematically, and using simulations) and applied to real biochemical systems (Hordijk, Hein, & Steel, 2010; Hordijk, Kauffman, & Steel, 2011; Hordijk & Steel, 2004, 2016; Mossel & Steel, 2005), as well as cognitive systems (Gabora & Steel, 2017, 2020a, 2020b). Because of the deep structural or algorithmic similarity between the OOL and the OOC (as discussed above), much of this analysis can be

readily imported from the former to the latter.

Hierarchical RAF structure

There are three ways in which subRAFs can combine and expand (Steel, Xavier, & Huson, 2020):

- (1) If a CRS (such as a network of catalytic molecules or a conceptual network) contains a RAF, then the collection of all its RAFs forms a partially ordered set (i.e., a poset) under set inclusion, with a unique maximal element: the *maxRAF*. A CRS need not have a RAF, but when it does there is a unique maxRAF. The fact that the maxRAF may contain many RAFs enables RAFs to evolve, as demonstrated (both in theory and in simulation studies) through selective proliferation and drift acting on possible subRAFs of the maxRAF (Hordijk & Steel, 2016; Vasas et al., 2012). The remaining RAFs are called *subRAFs*. The union of any two (or more) subRAFs is a RAF (which explains why there is a unique maximal RAF). There may be a large number of blueirreducible RAFs, referred to as *irrRAFs*: RAFs that cannot be broken down into smaller RAFs. It is computationally straightforward to determine if a conceptual network has a unique irrRAF.
- (2) Some RAFs have the additional property of being *closed*, meaning that they are stable unless the reaction or foodset changes. Formally, a closed RAF is a RAF that contains every reaction in the network that has each of its reactants and at least one catalyst present either in the foodset or as a product of some reaction in the RAF. The maxRAF is always closed. A subRAF that is not closed is said to be *transient*. A transient RAF may add additional reactions until it becomes closed. The *closure* of any subRAF will contain the original subRAF, and be larger (unless the original subRAF was already closed). In Figure 4, the RAF $\{r_1\}$ is not closed, but $\{r_1, r_2\}$ is (since reaction r_3 does not become available until later, as it takes place in the mind of a different individual).
- (3) R' may combine with a *co-RAF*: a nonempty set of reactions that is not a RAF on its own, but that forms a RAF when combined with R' .

These are the only intrinsic processes by which RAFs can expand. Expansion can also be extrinsically driven by a change in the foodset, due to the presence of a new environmental stimulus, or through social learning processes. Extrinsic expansion can also arise due to a change in the permitted reactions, such as when participants in an experiment are instructed to ‘think creatively’. Computationally, it is straightforward to determine if a RAF is the union of two smaller RAFs, and if a set of reactions is a co-RAF (Smith, Steel, & Hordijk, 2014).

Application of RAF networks to creative cognition

We now introduce the mathematical framework and terminology used in applications of RAF theory to creative cognition. We emphasize that this is not merely an *analogy* to the OOL scenario. The mathematical framework could have been applied to cognition before it was too early life; it is simply a way of formalizing the emergence of structure with the above-mentioned key properties, irrespective of the specifics with which that structure is realized.

All mental representations (MRs) in a given individual i are denoted X_i , and a particular MR $x = x_i$ in X_i is denoted by writing $x \in X_i$. The *foodset*, for individual i , denoted F_i , encompasses MRs that are either innate, or that result from direct experience in the world, including natural, artificial, and social stimuli. F_i includes everything in individual i ’s long-term memory that did not directly result from thinking something through for oneself.

F_i also includes information obtained by i through individual learning by nonsocial means, so long as this information retains the form in which it was originally perceived, and is not redescribed or restructured through abstract thought. The crucial distinction between foodset and non-foodset items is not whether another person was involved, nor whether the MR was originally obtained through abstract thought (by *someone*), but whether the abstract thought process originated in the mind of the individual i in question. Thus, F_i has two components:

- S_i denotes the set of MRs arising through direct experience encoded in individual i ’s memory, including MRs obtained through social learning of an MR x_h by another

individual h , denoted $\mathbb{S}_i[x_h]$, and MRs obtained through individual learning, denoted $\mathbb{S}_i[l]$.

- I_i denotes any *innate knowledge* with which individual i is born.

The term *foodset derived*, denoted $\neg F_i$, refers to mental contents that are *not* part of F_i (i.e. $\neg F_i$ consists of all the products $b \in B$ of all reactions $r \in R_i$). They are the result of abstract thought processes such as deduction, induction, divergent thinking, analogical thinking, and cross-domain transfer. A single instance of conceptual restructuring, or *representational redescription* (RR) in individual i is referred to as a *reaction*, and denoted $r \in \mathcal{R}_i$. A stream of abstract thought, involving the generation of representations that go beyond what has been directly perceived, is modeled as a sequence of catalytic events. Following Gabora and Steel (2017), we refer to this as a *cognitive catalytic process* (CCP). Self-organized criticality, discussed above, describes a CCP consisting of recursive RR events, resulting in a cascade of conceptual change.

The set of reactions that can be catalyzed by a given MR x in individual i is denoted $C_i[x]$. The entire set of MRs either *undergoing* or *resulting from* r is denoted A or B , respectively, and a member of the set of MRs undergoing or resulting from reaction r is denoted $a \in A$ or $b \in B$. In particular, $\neg F_i$ includes the products of any reactions derived from F_i . Its contents come about through mental operations—RR events—on the foodset. Thus, $\neg F_i$ includes everything in memory that *was* the result of one’s own CCPs. $\neg F_i$ may include a MR in which social learning played a role, so long as the most recent modification to it was a catalytic event, i.e., involved RR.⁶ Note that F_i includes information obtained through social learning from *someone else* who obtained it through RR. Thus, if h invents a game and teaches it to i , this knowledge is part of F_i but it is part of $\neg F_h$.

The set of *all* possible reactions in individual i is denoted $\mathcal{R}_i$. The mental contents of the mind, including all MRs and all RR events, is denoted $X_i \oplus \mathcal{R}_i$. This includes F_i and $\neg F_i$. Recall that the set of all MRs in individual i , including both the foodset and elements derived from that foodset, is denoted X_i . $\mathcal{R}_i$ and C_i are not prescribed in advance; because C_i

⁶ This distinction between foodset MRs and foodset-derived MRs may not be so black and white, but for simplicity, we avoid that subtlety for now.

includes reminders and associations on the basis of shared properties, different CCPs can occur through interactions amongst MRs. Nevertheless, it makes perfect mathematical sense to talk about $\mathcal{R}_i$ and C_i as sets. Table 2 summarizes the terminology and correspondences between the OOL and the OOC.

Cognitive RAFs include elements of cognition that have no obvious parallel in the original application of RAFs to the OOL. We denote the subject of attention at time t as $\dot{w}_t$. It may be an external stimulus, or a MR retrieved from memory. Any other contents of $X_i \oplus \mathcal{R}_i$ that are accessible to working memory, such as close associates of $\dot{w}_t$, or recently attended MRs, are denoted W_t , with W_t constituting a very small subset of $X_i \oplus \mathcal{R}_i$.

Modelling insight

We now show how creative insight through analogical transfer can be modeled using RAF networks. The focus is not on the mechanisms underlying analogy itself; thus, the model is consistent with theories that emphasize different aspects of analogy such as structure mapping (Gentner, 1983), constraint satisfaction (Holyoak & Thagard, 1996), and potentiality and honing (Scotney et al., 2020). We keep the internal structure of MRs and their associative links—which can be modeled using existing methods—to a minimum, so as to lay bare the proposed approach to importing network science into creativity research.

Cross-domain transfer in discovery by analogy

We explain the approach using a simple, striking, and well-known example: Kekulé’s realization that benzene must consist of a chain of carbon molecules that is circular in shape, inspired by a reverie in which he envisioned a snake biting its tail (Findlay, 1965). This insight marked a significant departure from earlier thinking in chemistry, and was critical to founding the field of organic chemistry. We now present a RAF model of Kekulé’s insight process. This innovation process is summarized in Figure 2, and depicted in Figure 3.

Absorbing the relevant background knowledge. Kekulé’s insight was preceded by a lengthy period sometimes referred to in the creativity literature as *preparation*. This stage involves the acquisition of knowledge of the target domain. In the case of analogy, it would also include learning of the source domain from which the analogy will be obtained. In

our example, the target domain is organic chemistry, specifically, knowledge of the structure and behaviour of carbon-based molecules. One source domain is herpetology, specifically, knowledge of the elongated shape and flexibility of snakes. Another source domain is geometry, specifically the knowledge of circular shapes. Since information from the reptile domain, specifically knowledge of the physical features of snakes featured prominently in Kekulé’s insight, they are part of the cultural lineage to which he contributed.

We represent the social transmission of general knowledge of snakes and circles, denoted S and C respectively, from a parent, individual h , to young Kekulé, individual i , as follows:

$$F_i \mapsto F_i \cup \{S_i, C_i\}, \quad S_i, C_i \in \mathbb{S}_i[S_h, C_h]. \quad (1)$$

There is no evidence that Kekulé was exceptional with respect to his knowledge of snakes, but he was undeniably exceptional with respect to his knowledge of organic chemistry, and his drive to understand what kind of structure benzene must have to explain its anomalous behaviour. We represent the social transmission from the teacher, individual g , to Kekulé, individual i , of knowledge pertaining to organic chemistry, denoted O , as follows:

$$F_i \mapsto F_i \cup \{O_i\}, \quad O_i \in \mathbb{S}_i[O_g]. \quad (2)$$

Kekulé also engaged in individual learning of organic chemistry, and this is represented as follows:

$$F_i \mapsto F_i \cup \{O_i\}, \quad O_i \in \mathbb{S}_i[l]. \quad (3)$$

Kekulé probably also engaged in individual learning of information pertaining to snakes and circles, but to a lesser degree than individual learning of organic chemistry, so we omit that detail here.

Insight. Creative honing is often (though not always) stimulated by *problem finding* (Abdulla, Paek, Cramond, & Runco, 2020), which begins with a sense of uncertainty, incompleteness, inconsistency, or need, accompanied by a lingering feeling that compels the exploration and expression of ideas (Feinstein, 2006). Insight by way of an analogy occurred

through mapping correspondences between the long, skinny shape of a snake, and its potential to bite its tail and thereby form a circular shape, to the long, skinny shape of the benzene molecule, and its potential to similarly close in on itself and form a circular shape. We model this as a CCP consisting of two reactions. The first involves modification of the MR of a snake to give a snake biting its tail. The second involves redescription of the MR of benzene as a linear molecule with a chain of carbon atoms as its backbone, resulting in a circular structure for benzene. (Note: it is quite possible that Kekulé had been familiar with the symbolic image of a serpent swallowing its tail, commonly referred to as the *uroboros* (or *ouroboros*). Were that the case, the uroboros MR would already be part of his foodset, and there would be no need to derive it from scratch.)

A reaction results in a new non-foodset item, i.e., it transforms an element of F_k into an element of $\neg F_k$. In general, for an individual k (where $k = i, j$), we write $a_k \xrightarrow{b_k} c_k$ to denote the reaction that transforms one MR (a_k) to a resulting MR c_k (in $\neg F_k$) by catalyst b_k , and we let $a_k + a'_k \xrightarrow{b_k} c_k$ denote a reaction that combines and transforms two MRs (a_k, a'_k) to a resulting MR c_k (in $\neg F_k$) by catalyst b_k . In our example, both catalysis events were provoked by the context: *need to explain the behaviour of the benzene molecule*, which we denote n_i . In the first reaction, r_1 , the concepts SNAKE and CIRCLE denoted S_i and C_i , serve as the reactants. They undergo restructuring during a RR event resulting in a product, the MR of a snake biting its tail, i.e., a UROBOROS, denoted $U_i \in \mathbb{S}_i(l)$. We describe this as follows:

$$S_i + C_i \xrightarrow{n_i} U_i, \quad F_i \mapsto F_i \cup \{U_i\}, \quad (4)$$

The second reaction involves modification of the *reactants* U_i and LINEAR-BENZENE, denoted L , to generate the *product*: RING-SHAPED-BENZENE, denoted $R_i \in \mathbb{S}_i(l)$. We model this as a reaction, r_2 , and describe it as follows:

$$U_i + L_i \xrightarrow{n_i} R_i, \quad F_i \mapsto F_i \cup \{R_i\}, \quad (5)$$

Since RING-SHAPED-BENZENE is associatively linked to the more abstract concept MOLECULE, this event expands the affordances of MOLECULE (i.e., it is now perceived as

something that can be circular). Affordances are a kind of association and, as such, they increase the connectivity of the conceptual network.

Conceptual network expansion. Kekulé’s insight bridged the domains of geometry, herpetology, and chemistry, increasing the density of connections in his conceptual network. More importantly, it integrated other isolated fragments of conceptual structure—specifically, mental representations of seemingly inexplicable experimental results into Kekulé maxRAF. The inertness of the benzene molecule was due to the stability afforded by its ring-shaped structure. In light of this, his experimental results with benzene now made sense.

The insight was followed by a *honing and verification* stage: development of the idea, as well as evaluation and assessment. This involved assessing and testing the hypothesis that benzene is circular in shape. The thought processes underlying these activities brought about further expansion of the maxRAF.

Teaching and social learning. Once Kekulé was confident in the correctness of the idea, the next stage was to share it with others. This involved teaching and social learning of the knowledge of benzene’s shape with peers and students. Kekulé’s insight altered the structure of their maxRAFs, as it had for Kekulé himself. Note, however, that in the RAF approach, the impact of this insight is described and depicted quite differently for Kekulé’s students and peers than for Kekulé himself. For Kekulé the knowledge of benzene’s ring-shaped structure was obtained through RR, modeled as a ‘reaction’ and therefore it expanded the set of foodset-derived MRs, $\neg F$. In contrast, for Kekulé’s peers and students, this knowledge is obtained as a result of social learning, i.e., it expands the set F of foodset MRs.

Not only did Kekulé’s insight impact the field of chemistry, but it impacted thinking in psychology as well, by influencing Jung’s views on the role of the unconscious in creative thought (Jung, Von Franz, Henderson, Jaffé, & Jacobi, 1964). Let us look briefly at this branch of the benzene–uroboros cultural lineage.

Jung had extensive training in psychology, acquired through both social learning and individual learning. We omit these here, but they could be described exactly as was done above for individual i . We denote Jung’s knowledge of psychology prior to his contemplation

of the benzene discovery as P_j . Jung intuited that the unconscious plays a role in creativity; his uncertainty stemmed from lack of a concrete example of the unconscious presenting its knowledge to the conscious mind. We denote his new understanding of psychology as a result of the benzene–uroboros story, as P'_j . The context that ‘catalyzed’ these insights was his *desire to understand the role of the unconscious on creativity*, which we denote n_j . We model this line of influence as a reaction, r_3 , and describe it as follows:

The set of nested subRAFs depicting Kekulé insight into benzene is illustrated in Figure 4. Recall that an irrRAF is a RAF that cannot be broken down into smaller RAFs. Both $\{r_1\}$ and $\{r_3\}$ are irrRAFs. ($\{r_3\}$ is a RAF because R_j and P_j are in the foodset of Jung’s maxRAF). Note that $\{r_2\}$ by itself is not a subRAF, since it requires r_1 to first generate U_i in the mind of Kekulé in order to form. Other subsets also fail to be subRAFs (for example, $\{r_2, r_3\}$). The subRAF $\{r_1, r_2\}$ is a closed subRAF, since n_j involves a different individual (Jung) from the individual (Kekulé) associated with $\{r_1, r_2\}$. On the other hand, $\{r_1\}$ is not a closed subRAF (since n_i and the reactants of r_2 were available to Kekulé after r_1 occurred, so r_2 could proceed).

Note that although the RING-SHAPED-BENZENE MR was not a member of the foodset in Kekulé’s conceptual network (because it originated in his mind), it *was* a member of the foodset in Jung’s mind (since he acquired it through social learning and did not have to derive it from scratch). Note also that this is not a definitive model of how the discovery of benzene’s ring structure occurred. The uroboros story may have induced SOC, such that it had a ‘snowball effect’ on Jung’s understanding of psychology, with one insight giving rise to another. The process may also have been more bidirectional than implied here; it is possible that the image of a line bending around to form a circle was initially interpreted by Kekulé’s dreaming mind as a snake, and only afterward understood to be the benzene molecule. The important point is that the concept of the uroboros played a key role in this discovery, which fundamentally revolutionized our understanding of organic chemistry, and impacted psychology, and thus constitutes a critical juncture in our collective history. More generally,

analogies have played a significant role in the insights and discoveries that mark transition points in human history, and this model provides a means of formally describing them. A valuable feature of this model is that, by enabling MRs to align with needs and desires, and thereby play the role of a catalyst that prompts the occurrence of a particular thought process, each new idea (or modification of an existing idea) can be traced to its point of origin. Thus, each contribution to a particular ‘lineage’ of creative thought retains its connection with a particular innovator in a particular set of motivating circumstances, despite that it may change form drastically as it ‘encounters’ the conceptual networks of other individuals.

Collaborative Learning Environments as higher-level RAFs

A complementary process to that of learning *from* others, is learning *with* others (Kittur et al., 2019; Shteynberg, Hirsh, Bentley, & Garthoff, 2020; Tomasello, Kruger, & Ratner, 1993), as often occurs in an educational context. Transitions in understanding often result not from the efforts of single individuals but from diads or larger groups, with or without an explicit teacher, and RAFs are well-suited to model this. The RAF approach lends itself to a formal model of this.

An illustrative example of creative insight in a collaborative learning context involving three individuals, A, B, and C, is illustrated in Figure 5. This collaborative innovation is described by four reactions: the first occurs entirely in the mind of Individual A. The next reaction involves all three individuals. The third reaction involves B and C, and the fourth takes place in the mind of B but also involves A. The resulting group-level collaboration is described by a maxRAF consisting of several subRAFs.

Insights arising from a Continuous Process

The RAFs examined thus far have provided a mechanism for the sudden transitions in understanding, yet the underlying model involves dynamics based on continuous-time cognitive processes. This is not so surprising in light of self-organized criticality (discussed earlier), and related phenomena associated with nonlinear processes. The emergence of a new creative insight occurs when certain critical factors coinciding within an individual mind—or in the case of collaborative learning environments, a group of minds—to allow a cognitive

RAF to proceed once, after which it serves as catalyst and/or reactant) in subsequent interactions (within an individual, or between individuals). For example, in Kekulé’s mind, the fortuitous occurrence of the first reaction r_1 allowed the second r_2 to proceed; for the collaboration shown in Fig. 5, several interactions of particular MRs, in a certain order were required in the path towards the new concept v . We now describe more formally how a simple stochastic model based on RAFs predicts discontinuous bursts (or cascades).

A simple null model of continuous cognitive change over time would model the appearance of new creative outputs by a standard Poisson process at a constant rate λ . In this case, the times between events are independently exponentially distributed with an expected value of $\frac{1}{\lambda}$.

Modifying this null model to incorporate cognitive RAFs allows for the emergence of ‘bursts’ of cognitive change, i.e., self-organized criticality. To see this, first observe that the null model ignores the possibility that a new creative output from some individual in $\mathcal{G}$ (or collaboration within $\mathcal{G}$) can rapidly become the catalyst (or a reactant) for a cognitive reaction within the mind of one (or more) other individuals in $\mathcal{G}$, resulting in a new creative output that, in turn, can spark further cognitive reactions (and so on). A simple way to model this cascade effect is to assume that each new creative output has a probability q of enabling a new cognitive reaction to take place in the mind of at least one other individual in $\mathcal{G}$ (and the resulting creative output can then give rise to a further reaction in at least one other individual, again with probability q , and so on). Thus, each new creative output arising from a background Poisson process (with rate λ) gives rise to a burst of $N = 1, 2, 3, \dots$ creative outputs (including the original item from the Poisson process). The size of these bursts (N) is variable; in fact, under this simple model N follows a geometric distribution with parameter $p = 1 - q$. It is therefore described by $\mathbb{P}(N = k) = p^{k-1}(1 - p)$, and so N has an expected value $\mu = 1/p = 1/(1 - q)$ and a variance of $\sigma^2 = (1 - p)/p^2 = q/(1 - q)^2$ (Grimmett & Stirzaker, 2020). In particular, for q close to 1, the bursts can be large, and highly variable in size.

Discussion and Conclusion

To understand how a creator dissolves uncertainty by thinking ideas through from new perspectives requires a theoretical framework that incorporates individual differences in the structuring of knowledge and experience. This chapter presents a new approach to modeling creative insight and the process by which new ideas emerge from ones, using RAF networks. RAFs provide a means of formalizing the notion that conceptual change is prompted by uncertainty, cognitive dissonance, questions, or gaps in understanding, which trigger the restructuring or redescription of representations, modeled as conceptual interactions, or ‘reactions.’ The approach differentiates between (1) foodset MRs (knowledge that is either innate, or that results from social learning or individual learning of *existing* information), and (2) non-foodset MRs (*new* information that results from abstract thought, or interactions amongst other MRs). Treating MRs as ‘reactants’, ‘catalysts’, or ‘products’ enables us to model the ideation process step by step. This in turn enables us to track trajectories of conceptual change within and across individuals, and to better understand what prompts creative thought, and how ideas shape, and are shaped by, the conceptual structure that bear them. Creative thinking alters the hierarchical structure of RAFs, and provides avenues by which isolated fragments of knowledge can be assimilated into the maxRAF.

New Directions and Testable Predictions

We mentioned that the RAF approach has been used to model key transitions in the origins of cultural evolution Gabora and Steel (2017, 2020a, 2020b). In keeping with the dictum ‘ontogeny recapitulates phylogeny,’ the application of autocatalytic networks to the cognitive underpinnings of cultural evolution lays the groundwork for an autocatalytic framework for conceptual change (Gabora & Beckage, submitted). In the course of development, a child must acquire a coherent, integrated, flexible conceptual network capable of adapting ideas to new situations, and combining knowledge in appropriate and meaningful ways. Thus, RAFs can be used to model transitions in cognitive development during childhood, culminating in the emergence of new participants in the creative culture-generating ‘machinery.’

The RAF approach could also be used to investigate a number of other issues related to creativity, learning, and development, such as the relationship between expertise and creativity, the impact of individual differences on creativity, the relationship between creativity and the half-life of a creator's contributions, and whether conceptual network diversity among members of a social group affects the rate or quality of innovation. It could be used to model the impact of different kinds of learning experiences on conceptual network structure, and the impact of this structure on creativity. It would be interesting to study whether individual differences in reliance on foodset versus foodset-derived information sources (i.e., the propensity to think things through for oneself) sources may culminate in different kinds of conceptual networks and concomitant learning and/or personality trajectories.

The RAF approach also generates testable predictions. It predicts that the conceptual networks of highly innovative individuals have large maxRAFs, and a high ratio of foodset-derived to foodset MRs. It further predicts that in successful groups and organizations, such individuals are counterbalanced by individuals with a low ratio of foodset-derived to foodset MRs, so as not to compromise the capacity to preserve proven effective solutions.

The RAF framework also predicts that there may be individuals whose conceptual networks contain roughly the same *concepts* and *associations* but who nevertheless think and behave quite differently, resulting in markedly different creative outputs. A possible example would be Mozart and Salieri; while they had similar environments, experiences, and social circles, their compositions and impact were markedly different. Standard node-and-link semantic network approaches provide no obvious way of explaining this because there is no uncertainty-driven 'catalysis' of MRs. In the RAF approach, individual differences in MR reactivities and in relative reliance on foodset versus foodset-derived knowledge are expected to impact the shape of their conceptual networks, and the result in differences in the number, kind, and impact of creative outputs. We suggest that the difference between Mozart and Salieri reflects, not differences in their knowledge of music and understanding of the rules of composition, but in their proclivity to access life experiences and emotions, mull them over,

and allow them to serve as reactants and catalysts in the domain of music. Differences in reactivity may help explain why intelligence is a necessary but not sufficient condition for creativity (Jauk, Benedek, Beate Dunst, & Neubauer, 2013).

It has been suggested that creative breakthroughs are more likely to arise from conceptually distant sources than from conceptually close ones (Gentner & Markman, 1997; Holyoak & Thagard, 1996; Poze, 1983; Ward, 1998). There is evidence that individuals from different (often adjacent) fields produce the most creative solutions (Franke, Poetz, & Schreier, 2014; Jeppesen & Lakhani, 2010; Wiley, 1998). Some studies have also shown an advantage of conceptually distant sources over near ones with respect to novelty, quality, and flexibility of ideation (Chan et al., 2011; Chiu & Shu, 2012; Dahl & Moreau, 2002; Goncalves, Cardoso, & Badke-Schaub, 2013; Hender, Dean, Rodgers, & Jay, 2002), while other studies concluded that there was no such advantage (Chan & Schunn, 2015b; Dunbar, 1997), or were inconclusive (Enkel & Gassmann, 2010; Malaga, 2000). Despite their potential benefit, distant sources may be harder to find, and require more iterative processing (Chan & Schunn, 2015b). A related prediction derived from the RAF approach is that creative collaborations require not just overlapping yet different conceptual structures, but compatible levels of MR reactivity. If their joint reactivity is too low, new ideas fail to emerge, but if it is too high, they lose the thread of continuity necessary for cumulative change.

RAF networks have been used to model the origins of evolutionary processes, biological (the origin of life) as well as cultural (the origin of cumulative innovation). We think this is not coincidental; indeed, elsewhere, we showed that both the evolution of early life and cultural evolution are instantiations of a primitive form of evolution—i.e., cumulative, adaptive, open-ended change—referred to as Self-Other Reorganization (SOR) (Gabora, 2019). Instead of replication using a self-assembly code, SOR entails internal self-organising and self-maintaining processes within entities, as well as interaction between entities. The argument for SOR bolsters the argument that they share a deep structure, and thus strengthens the rationale for applying RAFs in both domains. In any case, the RAF approach to creative cognition is consistent with the theory that humans possess two levels of complex, adaptive, self-organizing structure: an organismal level and a psychological level (Barton,

1994; Gabora, 2004; Maturana & Varela, 1973). Psychological research tends to be data rich and theory poor (Fried, 2020), and psychological theorizing remains fragmented (Teo, 2018). Theories about creativity are disconnected from theories about the relationship between domain-specific abilities and general intelligence (e.g., (Kovacs & Conway, 2016)), which are disconnected from theories about how attitudes or personality traits relate to the structuring of knowledge and experience (Dalege, Borsboom, van Harreveld, & van der Maas, 2018). The RAF approach offers an established mathematical framework for integrating research on creative cognition with research on semantic networks, and the kinds of structures that exhibit cumulative, adaptive, open-ended change, i.e., that evolve. Though still in its infancy, the RAF approach has the potential to provide a new way of understanding how human history unfolds, one that embeds creativity research in the formal study of self-organizing structures and their role in evolutionary processes, and that places creative cognition centre stage.

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Table 1

Application of graph theoretic concepts in biology and culture, respectively.

RAF Theory	Biology	Culture and Cognition
node	catalytic molecule	mental representation (MR)
edge	reaction pathway	association
cluster	molecules connected via reactions	MRs connected via associations
connected graph	autocatalytic closure (Kauffman, 1986, 1993)	conceptual closure ⁷ (Gabora, 1998)

Table 2

Terminology and correspondences between the application of RAFs in biology and cognition / culture.

Term	Biology)	Cognition / Culture
X_i	all molecule types in protocell i	all mental representations (MRs) in individual i
$x \in X_i$	a molecule in X_i	a MR in X_i
F_i	foodset for protocell i	MRs that are innate or directly experienced by i
$r \in \mathcal{R}_i$	a particular reaction in i	a particular representational redescription (RR) in i
$C_i[x]$	reactions catalyzed by x in i	RR events ‘catalyzed’ by x in i
$(x, r) \in C$	x catalyzes r	x ‘catalyzes’ redescription of r
$a \in A$	member of the set of reactants in r	member of the set of MRs undergoing r
$b \in B$	member of the set of products of r	member of the set of MRs resulting from r
$\neg F_i$	non foodset for i (i.e., all B of $\mathcal{R}_i$)	MRs resulting from R_i (i.e., all B of R_i)

Figure 1. Photograph of ambiguous wood-cuttings taken from the front cover of (Hofstadter, 1979). The top ‘trip-let’ (as the author refers to them) is not simply a rotated version of the one below it; it is a different shape. (Used with permission.)

Figure 2. Left: Sources of mental representations and symbols used here to depict them. Right: MRs participating in the fragment of the benzene-uroboros ‘lineage’ described in the text. Each instance of social learning (Column 2) has been preceded at some point in the lineage by a relevant instance of individual learning (Column 3) or representational redescription (RR) (Column 4).

Figure 3. Steps in the discovery of the ring-shaped structure of benzene. The meanings of symbols are provided in Figure 2. Steps 4, 5, and 7 are representational redescription events, modeled as reactions, corresponding to Insights 1, 2, and 3 in Figure 2. Note that since individual *i* obtained R_i through representational redescription, it is food-set derived item in *i*, but that very same MR is a food-set item in *j* because *j* obtained it from *i* through social learning. Note also when R reaches individual *j* (as R_j), it participates in a reaction (step 7) that was not possible in individual *i*.

Figure 4. The RAF consisting of the three reactions r_1, r_2, r_3 described by Eqns. (4), (5) and (6), respectively. Square boxes indicate representational redescription events (modeled as ‘reactions’), and dashed blue arrows indicate the needs, desires, or contexts that elicit these events (modeled as ‘catalysis events’). (For example, n_i is the desire to explain the behaviour of the benzene molecule), and n_j is the desire to understand the role of the unconscious on creativity.) This RAF contains precisely three subRAFs, namely $\{r_1\}$, and $\{r_1, r_2\}$, and $\{r_3\}$, which describe nested CCPs in the mind of Kekulé and the CCP in the mind of Jung.

Figure 5. An example of a hypothetical collaboration among three people, described by a RAF that involves four reactions. Individual A is motivated to consider a concept x from a new perspective, which results in x' (with the motivation being an internal catalyst α , such as ‘curiosity’). This first reaction ($x \xrightarrow{\alpha} x'$) in Individual A is verbally transmitted to Individuals B and C. A question raised by C catalyzes a further realisation that combining the new idea, x' , with a concept from B (namely, y) yields the superior idea, (z) . This second reaction ($x' + y \xrightarrow{\beta} z$) results from the collaboration among three individuals, with C providing the catalyst, and the other two supplying the reactants. As such, we can consider the RAF that extends across the three group members. Further prompting by B (catalyst γ) allows for this idea to be developed further (with input from C via w) to produce an even better idea, u . This third reaction ($w + z \xrightarrow{\gamma} u$) involves interaction between B and C. Input from A (catalyst δ) leads B to realize that an even better idea, v , is possible. This fourth reaction ($u \xrightarrow{\delta} v$) takes place in the mind of B, following catalysis by A. This final ‘product’ is then communicated to the rest of the group (as v').