Using system analogies to model psychological variables for artificial intelligence applications

Sonny Mattek^{1,2}

¹Inner State, Inc., ²Interdisciplinary Research Institute at Shasta

Correspondence:
sonny@enteris.me

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Abstract

Artificial intelligence (AI) aims to model feats easily accomplished by a human child, such as object recognition and speech comprehension. However, there are many limitations in the current capacity of AI to execute these tasks in practical settings. From a developmental psychology perspective, skills such as object recognition and speech comprehension are built on more fundamental underlying processes of attention and perception. However, these more fundamental psychological variables are difficult to model from a computational perspective. This paper proposes a novel mathematical model of basic psychological variables, including attention, salience, and surprise, using the method of system analogies. Established system analogies are still routinely used for pedagogical purposes, but they were originally applied to understand the basic laws of newer scientific domains. Here, analogy is employed to extend the well-established linear dynamic laws of physics and engineering into the domain of psychology. The fundamental principles that govern these psychological variables may provide building blocks that will afford the field of AI to more effectively model the workings of the human mind.

A key component of human intelligence involves reducing data into simpler conceptual frameworks. For example, although the human eye has an N-dimensional input, where N is the number of retinal cells (on the order of millions), the brain reduces this information into a 3-dimensional visual space that follows the rules of Euclidean geometry. Thus, childhood insights, such as “object permanence,” arise from triangulating simple rules out of observing the same patterns repeated over and over.

Ultimately, some insights can never be achieved by collecting more and more data, since the data just repeat the same pattern. As the brilliant French physicist Poincaré noted, “Before Newton, plenty of people had seen apples fall, but not one of them could draw any conclusions from it. Facts would be sterile were there not minds capable of choosing among them, discerning those behind which something was hidden, and in recognizing what was hiding” (Poincaré, 1905). Rather than collecting more and more data, the insights seem as if they came from staring at the same data over and over again.

Here is where psychological concepts like attention and surprise come into play. Consider a simple everyday example, a meditation practice that involves repeating the same rituals over and over. Why does a monk wake up every day to repeat the same mantras in the same posture over and over again? The reason is similar to why Johannes Kepler meditated on the path of Mars over and over again. Both parties eventually arrive at a deep insight, an “ah-hah” moment. In science, this is sometimes called “eureka”, and in Zen, it is sometimes called “satori”, or enlightenment. These examples point to a fundamental relationship between sustained attention and surprise.

In this paper we will explore how the psychological variables of attention, salience, and surprise might operate according to basic mathematical principles. Ultimately, these processes are much simpler compared to the complex mental operations such as language comprehension and object recognition that AI is currently trying to tackle. We will arrive at mathematical formulation of these variables by referring to the mathematical equation describing a basic associative learning curve. By using mathematical analogy, deeper insights can be extracted from this well-established learning phenomenon. Stepping back to these more basic operations of the mind may provide important building blocks that can restructure our approach to certain AI problems.

Background.

The field of psychological science as we know it today emerged at the same time that the field of physics was transitioning from classical mechanics to quantum mechanics, which was around the beginning of the twentieth century. Scientific progress unfolds at a painstaking rate: over a century had to pass before Newton recognized that Kepler’s model of planetary motion was a specific example of the general two-body problem, and even more centuries passed after that before Poincaré would finally solve the three-body problem that was left open by Newton¹.

Nonetheless, the rate of progress for the adolescent scientific field of psychology can perhaps be accelerated if we learn by example. Although psychology is a unique science with its own unique problems of theory and method, this field can still perhaps benefit from tapping into some of the more fundamental principles that other scientific domains have worked so hard to

¹ Poetically, Newton called the “three-body problem” the “problem of the moon”. Poincaré (1905)

uncover. This approach can be implemented through the use of mathematical analogy, which was most elegantly demonstrated in the timeless work of the physicist James Clerk Maxwell^{2,3}:

“The most obvious case is that in which we learn that a certain system of quantities in a new science stand to one another in the same mathematical relations as a certain other system in an old science, which has already been reduced to a mathematical form...[by detecting] the analogy between any system of quantities presented to us and other systems of quantities in known sciences, we should lose no time in availing ourselves of the mathematical labours of those who had already solved problems essentially the same.” (Maxwell, 1869)

“Recognition of the formal analogy between two systems of ideas leads to a knowledge of both, more profound than could be obtained by studying each system separately.” (Maxwell, 1870)

To apply the technique of mathematical analogy to psychology, we will take as our focus an observed phenomenon that has been replicated in thousands of psychological experiments, namely, the phases of associative learning. The discovery of the two major phases of associative learning, routinely referred to as (1) “conditioning” and (2) “extinction”, can be traced back to the groundbreaking observations of Ivan Pavlov (1927), and hence the phenomenon is sometimes referred to as “Pavlovian conditioning”. During the conditioning phase, a motivationally-neutral (conditionable) stimulus, such as a brief tone or light flash, is displayed by an experimenter to signal to a subject the subsequent occurrence of an unconditionably motivating stimulus, such as an aversive shock or appetitive food. The psychological association between the tone (conditionable stimulus, or CS) and the shock (unconditionable stimulus, or US), which manifests in the observable behavior and physiological responses of the subject, increases according to a saturating exponential that asymptotes at some maximum value. This maximum value, as well as the amount of time it takes to get there, both depend on psychological properties of the CS and the US. Following the conditioning phase, an extinction phase can be induced by presenting the tone in the absence of the reinforcing shock. In this second phase, the association between the CS and the US, again as measured by observable behavior and physiological response, decreases according to a decaying exponential.

The mathematical structure of the associative learning phenomenon implies that it can be modeled with a first-order ordinary differential equation that fits the general solution. Indeed, several models of this form have been proposed (Kendler, 1971; Rescorla & Wagner, 1972; Mackintosh, 1975; Pearce & Hall, 1980). Most of these models define the asymptote of the equation with a single term (λ) that is related to the US, but Konorski’s summary of Pavlov’s work suggests that this asymptote value is determined by both the CS and the US (1948), not just the US. Only Tracy Kendler’s model defines the asymptote in this way (1971). However, Kendler’s model under-specifies the rate parameter as being only tied to the CS, whereas other models define the learning rate as being tied to qualities of both the CS and the US (e.g., Rescorla & Wagner, 1972). In an attempt to synthesize these various existing models, equation (1) is proposed as a revised mathematical model of associative learning:

² “We all know that it was Maxwell who connected by a slender tie two branches of physics—optics and electricity—until then unsuspected of having anything in common.” Poincaré (1905), p. 237

³ “The equations of electrical circuit theory may be based on Maxwell’s dynamical theory in which the currents play the role of velocities. Expressions for the kinetic energy, potential energy, and dissipation are deducible from general dynamic equations. In other words, an electrical circuit may be considered [more generally] to be a vibrating system.” Olson (1943), p. 1

$$(1) \quad \frac{dV}{dt} = \alpha \beta \left(\frac{\alpha}{\gamma} - V \right)$$

Equation (1) is a first-order ordinary differential equation, wherein the change in associative value over time (dV/dt) is a function of the current associative value (V). Here, associative value is defined as the psychological connection of a CS with a US, which can be measured via overt behavioral and physiological response patterns that correspond to experimentally controlled CS-US contingencies that occur over time. In this equation, α is defined as the amount of attention paid to the CS, β is defined as the salience of the US, and γ is the familiarity of the US. Note that in this equation, the attention paid to the CS (α) is included in both the rate parameter and the asymptote. This is in contrast to most previous models (with the exception of Kendler, 1971), which have tended to define the asymptote with a single term λ , which traditionally has been labeled only literally as the maximum associative value, rather than being attached to a psychological construct (see Pearce & Bouton, 2001 for a review).

We will focus on three exciting points that can be gleaned from this discussion of associative learning: (1) *identifying the optimal parameters for the associative learning equation is still an important open question in the field of psychological science*, and (2) *solving this open question by identifying these parameters will provide us the opportunity to generate mathematical definitions of psychological variables (attention, salience, and surprise)*, and (3) *once the parameters of this equation are precisely established, we will be able to identify the deeper relationships between the parameters themselves*. This third point will be discussed in the following section, demonstrating the insights that can be revealed given equation 1.

A Formal Analogy.

Due to the mathematical structure of equation 1 (regardless of the particular parameters proposed here), the system underlying the phenomenon of associative learning can be classified as being one of several important linear dynamic systems. That is, there are ideal systems in various physical domains (mechanics, electronics, heat) that exhibit behavior with this same elegant mathematical structure (Figure 1). Drawing an analogy with these systems can allow us to see into a deeper level of the intriguing phenomenon of associative learning.

Mechanical system: force applied to a mass with friction

Electrical system: battery in series with a resistor and capacitor

Psychological system: association between a conditioned and an unconditioned stimulus

Figure 1. Analogous systems in different domains of science: mechanical (blue, A), electrical (magenta, B), and psychological (green, C). These systems can all be described by first-order ordinary differential equations, and thus share a common mathematical structure that opens the possibility of mathematical analogy.

Importantly, first note that the behavior of the ideal mechanical (blue, A) and electrical (magenta, B) systems outlined in Figure 1 can both be described by differential equations that are constructed from more basic underlying principles. For example, the differential equation describing the behavior of an electrical circuit that has a battery in series with a resistor and capacitor (a classic RC circuit, Figure 1B) can be derived using Ohm's Law, Kirchoff's Laws, and the lawful relationship between voltage and capacitance. In other words, *by knowing the basic laws, one can predict what behavior this particular system of arranged components will produce*. In the mechanical circuit, basic principles lie at the foundation of that differential equation as well (i.e., Newton's laws).

The theoretical debate surrounding existing models of associative learning can help us see how the psychological system might be deconstructed in a similar fashion. That is, there has been some question as to whether the learning rate term should be defined by "attention" instead of "salience" (Mackintosh, 1975; Pearce & Hall, 1980). This is an exceedingly difficult theoretical question to answer, precisely because we have an intuition that attention and salience are not the same, but are somehow fundamentally related. These terms are difficult to define with a verbal, dictionary definition. Again, the crux of the theoretical work here is to figure out how to effectively label the coefficients of the mathematical description of the phenomenon as a way

toward achieving mathematical as opposed to verbal definitions⁴. In other words, the systems in mechanical and electrical domains teach us, by analogy, that the variables in equation 1 likely have a profound fundamental relationship. We already have an intuition about the proportionality of salience and attention, which is *like* our intuition about the proportionality of force and acceleration. How can we separate acceleration from force? We cannot—they are inextricably related. Following this logic, a principle that is analogous to Newton’s second law in mechanics and Ohm’s law in engineering can be formulated using psychological variables:

$$(2) \alpha = \beta\gamma$$

where α = attention, β = salience, and γ = surprise. This mathematical statement posits a law that governs the nature of attention: *the amount of attention is proportional to salience and the degree of surprise*. This explains how Kepler could generate an “ah-hah” insight (i.e., a surprise), by readdressing his data over and over again, and how Newton could generate his insight by watching the proverbial apples fall over and over again: by increasing attention (increasing α) on the same stimulus (constant β), surprise will increase (increasing γ). However, no causal direction is implied, as explicit (“top-down”) attention to any event feature can make that feature more salient (by definition here), and in this case surprise will decrease (i.e., familiarity increases with repetition), until the salience is transformed to surprise. Whether attention translates to salience or surprise is perhaps a question for neuroscience, and seems to have an inherent nonlinear process of bifurcation beyond the scope of this paper.

The structure of equation (2) is such that it can be considered a specific case of the law of conservation of energy, analogous to Ohm’s Law and Newton’s second law. This implies that the law of conservation of energy generalizes to psychological variables. Attention is a form of energy that is not created or destroyed, but rather transformed. Salient events call forward our attention, as the planets in the sky captured the attention of early scientists, and through our attention this energy is transformed into surprise, or insight, about the elegant processes of nature.

Discussion.

There are many possibilities that arise from acknowledging mathematical principles in the psychological domain. This opens the door for integrating psychological variables with many known systems, and relying on these systems to find more nuanced analogies can deepen our understanding of specific psychological processes. This is not a process of reducing psychological variables to physical mechanisms, but rather, acknowledging a shared mathematical structure in the psychological and physical domains, and then harnessing this shared structure to deepen our understanding.

Additionally, there is an interesting “meta” principle at play that seems to open up a pathway to accelerate scientific progress, in the sense that scientific progress is a psychological process engaged by scientists. Specifically, one of the implications of the model proposed here is that meditating on existing data over and over can generate insights. This is a very different

⁴ “When we say force is the cause of motion, we are talking metaphysics; and this definition, if we had to be content with it, would be absolutely fruitless, would lead to absolutely nothing. For a definition to be of any use it must tell us how to measure force; and that is quite sufficient, for it is by no means necessary to tell what force is in itself, nor whether it is the cause or the effect of motion.” Poincaré (1905), p. 110

approach to the current frenzy to generate more datasets, increase sample size, and publish more papers, as a way forward for science.

On another note, there is a convenience to be found in translating dynamic systems of any domain into the domain of electrical circuits (Olson, 1943), because these systems can be easily simulated computationally. This translation by analogy could prove useful for AI. Here, we have seen how a simple psychological conditioning paradigm can be modeled with a classic RC circuit, opening the door to potentially designing equivalent circuits of more complex behaviors that have more parameters. This approach to formalizing complex behaviors could be useful for simulating particular behaviors in artificial agents.

Importantly, the variables used in equation 2 (attention, salience, surprise) will certainly influence circuits at multiple levels of the system, and this relationship could be further specified. Not only can classical conditioning be modeled as an RC circuit, but also sensory mechanisms in the nervous system periphery. For example, looking at the inner ear we find analog sampling filters that can be modeled with the same RC circuit as the global associative learning process highlighted in this paper. That is, a single cilia of an inner ear hair cell *is* literally an analog sampling filter, operating as an ideal bandpass filter that resonates at a single frequency. In turn, the operation of an inner ear cilia can be “artificially” simulated using classic RC circuit components (i.e., a switching means and a memory means; McCullough, 2000a, 2000b, 2001), which pass a single frequency. There are only two control parameters of an analog sampling filter: sampling frequency and sampling time. The control parameters of an analog sampling processor are almost identical: sampling frequency, sampling time, and signal detection. The latter, signal detection, is useful to initiate feed-forward signal processing functions or initiate further signal processing feedback for control and correction in real-time (i.e., in the moment), hence enabling an analog sampling processor to electrically model the variables used in equation 2 (attention, salience, surprise). Given this example, we can see how circuits can conveniently model multiple levels of nervous system functioning, and tapping into these models can open up a whole field of potential models for psychological processes that is currently unexplored.

In general, *the nervous system filters input that is analog*, and there is no analog to digital conversion step. Therefore, electrical circuits might be a better analogy for the nervous system as opposed to deep neural networks. The filtered input of the nervous system is processed according to task demands that change in real-time according to psychological parameters. That is, tasks are selected by the organism in real-time to maintain psychological states such as safety, satiety, etc (Barrett & Quigley, 2021). In this sense, the human body *is* an analog sampling filter, which uses many analog sampling filters to sample (attend/switch) and hold (remember) data in real-time. Complex tasks in AI such as object recognition and speech comprehension are built on capacities of attention switching and memory. Furthermore, analog sampling filters are pervasive at many levels of biological systems, and even the neuron membrane itself is effectively modeled as an RC circuit (e.g., see Koester & Siegelbaum’s chapter in Kandel, Schwartz, & Jessell, 2013).

In summary, there is a possibility that artificial agents designed to continuously sample the environment would benefit from the efficiency inherent to analog sampling processors, as the nervous system itself benefits from this efficiency, described in the previous paragraph for the domain of hearing. The proposed framework offers a strategy for modeling the sampling process in terms of psychological variables. Most input can be ignored due to redundancy, so it is convenient to have an input that consists of tiny filtered fragments of signal (e.g., single

frequencies), that can be flexibly processed according to goals. Psychological parameters (e.g., attention) determine which aspects of sensory input are further processed, in order to effectively guide behavior while minimizing unnecessary or undesirable processing costs. As a higher level example, phenomena such as excessive worry or addiction can potentially be seen as such undesirable processing costs that can be minimized by effectively channeling the energy of attention in a more optimal and functional way. Understanding the circuits through which this energy flows can help us model these mechanisms in a new way. Considering these final points, along with the other principles proposed in this paper, circuit theory models of behavioral phenomenon could provide a promising framework for linking psychology variables to physiology and behavior, including simulating behaviors with artificial agents.

Conclusions.

Associative learning is a fundamental phenomenon taught in psychology textbooks. Nonetheless, the precise variables that belong in the associative learning equation (equation 1) remain an important open question for psychological science. By analogy, this paper suggests that the variables that contribute to associative learning have a lawful relationship (equation 2). This lawful relationship reflects how the law of conservation of energy generalizes to the psychological domain. This proposed lawful relationship can explain why prolonged attention to the same stimulus may lead to surprising insights, both in science and in spiritual practices. Lawful mathematical relationships between psychological variables may be important building blocks for AI applications that aim to model the processes of the human mind. The ability to model psychological variables with electrical circuit parameters may prove to be an important technique for simulating complex behavior. Future work will be required to explore additional analogies and applications of these concepts.

All behavior is part of a natural circuit. Stimulation comes in through breathing, eating, and the special senses, and the nervous system transforms the incoming signal into behavioral and physiological output, the status of which is always fed back to the system. Translating elements of this signal processing flow into terms of electrical circuits could be a promising modeling strategy for understanding psychological constructs and simulating them in artificial agents, if circuit parameters can be tied to psychological variables as demonstrated in the example here.

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The author declares no conflict of interest.

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