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The role of design and training in artifact expertise: The case of the abacus and visual attention

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Abstract

Previous accounts of how people develop expertise have focused on how deliberate practice transforms the cognitive and perceptual representations and processes that give rise to expertise. However, the likelihood of developing expertise with a particular tool may also depend on the degree to which that tool fits pre-existing perceptual and cognitive abilities. The present studies explored whether the abacus – a descendent of the first human computing devices – may have evolved to exploit general biases in human visual attention, or whether developing expertise with the abacus requires learning special strategies for allocating visual attention to the abacus. To address this question, we administered a series of visual search tasks to abacus experts and subjects who had little to no abacus experience, in which search targets and distractors were overlaid atop abacus “beads.” Across three studies, we found that both experts and naïve subjects were faster to detect targets in semantically-relevant components of the abacus, suggesting that abacus training is not required to exhibit attentional biases toward these components of the abacus. This finding suggests that the attentional biases that scaffold numerical processing of the abacus may emerge from general properties of visual attention that are exploited by the design of the abacus itself.

Keywords: visual attention; mental abacus; expertise; artifact design

One hallmark of human cognition is our ability to create complex artifacts and to develop expertise in using them. For example, auto mechanics are experts at using specialized tools like car jacks and fender rollers, and neuroscientists are experts at using Magnetic Resonance Imaging and Transcranial Magnetic Stimulation. Previous studies have explored how attaining expertise might depend on pre-existing individual differences (e.g., Gobet & Erekü, 2007; Smith, Tsimpli & Ouhalla, 1993), but also on how it can be developed through extensive, deliberate practice (Charness, Krampe & Mayr, 1996; Ericsson, Krampe & Tesch-Romer, 1993; Ericsson & Lehman, 1996; Platz et al., 2014; Starkes et al., 1996), and how such practice transforms the cognitive and perceptual representations and processes that give rise to expertise (e.g., Chase & Simon, 1973; Chi, Glaser & Farr, 1988; Chi, Glaser & Rees, 1982; Ericsson & Lehman, 1996; Gauthier, Skudlarski, Gore & Anderson, 2000). However, the likelihood of developing expertise with a particular tool may also depend on the degree to which that tool has been designed to fit the pre-existing perceptual and cognitive abilities of novices.

In the present study, we consider how the design of the *abacus* – a physical artifact that represents exact numerical quantities and arithmetic operations over those quantities via the positions of beads in columns – may play a role in explaining the remarkable levels of expertise found in abacus experts. Individuals who have received training using the abacus are able to rapidly decode the number represented by the abacus despite the fact that this requires attending to the precise locations of a large number of abacus beads (Du, Yao, Zhang & Chen, 2014; Frank & Barner, 2012). Additionally, abacus users who have learned a more advanced technique known as *mental abacus* – which involves visualizing the physical abacus – are able to perform mental calculations with staggering speed and accuracy (Frank & Barner, 2012; Hatano, Miyake & Binks, 1977; Stigler, 1984). This expertise is intriguing because it appears to require users to

mentally represent and manipulate the locations of numerous abacus beads, a surprising feat in light of previous studies, which have suggested that visual working memory is subject to restrictions, e.g., such that only three or four objects can be simultaneously tracked (e.g., Cowan, 2001; Irwin, 1992; Luck & Vogel, 1997; Todd & Marois, 2004; Vogel, Woodman & Luck, 2001; Vogel & Machizawa, 2004), or that if more objects are tracked, they are represented with less precision (Alvarez & Cavanagh, 2004; Bays & Husain, 2008).

Our studies address the nature of abacus expertise by testing whether the extensive training abacus experts receive might lead them to develop special strategies for allocating their visual attention to semantically-relevant aspects of the abacus, which could allow experts to efficiently process the abacus and help explain their remarkable computational abilities. We contrast this hypothesis with the idea that abacus expertise does not solely result from extensive training and associated enhancements to perceptual and cognitive resources and processes (Dong et al., 2016; Hu et al., 2011; Li et al., 2013a; Li et al., 2013b; Wang et al., 2013; Yao et al., 2015), but is also scaffolded by the design of the abacus, and its efficient use of pre-existing cognitive resources (Frank & Barner, 2012; Barner et al., 2016; Barner et al., in press). By this latter account, the design of the abacus may exploit general biases in visual attention, leading even naïve subjects who have no familiarity with the abacus to selectively attend to semantically-relevant aspects of the abacus. Our studies address how abacus training might shape the allocation of attention over the abacus by exploring how expert abacus users and naïve subjects attend to abacus-like displays.

The idea that the design of the abacus is well-adapted to human perception and cognition is motivated by the fact that its structure has gradually evolved over thousands of years, in multiple human cultures, descending from the oldest human computing devices (Ifrah, Harding,

Bellos & Wood, 2000; Menninger, 1969). Over this long history, different designs for these physical calculators arose, beginning with stone counting boards and “calculi” (stones that represented number), and including designs that varied the number of beads required to represent a number, the orientation of beads (vertical vs. horizontal), and even the size and shape of beads. This protracted history of experimentation with different representational formats raises the possibility that, over time, the design of the abacus – like that of other artifacts (Basalla, 1988; Petroski, 1993) – became tailored to properties of human perception and cognition.

(Figure 1 about here)

The most commonly used abacus today, the Soroban abacus, chunks beads into columns of 5 (see Figure 1). Each column represents a digit with a specific place value, which increases from right to left (i.e., from ones to tens to hundreds, etc.). Each column is divided by a horizontal beam: below the beam are four *earthly beads*, and above the beam is one *heavenly bead*. When the beads in each column are moved toward the horizontal beam to be *in-play*, they count toward the value of the column, and can be used to represent a digit between 1 and 9. Specifically, when in-play, the heavenly bead in each column counts as a multiple of ‘5’ based on its place value (e.g., 5, 50, 500, etc.) and each of the four earthly beads counts as a multiple of ‘1’ (e.g., 1, 10, 100, etc.). When all of the beads in a column are *out-of-play* – and moved away from the horizontal beam – the column represents 0. Columns representing zero are important for determining cardinality when the zero is *trailing* (e.g., the ‘0’ in 250) but not when it is *leading* (e.g., 025).

The abacus experts included in our study were all students who had received extensive training in using the physical abacus as well as training in mental abacus. Mental abacus students are first taught how the physical abacus represents number, often for up to one year before

learning the mental technique. This includes instruction on how to perform basic arithmetic operations like addition and subtraction, using highly practiced physical procedures. It also includes extensive practice with abacus “flashcards” – i.e., pictures of abacus states like the one in Figure 1 – that are used to practice rapid perceptual decoding from the abacus format into Arabic numerals or verbal responses. Previous studies indicate that through training, abacus students gain impressive facility with this kind of decoding (Frank & Barner, 2012), to the point that it becomes automatic. For example, compared to naïve subjects, abacus experts are slower to decide which of two abaci have a larger number of beads when the abacus with the larger number of beads also represents a smaller numerical value (Du et al., 2014), suggesting that abacus experts immediately and involuntarily process the number represented by the abacus. This automatic decoding is impressive in part because it requires users to attend to the precise locations of a large number of abacus beads in order to represent and manipulate exact numerical values. For example, representing a cardinality like ‘699’ minimally requires attending to the locations of 12 beads, in order to recognize whether these beads are in-play or out-of-play, heavenly or earthly, and what their specific place value is. Failing to attend to the location of any of these beads – and thus, e.g., their in-play/out-of-play or heavenly/earthly status – would result in decoding errors.

After students have become experts at using the physical abacus, they begin to receive training in the mental abacus technique. Mental abacus training generally begins with the explicit instruction to create a mental image of the physical abacus and to manipulate imagined beads to perform mental arithmetic. The mental representation of the abacus that experts create appears to be roughly isomorphic to the physical abacus itself (Hatano et al., 1977; Stigler, 1984; Stigler et al., 1986). For example, Stigler and colleagues provided evidence that when mental abacus users

are performing computations, they proceed through the same set of intermediate states in calculation that the physical abacus does (e.g., when adding $10 + 3$, the abacus passes through states for ‘10’, ‘11’, ‘12’, and ‘13’ for each bead that is moved). Also, recent evidence suggests that the physical gestures produced by expert users during mental calculation closely resemble those that would be made were the users to move their fingers to conduct the same operations on a physical abacus (Brooks et al., in press).

Broadly speaking, it is clear that both abacus training and design each play a role in the development of abacus expertise. For example, several studies have suggested that abacus training leads to structural changes in the brain, including enhancements of the white matter tracts that link visuo-spatial and motor areas, and modulation of neural activity in right fronto-parietal areas and the left occipitotemporal junction (Dong et al., 2016; Hu et al., 2011; Li et al., 2013a; Li et al., 2013b). These cortical changes are thought to lead to enhanced computational resources, including more automatic numerical processing and enhanced verbal and visuospatial working memory (e.g., Dong et al., 2016; Yao et al., 2015; Wang et al., 2013). Additionally, and not incompatible with the idea that abacus training leads to augmented visuo-spatial resources, other studies have suggested that the abacus design has evolved to make efficient use of existing cognitive and perceptual resources in typical populations (Frank & Barner, 2012; Barner et al., 2016; Barner et al., in press). For example, both abacus experts and subjects who have no knowledge of abacus are better at numerical estimation when dot arrays are configured similarly to the abacus – e.g., when dots are organized into vertical columns – suggesting that the design of the abacus may be tailored to visuo-spatial processing (Frank & Barner, 2012). Also, previous studies suggest that mental abacus users may appear to exceed limits on visual working memory in their tracking of numerous abacus beads (e.g., 12 in the case of ‘699’) because the design and

structure of the abacus allows beads to be chunked into columns, such that each column of 4-5 beads can be treated as a single unit in visuo-spatial working memory (Frank & Barner, 2012).

Although both abacus design and training play a role in explaining abacus expertise, broadly construed, the goal of the present studies was to tease apart these factors in explaining one phenomenon: How abacus experts deploy their visual attention to the abacus. In particular, we were interested in whether abacus expertise might require users to develop special strategies for allocating their visual attention toward the abacus. For example, through training, abacus students might learn to preferentially allocate their attention toward semantically-relevant elements of the abacus, like in-play beads (as opposed to out-of-play beads) and beads in columns representing trailing zeroes (as opposed to leading zeroes; Figure 1). Through practice, such attentional biases might also become automatic, given the evidence reviewed above that abacus experts rapidly and spontaneously decode the number represented on an abacus (Du et al., 2014; Frank & Barner, 2012). Such attentional strategies could also play a role in explaining how mental abacus experts are able to deploy chunking strategies to represent the abacus within the limits of visual working memory (Frank & Barner, 2012). Critically, this account predicts that naïve participants who do not know how the abacus represents number should fail to exhibit equivalent attentional biases, and might instead allocate their attention evenly across the abacus.

Alternatively, abacus expertise might not require learning special strategies for allocating visual attention to the abacus, but might instead be present in experts and non-experts alike, and stem from general attentional biases that are leveraged by the design of the abacus. Specifically, the structure of the abacus – having evolved over many centuries (Ifrah, Harding, Bellos & Wood, 2000; Menninger, 1969) – might exploit existing attentional biases such that semantically-relevant beads occupy regions of visual space to which even individuals who have

no abacus training are most likely to spontaneously allocate their attention. This role for the design of the abacus would fit with previous evidence that the visual form of mathematical tools and notation can scaffold or interfere with the comprehension of abstract mathematical concepts (Heeffer, 2011; Kirshner, 1989; Tall, 1996).

The present studies

In the present studies, we set out to explore whether abacus experts exhibit attentional biases toward semantically-relevant aspects of the abacus, and if so, whether these biases require abacus training, or if they might instead be present in naïve subjects and exploited by the design of the abacus itself. To measure how attention is allocated to the abacus, we conducted a series of visual search tasks, and presented abacus experts and naïve subjects with abacus-like displays in which search targets and distractors were overlaid atop abacus “beads”. We reasoned that participants would be faster to detect targets in locations toward which they preferentially attend. Using this method, we asked several questions.

In Experiment 1, we asked how abacus experts allocate their attention while reading the abacus. To address this question, experts were asked to complete a visual search task while simultaneously reporting the number represented by the abacus. Experiment 2 then asked whether the attentional biases exhibited by abacus experts in Experiment 1 only emerge when experts are explicitly asked to read the abacus, or if instead these biases may be more automatic. To test this question, experts were asked to complete only the visual search task, and were not asked to report the number represented by the abacus. Finally, in Experiment 3 we explored whether the ways in which experts allocate attention to the abacus require extensive practice, or if instead the structure of the abacus itself might mediate how visual attention is allocated. To test this final question, we explored whether subjects with little to no experience or knowledge of

the abacus differ from experts in how they attend to semantically-relevant aspects of abacus structure, by having them perform the visual search task from Experiments 1 and 2.

Experiment 1

The goal of Experiment 1 was to explore how abacus experts allocate their attention toward the abacus when they are reading the number it represents, as an index of which aspects of abacus structure they find most semantically-relevant. To test this, subjects were asked to perform two concurrent tasks. First, they performed a visual conjunction search task (Treisman & Gelade, 1980) in which search targets and distractors were overlaid atop schematic abacus beads. We manipulated whether search targets were located on in-play or out-of-play beads, and whether targets were located on beads in columns representing trailing or leading zeroes (e.g., 054 vs. 540). Second, on each trial of the visual search task, subjects were also asked to read and report the number represented by the abacus, which changed from trial to trial. We reasoned that if in-play beads and beads in trailing zero columns are especially relevant to extracting the number represented by the abacus, then when experts are simultaneously reading the abacus, they should preferentially attend to and thus be faster to detect targets at those locations, compared to when targets are on out-of-play beads or on beads in leading zero columns.

Method

Participants. The participants were 63 abacus experts. Due to non-reporting by parents, we lack demographic information for 11 of these participants. The remaining participants had a mean age of 10.62 years (range: 8.42 to 15.0). All participants in this experiment and in Experiment 2 were enrolled in Universal Computation Mental Arithmetic System (UCMAS) franchise schools, located in Gujarat Province, India, where abacus training is a popular after school activity. These students were included in our subject pool if they 1) had completed Level

4 UCMAS training (which comprises training using the physical abacus, and an introduction to mental abacus), 2) were judged by their instructor to be among the best students in their class, and 3) could travel to our test site in Vadodara, India. Thus, all participants were proficient at physical abacus, and were in the process of learning mental abacus. Seventeen of the participants from Experiment 1 also participated in Experiment 2, with experiment order counterbalanced.¹ Some participants were also tested in another set of experiments during the same session, the results of which are reported in Brooks et al. (in press).

Materials and Procedure. Participants completed two concurrent tasks. In the *Visual Search* task, subjects were shown an abacus schematic in which targets and distractors were overlaid atop abacus beads. Participants were asked to find the target item (i.e., a large circle) and report whether it was red or blue, while ignoring distractor items (i.e., large squares and small circles that could be either red or blue). Concurrently, in the *Abacus Reading* task, subjects were asked to report the number represented by the abacus schematic that was presented on each trial.

Participants were seated in front of Macintosh laptop screens, wore headphones throughout the task, and provided responses using the laptop keyboards and attached USB numeric keypads. Stimuli were presented to subjects using custom software designed using the Psychtoolbox 3 package for MATLAB (Kleiner et al., 2007). Prior to the task, small groups of participants were given instructions by the experimenter in English (the language of instruction at UCMAS), and these instructions were illustrated using examples of several trials of the Visual Search and Abacus Reading tasks. Each participant then completed a brief training phase before

¹ There were no main effects or interactions involving experiment order among the subjects who participated in both Experiments 1 and 2.

beginning the main task. During both training and test phases, feedback for a correct answer was given with a green cartoon smiling face accompanied by a high pitch double tone, and feedback for an incorrect answer was given with a red frowning face accompanied by a low pitch single tone.

Training phase. The training phase included two stages. In the first stage, participants received training on how to indicate the color of the target item during the Visual Search task. Participants first received two trials in which they were shown either a blue or red large circle, in succession, and were asked to press a button on the left or right side of their keyboard that matched the circle in color (e.g., “Please press the “blue” button on the left”; the “blue” button was the ‘z’ key, marked with a blue sticker, and the red button was the ‘/’ key, marked with a red sticker). After this, participants received an additional six trials in which they were shown a large circle and were asked to press the color-matching button (the circle was red on three trials and blue on three trials). If a participant responded incorrectly on any trial, that trial was repeated. Once all six trials were completed, participants proceeded to the second stage of training.

In the second stage of training, participants were introduced to the Visual Search and Abacus Reading tasks. First, they were shown a schematic of an abacus, which served as the search array. All of the abacus beads in the array except the target bead were overlaid with distractor items. These distractors were randomly-generated, and could be either big squares or small circles, and red or blue. The target bead was overlaid with a big circle, which was also randomly assigned to be either red or blue. Participants were told that they would have to find the big circle and then report the number of the abacus. The bead that contained the target item – i.e., the big circle – was then identified with a white ring, and participants were asked to indicate its color by pressing the appropriate “red” or “blue” button. After participants received feedback

for their response, the abacus search array was removed from the screen, and participants were signaled to enter the number that had been depicted on the abacus, using their numeric keypad. Participants received 10 additional trials of a similar structure, in which the abacus represented a random 2 or 3 digit number, with the search target and distractors randomly distributed. For the last eight of these trials, search targets were not identified for the participants with white rings, but instead needed to be found. If participants responded incorrectly on any of the ten training trials – e.g., by incorrectly indicating the target’s color or the abacus number – they began the ten trial loop again. All participants successfully completed the training phase.

Test phase. On each of 128 trials, participants viewed either a two- or three-column abacus schematic that depicted a number between 1 and 999 (see Figure 2).² As in training, participants were instructed to find the big circle and judge whether it was red or blue, while ignoring the big square and small circle distractor items (which were randomly assigned to be blue or red). Reaction time and accuracy were recorded once the participant indicated the color of the target; after this, the participant entered the number represented by the abacus and we provided feedback and recorded their accuracy. As described below, we manipulated the location of the target bead to explore how abacus experts allocate visual attention between beads in leading vs. trailing zero columns, and between in-play and out-of-play beads (see Figure 2).

(Figure 2 about here)

Comparison of in-play vs out-of-play beads: In 64 of the trials, the target bead appeared in a randomly-chosen column that represented a digit between 0 and 9. Across these trials, the target bead appeared equally often in in-play vs. out-of-play positions, in two- vs. three-column

² Single digit numbers (e.g., ‘1’, ‘9’, etc.) were represented with two abacus columns, using a leading zero (e.g., ‘01’, ‘09’).

abacus schematics, and was also equally likely to appear in each of the rows of beads. Because the abacus consists of one heavenly row of beads and four earthly rows of beads, this means that the target was four times as likely to appear on an earthly bead as on a heavenly bead.

Comparison of beads in leading vs trailing zero columns: In the other 64 trials, the target bead appeared in a column that represented either a leading or trailing zero, appearing in each of these columns an equal number of times, and equally often in two- or three-column abacus schematics. As in the in-play vs. out-of-play condition above, the target was also equally likely to appear in each of the rows. Although the beads in both leading and trailing zero columns are also necessarily out-of-play, we did not include trials in which targets appeared in these columns in the in-play vs out-of-play analyses described below.

Results and Discussion

To begin, participants correctly reported the number represented in the abacus displays on 95% of trials, indicating that they were engaged in reading these displays, as per our instructions. Thus, we have reason to believe that during the search task, participants were allocating their attention toward aspects of the abacus displays that are relevant to decoding number.

Our analyses for the visual search task were restricted to trials in which participants correctly indicated the color of the target, which occurred on 98% of trials. Reaction times were log transformed prior to analyses. Following standard practice, in all experiments reported here, all trials with log-transformed reaction times more than three standard deviations away from the mean were excluded from analysis. In the final dataset, participants took on average 3.76 seconds (95% CI: 3.55 – 3.98) to indicate the color of the target. We used a linear mixed-effects regression model (R version 3.3.0, lme4 version 1.1-12) to predict whether reaction times were

affected by the position of target beads: i.e., whether they appeared on in-play beads, out-of-play beads, in leading zero columns, or in trailing zero columns, treating position on in-play beads as the intercept. Additionally, our model tested for whether reaction times were affected by the trial number of the task (to test for practice effects) and whether the abacus display had two or three columns; We also tested for interactions between the number of abacus columns and position of the target bead. We used the maximal convergent random effects structure (a random slope of condition and a random intercept for each participant; following Barr et al., 2013). All p-values were computed using the lmerTest package version. Raw tabular data and full reports of analyses for all experiments reported in this paper can be found at:

https://github.com/langcog/abacus_attn.

Our analyses revealed that subjects were faster to detect targets on in-play beads ($M=3.63$, 95% CI: 3.40-3.86) than on out-of-play beads ($M=3.88$, 3.66-4.10; $\beta=.120$, $SE=.019$, $p<.001$; Figure 3), consistent with the idea that abacus experts preferentially deploy their attention toward in-play beads relative to out-of-play beads while reading the abacus. Further, while participants' speed to detect targets in trailing zero columns ($M=3.66$, 3.43-3.89) was similar to their speed to detect in-play targets, they were significantly slower to detect targets on beads in leading zero columns ($M=3.86$, 3.63-4.09; $\beta=.154$, $SE=.018$, $p<.001$; Figure 4), suggesting an attentional advantage for beads in trailing zero columns relative to beads in leading zero columns. Together, these results indicate that abacus experts preferentially deploy their attention toward in-play beads and beads in trailing zero columns when decoding the number represented by the abacus.

Also, our analyses indicated that participants were slower to detect targets when the abacus display had three as opposed to two columns ($\beta=.253$, $SE=.017$, $p<.001$), which is

unsurprising given that the three-column displays had more distractors. Interestingly, however, we also found significant interactions between number of abacus columns and target bead position (all $ps < .05$). Specifically, the attentional advantage for in-play and trailing zero targets was stronger in the two-column trials. We return to this finding when discussing the results of Experiment 2.

(Figures 3 and 4 about here)

Experiment 2

Experiment 1 suggests that, when asked to read the abacus, abacus experts are faster to detect search targets on in-play beads (than out-of-play beads) and on beads in columns representing trailing zeroes (than on beads in columns representing leading zeroes), suggesting that they preferentially allocate their attention toward these aspects of the abacus while decoding its number. In Experiment 2, we explored whether the biases observed toward in-play beads and beads in trailing zero columns only emerge when abacus experts are explicitly asked to read the abacus, or if instead these biases may be more automatic and arise even when the intention is not to read the abacus. As evidence for the latter possibility, recall that, in a Stroop-like paradigm, abacus experts are slower to decide which of two abaci have a larger number of beads when the abacus with the larger number of beads also represents a smaller numerical value (Du et al., 2014). This finding suggests that abacus experts spontaneously read the number represented by an abacus display, even when they are not instructed to do so. To test whether the attentional biases observed in Experiment 1 might emerge automatically, in Experiment 2 we asked abacus experts to complete only the visual search task: i.e., we did not also ask them to report the number represented by the abacus.

Method

Participants. The participants were 67 abacus experts. Due to non-reporting by parents, we lack demographic information for 11 of these participants. The remaining participants had a mean age of 10.44 years (range: 7.47 to 14.36). Abacus experts were included in our subject pool as described in the Participants section of Experiment 1.

Materials and Procedure. The methods and procedures were identical to those of Experiment 1 except that participants were not also asked to report the number represented by each abacus display, nor were they given training on this task.

Results and Discussion

As in Experiment 1, we restricted our analyses to trials in which participants correctly indicated the color of the target, which occurred on 96% of trials. On average, in included trials, participants took 1.97 seconds (1.85-2.10) to indicate the color of the target. As before, we used a mixed-effects model to predict whether reaction times were affected by the position of target beads (in-play, out-of-play, leading, or trailing), trial number, the number of abacus columns, and interactions between number of abacus columns and target bead position. This model used the same random effects structure as in Experiment 1: a random slope and intercept for each participant.

Consistent with the findings of Experiment 1, we found that subjects were faster to detect targets on in-play beads ($M=1.88$, 1.76-2.01) than on out-of-play beads ($M=2.09$, 1.96-2.22; $\beta=.119$, $SE=.020$, $p<.001$; Figure 3). Further, while participants' speed to detect targets in trailing zero columns ($M=1.89$, 1.76-2.01) did not differ from their speed to detect in-play targets, they were significantly slower to detect targets on beads in leading zero columns ($M=2.05$, 1.91-2.18; $\beta=.092$, $SE=.019$, $p<.001$; Figure 4). Finally, as in Experiment 1, participants were slower to detect targets in three-column than in two-column abacus displays (as in Experiment 1; $\beta=.163$,

$SE=.019$, $p<.001$); however, we did not find significant interactions between number of abacus columns and target position (all $ps>.54$), indicating that attentional advantages for in-play and trailing zero targets were equally strong across the two- and three-column trials. This finding contrasts with Experiment 1, in which in-play and trailing-zero effects were weaker in three-column than in two-column trials, raising the possibility that the load imposed by having to read three-column abaci in Experiment 1 may have suppressed attentional effects.

To directly assess the similarity of search patterns among abacus experts who had been asked to read the abacus (Experiment 1) and those who had not (Experiment 2) we used another mixed-effects model to predict all participants' reaction times from Experiments 1 and 2. This model tested the effect of experiment, effects of target position, and interactions of experiment with target position. Although participants were – unsurprisingly – faster to detect the target in Experiment 2 than in Experiment 1 (since the latter required them to perform two concurrent tasks; $\beta=-.797$; $SE=.049$; $p<.001$), the effects of target position condition (in-play, out-of-play, leading, or trailing) did not interact with experiment (all $ps > .21$). While this null effect is consistent with the idea that subjects in the two experiments exhibited similar search patterns and attentional biases, the logic of frequentist statistical analysis means that no inference of similarity is possible.

To quantify the evidence in favor of the null hypothesis (reflected by the lack of a significant experiment by target position interaction), we turned to a Bayesian hypothesis testing approach.³ Specifically, we computed a Bayesian mixed-effects regression using the package

³ Because of the importance of using random slopes in mixed effects analyses more generally (e.g., Barr et al., 2013), we specify our primary models using frequentist tools that allow for random slopes, but use Bayesian tools where appropriate to quantify support for the null.

BayesFactor (Morey & Rouder, 2015; see also Rouder et al., 2012). Our model had fixed effects as follows: main effects of experiment, trial number, and target position condition, and interactions of experiment by trial number and experiment by target position condition. It included random intercepts by subject (but not random slopes, as specifying this model is not possible in the BayesFactor package). We compared this model to one that did not include the interaction of experiment by condition, and found very strong evidence for the simpler model, with $BF_{10} = .00083$ (i.e., more than 1200 times greater likelihood of the data under the simpler model). This result supports the conclusion that the pattern of reaction times across conditions was similar between the experiments, such that participants in each experiment showed a similarly strong advantage for in-play beads, and beads in trailing zero columns.

Taken together, these results suggest that, even when they are not explicitly instructed to read the abacus, abacus experts automatically deploy their attention toward semantically-relevant aspects of abacus structure, like in-play beads and beads in trailing zero columns.

Experiment 3

Experiment 3 explored how the automatic attentional biases documented in abacus experts in Experiments 1 and 2 relate to the extensive training those experts have received. On one hand, knowledge of how the abacus represents number and sustained practice using the abacus could create the visual attentional biases observed in abacus experts, consistent with previous case studies showing that practice transforms the domain-relevant cognitive and perceptual processes that give rise to expertise (Chase & Simon, 1973; Chi, Glaser & Farr, 1988; Chi, Glaser & Rees, 1982; Ericsson & Lehman, 1996; Gauthier, Skudlarski, Gore & Anderson, 2000; McCormack et al., 2014; Sheridan & Reingold, 2007). By this account, extensive training with decoding the abacus could lead abacus experts to preferentially allocate their attention

toward semantically-relevant aspects of abacus structure, like in-play beads, and beads in columns representing trailing zeroes. This account would predict that naïve participants who do not know how the abacus represents number should fail to exhibit equivalent attentional biases, and might instead allocate their attention evenly across the abacus.

On the other hand, the biases observed in abacus experts – to attend toward in-play beads, and toward beads in trailing zero columns – may not require training or practice using the abacus, but may instead be present in experts and non-experts alike, and stem from general attentional biases that are leveraged by the design of the abacus. If this is the case, then even naïve subjects – who do not know how the abacus represents number – should preferentially attend to semantically-relevant aspects of the abacus. These general attentional biases could in principle facilitate the efforts of novice abacus students to learn how the abacus encodes number, and with practice, processing of the abacus’ numerical content could become automatic (Du et al., 2014).

To explore these possibilities, we conducted the visual search task of Experiments 1 and 2 with a group of adult participants from an American university community, and probed their familiarity with the abacus after they performed the task. This method allowed us to address whether biases toward in-play beads and beads in trailing zero columns are present in naïve subjects with no familiarity with the abacus, and if not, what levels of familiarity with the abacus might be required to develop such biases.

Method

Participants. Fifty-six adult participants (mean age = 21.24; range 18.42 – 33.27) were recruited from a participant database at the University of California, San Diego. All participants received course credit for their participation.

Materials and Procedure. All participants completed the visual search task prior to answering questions about their familiarity with the abacus.

Visual Search Task. The materials and procedures for the visual search task were the same as in Experiments 1 and 2.

Abacus Familiarity Assessment. After they completed the visual search task, participants were asked two questions to determine their level of familiarity with the abacus. First, we probed whether participants recognized that the display from the Visual Search Task represented an abacus: *Do you know what the display was?* Next, after telling them that the display depicted an abacus (if they had not said so themselves), we asked participants to describe their experience using an abacus: *Have you ever used an abacus, and if so, for how long?*

Based on their answers to these questions, we classified participants into three categories: “naïve”, “low familiarity” and “moderate familiarity”. Participants were considered “naïve” with respect to the abacus if they answered *no* to both questions ($n = 32$). They were considered to have “low familiarity” with the abacus ($n = 14$) if they stated that the display was an abacus (in addition to “abacus”, we accepted responses such as “counting beads” and “old calculator”) or reported using an abacus in the past despite failing to initially recognize that the display was an abacus. Finally, participants were considered to have “moderate familiarity” with the abacus ($n = 10$) if they *both* recognized that the display was an abacus *and* reported having used an abacus in the past. In general, even participants in the “moderate familiarity” group reported only sparse levels of prior experience with the abacus: e.g., four out of the seven participants in this group reported using the abacus only briefly as young children.

Results and Discussion

As before, we restricted our analyses to trials in which participants correctly indicated the color of the target, which occurred on 98% of trials. On included trials, participants took on average 1.20 seconds (1.12-1.27) to indicate the color of the target. We again used a mixed-effects model with the same random effects structure to predict whether reaction times were affected by the position of target beads (in-play, out-of-play, leading, or trailing), trial number, familiarity with the abacus (naïve, low, moderate), and by interactions between abacus familiarity and target bead position.⁴

Just as in Experiments 1 and 2 with abacus experts, participants were faster to detect targets on in-play beads ($M=1.15$, 1.07-1.24) than on out-of-play beads ($M=1.23$, 1.16-1.31; $\beta=.077$, $SE=.017$, $p<.001$; Figure 5). Also as before, while participants' speed to detect targets in trailing zero columns ($M=1.16$, 1.09-1.25) did not differ from their speed to detect in-play targets, they were significantly slower to detect targets on beads in leading zero columns ($M=1.23$, 1.15-1.31; $\beta=.056$, $SE=.017$, $p<.005$; Figure 6).

The subset of subjects in the “naïve” group, who reported never having used an abacus and could not recognize that the display represented an abacus, appeared to show just as strong of an advantage for in-play beads as did subjects in the low and moderate familiarity groups (Figure 5): There were no significant interactions between levels of abacus familiarity and target position on out-of-play (as opposed to in-play) beads (all $ps > .37$). As in the previous experiment, we turn to a Bayesian approach to test this claim of similarity directly. Again using the BayesFactor package, we specified a mixed effects model containing fixed effects of trial

⁴ A preliminary model that included number of abacus columns did not yield any significant interactions with effects of target bead position (as in Experiment 2); This factor was excluded from subsequent models.

number, target position condition (out-of-play vs. in-play), expertise, and an interaction of target position condition and expertise, along with a random intercept for each subject. We compared this to an identical model without the interaction term and found $BF_{10} = .0135$, indicating more than 70 times higher probability of the data under the simpler (no interaction) model. Thus, we are licensed in concluding that there is no interaction between target position condition and expertise, suggesting that all expertise groups showed a similarly strong advantage for in-play beads relative to out-of-play beads.

In our previous (frequentist) LMEM, there was, however a significant interaction between abacus familiarity and target position in leading vs. trailing columns, as subjects in the moderate familiarity group exhibited a greater trailing vs. leading effect, compared to naïve subjects ($\beta=.116$, $SE=.044$, $p=.010$). To test whether even the naïve subjects ($N=32$) showed the leading zero effect, we fit a follow-up model to the data from only these participants and again found a significant leading zero effect ($\beta=.056$, $SE=.015$, $p<.001$; Figure 6).

Together, these findings provide strong evidence that knowledge of how the abacus represents number, and practice using the abacus, is not required to exhibit attentional biases toward semantically-relevant aspects of abacus structure. Just as in Experiments 1 and 2 with abacus experts, we observed reliable attentional biases toward in-play beads and toward beads in trailing zero columns, even in subjects who have never used an abacus and cannot even recognize one. This provides evidence that the design and structure of the abacus may leverage general properties of visual attention, and lead even naïve subjects to attend to numerically-relevant components of the abacus.

(Figures 5 and 6 about here)

Finally, we assessed whether the attentional biases documented in Experiment 3 – in subjects who have little to no familiarity with the abacus – are *as strong* as those exhibited by abacus experts, and thus whether extensive practice with the abacus might strengthen these biases. To do so, we assessed the similarity in performance of the subjects of Experiment 3 and the abacus experts from Experiment 2, who each completed the same visual search task. We used a mixed-effects model to test the effect of experiment, effects of target position, and interactions of experiment with target position. Although participants from Experiment 3 were on average faster to detect the target than the abacus experts in Experiment 2 ($\beta = -.450$; $SE = .047$; $p < .001$), the effects of target position did not interact with experiment (all $ps > .05$), suggesting that we could not conclude that subjects in the two experiments exhibited different search patterns and attentional biases.

As above, to provide support for this null claim, we fit a Bayesian mixed effects model with random intercepts by subject and fixed effects of trial number, target position condition (in-play, out-of-play, leading, or trailing), and group (naïve subjects from Experiment 3 vs. abacus experts from Experiment 2). We compared this model to the identical model without the interaction term, and found strong support for the simpler (no interaction) model: $BF_{10} = .0032$, indicating that the observed data were more than 300 times as likely under the simpler model. Taken together, these results suggest that the advantages for in-play beads and beads in trailing zero columns observed in abacus experts (Experiment 2) are not stronger than those exhibited by subjects who have little to no familiarity with the abacus (Experiment 3).

In principle, the fact that abacus experts in Experiment 2 took longer to detect the target than did subjects in Experiment 3 could indicate that abacus experts spontaneously engaged in additional processing of the abacus stimuli, relative to subjects with little to no familiarity with

the abacus. For example, abacus experts in Experiment 2 may have not only searched for the target, but may have also decoded the number represented by the abacus, even though they were not instructed to do so (Du et al., 2014), leading to longer reaction times overall. However, the slower speed of abacus experts in Experiment 2 could also be explained by the fact that those subjects were children, while the participants in Experiment 3 were adults. Future studies could directly compare age-matched samples to determine if abacus experts and naïve subjects differ in their automatic processing of the abacus. This will also be important for understanding whether the greater quantity of visual experience that adults have had has led them to attend to perceptually salient properties of images, which could lead them to exhibit similar search patterns to abacus experts.

Post-hoc Analyses of the In-play Bias

Thus far, we have explored how abacus experts (Experiments 1 and 2) and participants with little to no familiarity with the abacus (Experiment 3) allocate their attention toward the abacus. Our findings indicate that, like abacus experts, even naïve subjects – who have never used an abacus and cannot recognize one – are faster to detect targets that appear on in-play beads (relative to on out-of-play beads), and on beads in trailing zero columns (relative to in leading zero columns). The fact that naïve subjects exhibited similar search patterns to abacus experts suggests that the design of the abacus may somehow take advantage of general properties of visual attention to channel attention toward semantically-relevant aspects of abacus structure. However, these findings do not speak to what these general attentional biases might be, and whether the same biases that explain how naïve subjects attend to the abacus also explain how abacus experts process the abacus. Here, we in a series of post-hoc analyses, we explore what more general attentional biases might underlie the advantage for in-play beads and whether the

same general biases might guide both abacus experts and participants with little to no experience with the abacus. As we explain in the General Discussion, our data cannot directly speak to the nature of the general attentional biases that might underlie the advantage for beads in trailing zero columns, and so we defer discussion of this topic to the next section.

To begin, we explored whether the advantage for in-play beads could be explained by a general attentional bias that favors targets that are closer to the horizontal beam over those that are further away. In particular, attention may be drawn first to the horizontal beam due to its relative salience (e.g., it has a distinct thickness and color, and contrasts in orientation with vertical columns of beads; see Koch & Ullman, 1987; see also Itti & Koch, 2001; Itti et al., 1998; Li et al., 2002; Treisman & Gelade, 1980), such that locations further from the beam are less selected for attention than locations closer to the beam. This would explain the in-play advantage, because in-play beads are generally closer to the horizontal beam than out-of-play beads. But critically, if this is correct, then in some cases out-of-play targets should be detected *faster* than in-play targets due to the in-play beads being further away from the beam, i.e., in cases where several earthly beads within a column are in-play, and the target in-play bead does not directly adjoin the beam.

To test the prediction that proximity to the beam explains the in-play advantage over and above in-play/out-of-play status, we used mixed-effect models to explore whether subjects' speed to detect the target in each experiment was affected by in-play/out-of-play status (we excluded leading / trailing trials from analysis), trial number, proximity of the target to the beam, and an interaction between in-play/out-of-play status and proximity to the beam. This analysis was confined to earthly beads, since in-play status and proximity to the beam are confounded for heavenly beads. Consistent with the idea that the in-play advantage can be explained by a bias to

attend toward targets that are closer to the horizontal beam, our models for Experiments 1 and 3 detected effects of proximity to the beam (Experiment 1: $\beta=.030$, $SE=.011$, $p=.005$; Experiment 3: $\beta=.044$, $SE=.009$, $p<.001$) but no effect of in-play/out-of-play status (Experiment 1: $\beta=-.005$, $SE=.077$, $p=.94$; Experiment 3: $\beta=-.090$, $SE=.069$, $p=.19$), or interactions between in-play/out-of-play status and proximity to the beam (Experiment 1: $\beta=.005$, $SE=.015$, $p=.75$; Experiment 3: $\beta=.009$, $SE=.013$, $p=.48$). These findings suggest that, for the abacus experts in Experiment 1 (who performed the dual search-reading task), and the subjects that had little to no familiarity with the abacus in Experiment 3 (who performed only the search task), beads that were closer to the beam were more likely to be selected for attention, suggesting that the in-play advantage may emerge from this more general attentional bias.

Interestingly, however, our model of the search patterns of abacus experts in Experiment 2 – who performed only the search task, just like the subjects of Experiment 3 – yielded a different pattern of effects. Here, proximity to the beam did not emerge as a significant predictor ($\beta=.004$, $SE=.011$, $p=.71$), though in-play/out-of-play status did ($\beta=-.279$, $SE=.084$, $p=.001$). The in-play effect was also qualified by an interaction between in-play/out-of-play status and proximity to the beam ($\beta=.059$, $SE=.016$, $p<.001$), such that participants were slower to detect out-of-play targets – but not in-play targets – that were further away from the beam. The fact that proximity to the beam did not affect abacus experts' speed to detect targets on in-play beads in Experiment 2 suggests that they automatically allocated their attention across these beads, and perhaps treated them as a “single object” of attention (Frank & Barner, 2012; for review, see Chen, 2012; Scholl, 2001; see also Duncan, 1984; Egly, Driver & Rafal, 1994; Luck & Vogel, 1997; Treisman, 1982; Vecera & Farah, 1994).

In sum, our post-hoc analyses provide some evidence that the in-play advantage observed in abacus experts in Experiment 1 and entirely naïve subjects in Experiment 3 might be explained by a more general attentional bias to attend toward salient objects (e.g., the horizontal cross-beam), such that other proximal objects are also likely to be selected for attention (e.g., in-play or out-of-play beads that are close to the beam), since effects of proximity to the beam emerged over and above in-play/out-of-play status in both experiments. However, the fact that abacus experts from Experiment 2 – who performed an equivalent task to the naïve subjects from Experiment 3 – did not show an effect of proximity to the beam on in-play targets suggests another intriguing possibility: As a consequence of training, abacus experts may automatically allocate their attention across in-play beads, by selecting these beads as a single “object” of attention.

General Discussion

The present studies explored whether the abacus – a descendent of the first human computing devices – evolved to exploit general biases in human visual attention, or whether abacus expertise requires learning special attentional strategies through practice. To address this question, we administered a series of visual search tasks to abacus experts and subjects who had little to no experience with the abacus. In these tasks, search targets and distractors were overlaid atop abacus “beads.” Using this method, Experiment 1 found that when asked to read an abacus, experts were faster to detect targets that were in-play, and targets in columns representing trailing zeroes, providing evidence that these aspects of abacus structure are relevant to extracting abacus number. Experiment 2 built upon these results and found that, for abacus experts, attentional biases toward in-play and trailing zero beads may be automatic, as they emerge even when experts are not explicitly asked to read the abacus. Finally, in Experiment 3

we found that, like abacus experts, subjects with no experience or familiarity with the abacus also show an advantage for in-play beads, and for beads in trailing zero columns. Together, these findings suggest that automatic biases to attend toward numerically-relevant components of the abacus do not require any experience with the abacus to develop, and may thus emerge from general properties of visual attention that are exploited by the design of the abacus itself. These findings contribute to a rich literature that has indicated that both abacus training and design play a role in explaining abacus expertise (Barner et al., 2016; Barner et al., in press; Dong et al., 2016; Frank & Barner, 2012; Hu et al., 2011; Li et al., 2013a; Li et al., 2013b; Wang et al., 2013; Yao et al., 2015).

Our findings also contribute to the broader literature on how expertise arises, more generally. In particular, evidence that the development of abacus expertise may in part be facilitated by abacus design provides a concrete example of how developing expertise with a particular tool may not solely depend on pre-existing individual differences (e.g., Gobet & Erekü, 2007; Smith, Tsimpli & Ouhalla, 1993), or extensive practice that transforms cognitive and perceptual abilities (Charness, Krampe & Mayr, 1996; Chase & Simon, 1973; Chi, Glaser & Farr, 1988; Chi, Glaser & Rees, 1982; Ericsson, Krampe & Tesch-Romer, 1993; Ericsson & Lehman, 1996; Gauthier, Skudlarski, Gore & Anderson, 2000; Platz et al., 2014; Starkes et al., 1996), but also on the degree to which that tool has been designed to fit the pre-existing perceptual and cognitive abilities of novices.

By showing that the design of the abacus plays a role in the development of abacus expertise, our findings also hold implications for how easily and widely abacus expertise can be attained, and thus its educational utility. In particular, by presenting numerically-relevant information in regions that even novices automatically attend to, the design of the abacus may

scaffold learning how the device represents number, and thus make expertise more easily attainable. Consistent with this, even children with typical levels of visuo-spatial ability can readily acquire physical and mental abacus expertise (Barner et al., 2016; Barner et al., in press). The effect of abacus design may hold broader lessons for the use of concrete manipulatives in mathematical education (Ball, 1992; Uttal, Scudder, & Deloache, 1997), as it suggests that such manipulatives will be most effective when their design takes into account the perceptual and cognitive abilities of their users. This conclusion also fits with previous evidence regarding how the visual form of mathematical tools and notation can scaffold (or interfere with) the comprehension of abstract mathematical concepts (Heeffer, 2011; Kirshner, 1989; Tall, 1996).

What general aspects of visual attention might give rise to the advantage for in-play beads and beads in trailing zero columns that we observed in both abacus experts and naïve subjects? Beginning with the in-play advantage, our post-hoc analyses of responses from abacus experts in Experiment 1 and naïve subjects in Experiment 3 suggest that the in-play bias may reduce to a bias to attend to locations that are closer to the horizontal beam. In particular, we suggested that attention may initially be drawn to the beam due to its contrasting orientation (relative to the abacus columns), thickness, and color (see Koch & Ullman, 1987; see also Itti & Koch, 2001; Itti et al., 1998; Li et al., 2002; Treisman & Gelade, 1980), making other neighboring areas, like in-play beads, more likely to be selected for attention. Future studies could directly test this idea by exploring whether subjects still show an advantage for detecting in-play beads when the horizontal beam has been removed, or has been otherwise made less salient.

With respect to the advantage for beads in trailing zero columns, one possibility is that it can be explained in part by hemispheric asymmetries in spatial frequency processing (Ivry &

Robertson, 1998; Christman, Ketterle & Hellige, 1991; Peyrin et al., 2003; Piazza & Silver, 2014; Sergent, 1982). In particular, prior studies have found that stimuli of higher spatial frequencies – e.g., sinusoidal gratings or natural scenes containing higher levels of detail – are more rapidly and accurately processed when they are presented to the right visual field (RVF), compared to when they are presented to the left visual field (LVF; Christman et al., 1991; Peyrin et al., 2003; Sergent, 1982). Applied to our studies, this suggests that participants should have been faster to find the big circle when it appeared in the RVF, potentially making sense of the advantage for detecting targets in trailing zero columns, which always appeared on the right side of the display in our studies. However, since we cannot be sure that the beads in trailing zero columns were presented to subjects' RVF – because we did not ask subjects to fixate on the center of the screen – our data are not conclusive with respect to this hypothesis. Future studies could explore this possibility by directly manipulating the visual field in which the abacus search display is presented.

Regardless of which general biases in visual attention might explain why naïve subjects preferentially attend toward in-play beads and beads in trailing zero columns, our results suggest that the design of the abacus capitalizes on such biases to direct attention toward semantically-relevant aspects of the abacus. This lends support to the idea that the structure of the abacus evolved over time, and became tailored to properties of visual attention and human cognition more generally (Frank & Barner, 2012). By this account, early versions of the abacus may not have been as easy to use or as powerful as current forms (see Ifrah, Harding, Bellos & Wood, 2000), and these perceived shortcomings may have driven further innovation and changes to the basic design of the abacus, as appears to be the case with the design of many artifacts (Basalla, 1988; Petroski, 1993).

Consistent with the idea that the design of the abacus could have evolved over time, it is known that current versions of the abacus, including the specific abacus studied here – the Japanese *Soroban* – have been modified from their early forms, e.g., by reducing the number of beads in each column (Frederic, 2005). Such modifications may have resulted in design features that have made the abacus tailored to properties of visual attention and human cognition more generally. For example, both the *Suan Pan* and the *Soroban* include a salient horizontal divider and vertically-oriented columns of beads, and these features could mediate how visual attention is allocated: As noted above, the salience of the horizontal divider could attract attention to in-play beads, and the vertical grouping of columns of beads could encourage columns to be treated as “objects” in working memory (Frank & Barner, 2012). Although little is currently known about the degree to which psychological factors have shaped historical modifications in abacus design across cultures, it seems likely that they have, since computations using the physical abacus are distributed (Hutchins, 1995; Zhang & Norman, 1994), and require coordination among the internal mental representations of abacus users and the external representations instantiated by the abacus to avoid interference (e.g., Kiyonaga & Egner, 2014).

Finally, although our data suggest that general properties of visual attention lead naïve subjects to attend toward numerically-relevant aspects of abacus structure, it is possible that sustained practice using the abacus further shapes how experts attend to and process the abacus. Clearly, only individuals with abacus training are able to interpret what they are attending to in order to decode the number represented by an abacus, and previous findings indicate that abacus training makes this decoding automatic (Du et al., 2014; Frank & Barner, 2012). In the context of our study, it is also possible that practice using the abacus leads to the formation of *specific* attentional biases toward in-play and trailing zero beads, in contrast to the more general

attentional biases of naïve subjects. For example, perhaps abacus experts – but not naïve subjects – would continue to exhibit an automatic in-play bias even if the horizontal beam were less salient, or a trailing zero bias even if the abacus was presented to the left visual field, because they have learned that these components of abacus structure are semantically-relevant. Another intriguing possibility left open by our data is whether abacus experts – but not naïve subjects – have developed an ability to extend their attention automatically across all in-play beads, such that they can treat the beam and in-play beads as a single “object” of attention (see Chen, 2012; Scholl, 2001; see also Duncan, 1984; Egly, Driver & Rafal, 1994; Luck & Vogel, 1997; Treisman, 1982; Vecera & Farah, 1994); This possible effect of abacus training could help explain how mental abacus users are able to chunk abacus beads into columns, and represent the abacus within the limits of visual working memory (Frank & Barner, 2012). In sum, our findings open new avenues for exploring the effects of abacus design and training on visual attention.

List of Figures

Figure 1. A schematic representation of a *Soroban* abacus with highlighted examples of in-play beads, out-of-play beads, and beads in leading zero and trailing zero columns. This abacus represents the number “6,780”. This schematic is similar to that used in our studies.

Figure 2. Examples of trials from the search tasks of Experiments 1 through 3. Left Panel: Example of a three-column trial in which the big circle (blue) is out-of-play and in a column representing a trailing zero. Right Panel: Example of a two-column trial in which the big circle (blue) is in-play.

Figure 3. Average reaction time to detect targets on beads that were in-play vs. out-of-play in Experiments 1 and 2 (Error bars indicate 95% CI).

Figure 4. Average reaction time to detect targets on beads that were leading vs. trailing zero columns in Experiments 1 and 2 (Error bars indicate 95% CI).

Figure 5. Average reaction time to detect targets on beads that were in-play vs. out-of-play by subjects in the naïve, low familiarity, and moderate familiarity groups of Experiment 3 (Error bars indicate 95% CI).

Figure 6. Average reaction time to detect targets on beads in leading vs. trailing zero columns by subjects in the naïve, low familiarity, and moderate familiarity groups of Experiment 3 (Error bars indicate 95% CI).

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