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Engaging Algebra Early through Manipulatives: reappraising Cuisinaire-Gattegno Rods

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Abstract The Cuisenaire-Gattegno (Cui) approach uses color coded rods of unit increment lengths embedded in a systematic curriculum designed to guide learners as young as age five from exploration of ratio through to formal algebraic writing. As the rods have had greater adoption as a teaching aide than the curriculum, we set out to investigate how fidelity to the seminal curriculum and pedagogy impacts learning via a meta-analysis and novel study of preparation for future learning. This meta-analysis of 23 studies (n=1968) revealed advantages of Cui over traditional arithmetic approaches (effect size = 0.55). Curriculum fidelity significantly predicted efficacy. Higher fidelity implementations were associated with large effects and lower fidelity resulted in small or null effects.To test how this curriculum prepares students for future learning, we carried out an 18-month longitudinal school-comparison study (n=114) executed to a similar fidelity level as the study with the largest treatment effect. Cui treatment accelerated learning rates measured during the school-year after treatment, and demonstrated transfer to novel tests of algebraic reasoning (effect size = 1.0). Tests of scholastic aptitude replicated aptitude by treatment interaction

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for both arithmetic and algebraic reasoning. While Cui provided significant learning benefits for children with higher aptitude, these benefits were significantly enhanced for children with lower aptitude. Together, these findings support the benefits of this approach, and further substantiate the importance of embedding these teaching aides within the theory-grounded curricula that gave rise to them.

Keywords preparation for future learning · aptitude-treatment interactions · arithmetic fluency · early algebra · NCTM pre-algebra · Cuisenaire–Gattegno · Cuisenaire rods

1 The reemergence of early algebra in school mathematics

Recent educational policy changes are shifting the focus of early mathematics education research from arithmetic (computation with numbers) towards algebra (computation with types) (Cai & Knuth, 2011; NCTM, 2000). Algebra encompasses the relationships between quantities, the use of notation, the modeling of phenomena, and the mathematical study of change. While the word algebra is not often heard in elementary school classrooms, the mathematical experiences and conversations of students in early grades frequently include elements of pattern recognition and algebraic reasoning. This raises the question of how and how well early algebra can serve as a preparation for future learning of algebraic reasoning. Narrative reports of small scale quasi-experiments suggest that even a limited exposure to these ideas can help children to out perform their peers when they take part in high stakes standardized tests, such as the Massachusetts Comprehensive Assessment System (MCAS) (Schliemann et al., 2007). MCAS provides an efficient opportunity to gather data on early algebra interventions (Kaput & Blanton, 2000). A recent longitudinal intervention study in Boston has shown that introducing algebra as part of the early mathematics curriculum is highly feasible. Specific representational tools – tables, graphs, numerical and algebraic notation, and certain natural language structures – can be employed to help students express functional relations among numbers and quantities and solve algebra problems (Carraher et al., 2008).

Large scale longitudinal studies of early algebra of the kind we report here are much less common. In this article we investigate a revival of interest in Cuisenaire-Gattegno (Cui) early algebra research. This was made possible by a window of opportunity created by new statutory curriculum adopted in the UK in 2014 (DfE, 2013). The national curriculum (NC) was one of the first in the world to mandate that all four arithmetic operations and fractions as operators with small numbers be studied together from Year 1 – an object-oriented algebraic structural approach. The NC designers recognized that the traditional practice of teaching arithmetic operations sequentially over several years was not working for a significant minority of learners. A few English schools were able to take advantage of this new freedom to innovate before recent government guidance silently rowed back from the new statutory mandate, to revert to a traditional symbol focussed concrete-pictoral-abstract progression (DfE, 2020).

We describe an efficacy study to explore how physical and diagrammatic set combination and mathematical writing interacts with domain general reasoning aptitude as a preparation for arithmetic proficiency. Cui educates learners' sensitivity to common patterns of mathematical relations by coordinating vision, audition, haptic, sensorimotor and introspective modalities through constructions with color-coded rods of unit increments. The integers are introduced as the names for a sequence of diagrams constructed by partitioning.

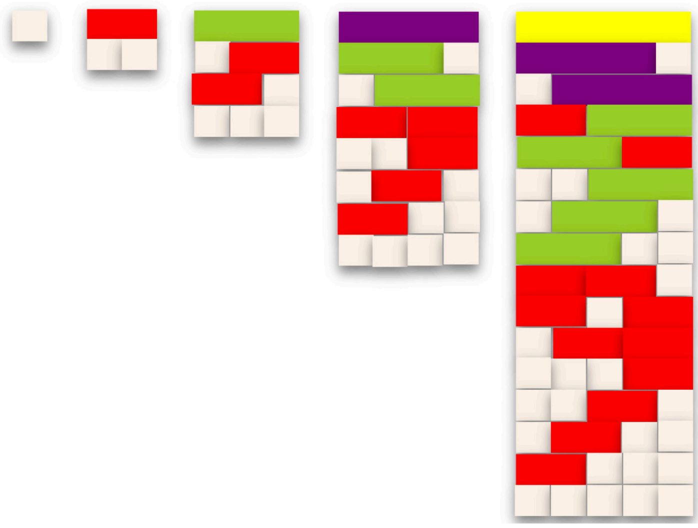


Fig. 1 *'Complete patterns' for small integers (Gattegno, 2010a, p. 80)*

This experience of number is enhanced by the use of mathematical vocabulary, symbols and notation. From the outset Gattegno introduces the concept of 'equivalence' as a generalisation of 'equivalent color' and 'equivalent length.' Each 'complete pattern' in the sequence of diagrams corresponds to an equivalence class of partitions of an integer (Figure 1). Other examples of equivalence are 'equivalent expressions' (such as 'w+r', 'r+w') and equivalent equations. Color codes and expressions are at the same time named integer values, computed by measuring the length of one rod by another, and recipes for colored rod constructions: '+' for example being the action of placing two rods end to end. Gattegno generalises the concepts of school algebra to encompass sensitivity to the dynamic that combines two objects of the same type ('w','r') to form a third of that type (a named rod construction) (Benson & Cane, 2017).

We based our experiment design on a three-year statistical Cui study conducted by William Brownell at UC Berkeley (Brownell, 1967b). He challenged the traditional criteria for measuring arithmetic understanding in terms of accuracy and speed only (Osburn, 1928). He argued that 'the element of method' is at least equally important (Brownell, 1928, p. 63). His motivation was "to make arithmetic less a challenge to the pupil's memory and more a challenge to his intelligence" (Brownell, 1935, p. 31). The success of the early adopters of the Cui approach in Scotland highlighted the limitations of traditional approaches to arithmetic teaching and led Brownell to note:

- The attention span of school beginners has been seriously under-estimated
- The ‘readiness’ of school beginners for systematic work in arithmetic has been seriously under-rated
- Children in the lower grades can learn more arithmetic than is being expected of them in the schools in the US (Brownell, 1960, p. 173-4).

The opinion that traditional approaches were not working was shared by many teachers, including the founders of the UK Association of Teachers of Mathematics (ATM) who set out to conduct research into the efficacy of Cuisenaire rods and other teaching aids. Brownell went on to lead the most comprehensive of these evaluations. He found that the use of the rods alone was problematic. What was needed was a rigorous curriculum and pedagogy. Only this would engage learners’ mathematical cognition by scaffolding and building mathematical understanding through the experience of rod constructions, the associated mathematical vocabulary and algebraic writing.

Our study investigated four questions motivated by these observations and conclusions. Firstly, we expected to find medium to large effects of Cui compared to a wide range of traditional approaches. Secondly, when Brownell investigated the early adopting Scottish schools he suggested that the treatment effect might be tightly coupled with the duration of the intervention and the consistency of its application. Thirdly, he presented evidence for an aptitude-treatment interaction in that the effects would be greater for children with lower scholastic aptitude. And finally, he proposed, but didn’t test, an hypothesis that Cui would be an excellent preparation for future learning.

In Section 2 we introduce the Cui program and Brownell’s research. We do this to highlight some of these important effects and to motivate both the meta-analysis and the present study which examines the long term transfer effects. In Section 3 we describe our meta-analysis of the literature on Cui effectiveness. We structure our analysis to test the hypothesis that it’s not just using the rods that leads to the significant effects, but fidelity to Gattegno’s curriculum and pedagogy. Section 4 reports the results of that analysis.

In Section 5 we discuss how we adapted Brownell’s post-test design to perform a replication-extension study with two English primary schools over two years. In seeking to replicate his attribute-treatment interaction we adopt a language neutral test of domain general reasoning in place of Brownell’s original verbal reasoning test of scholastic aptitude. Ours is pre-post-test but with non-experimental treatment and control. It is non-experimental in that we had a single school in each arm of the study which was not randomly selected. This is a common feature of the pre-post-tests in the meta-analysis. We describe how Brownell created a balanced quasi-experimental design, and how we adapted his approach in our statistical analysis. Finally, to test his future learning hypothesis, we look both at gains recorded during Cui training and gains recorded in the six months following the end of the intervention.

In the present study (N=120) students in treatment and business-as-usual control conditions completed assessments at regular intervals including Brownell’s original test of algebraic reasoning, together with novel tablet-based assessments of arithmetic fluency and relational reasoning. Six students missed more than one of the four observations, leaving a remaining sample of (n=114) participants. We measure two different aspects for each child: Aptitude and Growth in factual fluency. The early impact of the treatment was well established in meta-analysis. This investigation focuses on the year of growth following the nine-month introduction of Cui in UK Year 1 (aged 5 on entry). We measure factual fluency at four growth points ($g1..g4$) and study performance in algebraic reasoning in the final semester of Year 2.

An online resource accompanying this article documents the 37 studies from which the meta-analysis is drawn and the core 80-unit lesson sequence on which we based our treatment in Year 1 and the first term of Year 2. Our findings are reported in Section 6. Section 7 discusses the contribution of this work, its limitations and next steps.

2 Evaluating the Cuisenaire-Gattegno approach

Cuisenaire rods are cuboids, the length of each a multiple of the length of the smallest - a 1cm white cube. Each student has a box containing sufficient rods of different sizes to construct all the partitions of the smaller rods (Figure 1). Rods of the same size have the same color.

Gattegno was concerned to make teachers and pupils aware of the dynamic which transforms rod constructions, diagrams, written expressions and equations into equivalent forms. He contrasted this ‘algebraic understanding’ of the nature of number systems with traditional symbol manipulation in school algebra and with drill-based factual fluency. He uses operations with the rods – placing them end to end, side by side or stacked as towers – to model sets with structure such as the integer and rational number systems. In the Cui approach “all the operations with integers and fractions can be studied simultaneously (with colored rods); whole numbers being recognised as the equivalence class of their partitions and fractions as ordered pairs, one serving to measure the other, or as operators belonging to classes of equivalence which are the rational numbers involved in the operations” (Fedon, 1966, p. 201). Gattegno demonstrated that “Children of six or seven are thoroughly familiar with their tables, children of five conceive and compare fractions easily and accurately, children of eight solve simultaneous equations and at 10 they understand permutations and combinations which they themselves form and analyse” (Gattegno, 1956, p. 88).

The Cui programme has four distinctive characteristics. Firstly, it consists of a suite of exercises with permutations of rods that encourage the learner to pay attention to the relationship between quantities (Cuisenaire & Gattegno, 1962). These exercises give rise to a substantial experience with integers and rational numbers. Secondly, the exercises are organised in a directed graph of mathematical concepts and their inter-dependencies. The graph introduces learners from the outset to concepts such as equivalence, set, function and domain (Gattegno, 2010a, p. 193). Thirdly, from the outset the approach encourages written expressions and equations in all four arithmetic operations and fractions as operators – initially for computation with types and subsequently for computation with small numbers. Gattegno called this sequence ‘algebra first’ in contrast with traditional ‘counting first’ school mathematics. These writings gives rise to an appreciation (‘awareness’) of cryptomorphisms between the rod world and mathematics (Benson, 2011). Fourthly, the ‘subordination of teaching to learning’: a theory of learning based on conscious (or unconscious) awareness as the unit of study (Gattegno, 1987, 2010b; ATM, 2018). Young & Messum (2011) have reviewed this model of human learning and shown how it can be applied both inside and outside the classroom. Dehaene (2020) offers neurophysiological and computational evidence for this theory.

Gattegno’s work caught the attention of William Brownell, a pioneer of educational research and sometime president of the American Educational Research Association (Kilpatrick & Weaver, 1977). Brownell believed that “Children differ markedly in the ways in which they think of numbers and in the ways in which they learn number facts. No adequate measurement of degrees of development can be made, therefore, unless the measures of speed and accuracy are supplemented by a measure of the maturity of the processes employed in dealing with numbers” (Brownell, 1928). As Dean Emeritus of the Berkeley School of Education Brownell undertook several large scale quantitative and qualitative studies of Cui (Brownell, 1967a,b).

Our work drew on Brownell (1967b), an unusual design for this kind of evaluative research and one of the larger longitudinal studies. We will describe the study in some detail as it was the most comprehensive study to date. It was conducted in Scotland and California. Brownell administered pen and pencil tests to (n = 1109) learners who remained in the program after three years of schooling - at the end of Scottish Primary III. It was a post-test-only control quasi-experiment classified as design type 6 by Campbell & Stanley (1963). Brownell recruited classrooms from 24 schools. Half of the classes had followed a pure Cuisenaire-Gattegno course of study, and half the traditional ‘counting first’ curriculum. Teaching intensity averaged between 33 and 67 minutes per day. Accordingly Brownell divided his data into longer and shorter durations of study. Brownell assessed children’s domain general cognitive skills that fall outside mathematics via a standardized verbal reasoning test, although he conceptualized this scholastic aptitude as ‘IQ’ (sic) at the time. This test was administered at the end of the 3 years (Brownell, 1967b, p.8,37).

Learners were selected at random from each group, matched by age, gender and verbal reasoning skills. High and Low scholastic aptitude subjects were determined by removing the middle 20% from the verbal reasoning distribution. This resulted in a smaller sample of 405 X and 453 C. The data was then divided into eight cells based on treatment (X, C), scholastic aptitude (Hi, Lo) and teaching intensity (high, low). Teaching in the range 31-34 minutes per day was classified as low intensity, and the range 47-64 minutes per day was taken as high intensity (Brownell, 1967b, p.48). From these eight cells, one cell would have been identified as having the smallest sample which in this case was 38. For statistical inference testing, it is desirable to have equal sample sizes in each cell. The reason why Brownell does this is to eliminate unwanted correlations between the additional variables e.g. scholastic aptitude and intensity of teaching. By doing this, he ended up mimicking a balanced experimental design which in an ideal world would have been achieved before the tests were administered. Obviously in this case it was not practical since children are allocated to schools by their parents and local authorities and not by Brownell. To achieve a balanced design Brownell removed samples from the other seven cells at random until he had 38 pupils in each cell. His final sample size was 304. This meant 1003 of the original 1337 population were excluded. By setting aside data in this way Brownell introduced a potential risk that the excluded pupils might have given different results.

He tested material covered in both courses of study (the Common test), and content covered in only one of them (the CUI and TRA tests). Brownell used an ANOVA test to confirm that the differences and interactions between effects were significant. High teaching intensity studies showed evidence of a treatment effect in all three tests. The interactions between treatment and scholastic aptitude in all three tests were statistically significant. Referring to the aptitude-treatment interaction Brownell wrote that "it is reasonable to suggest that children identified as low in intelligence and exposed to a relatively long period of instruction in arithmetic will gain more through involvement in the Cui program" (Brownell, 1967b). In the case of the CUI test it is children who scored highest on his scholastic aptitude task who gained the most.

3 Meta-analysis

Observational studies of early adopters of Cui were generally positive and in British Columbia a Royal Commission recommended a large-scale study with a view to integrating the method into elementary teacher training programmes (Howard, 1957; Ellis, 1964). Such findings encouraged researchers to compare the Cui vs Conventional approach. Robinson cites 50 qualitative comparisons employing 15,000 students over several grade levels. He writes, "One could say that research reported to date has compared the effects of some 20,000 student years of Cuisenaire exposure to the effects of the equivalent amount of 'traditional' instruction" (Robinson, 1964).

Our meta-analysis consisted of 13 experiments which gave rise to 23 studies. To investigate the effect of fidelity to Cui we created a weighted ranking of the experiments, according to dimensions of fidelity suggested by Brownell.

3.1 Methods

We identified 37 studies in the literature that examined the impact of Cuisenaire rods on arithmetic development in children including those which reported a metric for arithmetic understanding. These tests quantify performance with arithmetic operations. They range from evaluating simple addition and subtraction expressions to missing number sentences to working with fractions. We looked for tests that could inform our research with the Woodcock-Johnson Maths Fluency subscale, a metric widely used in cognitive, educational and neuro-imaging studies. Details of search parameters are documented in the accompanying online resource. We excluded four foreign language dissertations that did not have an English translation (they reported a direction of effect Cui $>$ Control), observational studies and studies where the control did not follow a traditional curriculum. Our analysis required reported means and standard deviation or sufficient statistical detail to allow us to impute these values. One dissertation was excluded as it did not report means.

After systematic application of these inclusion criteria, thirteen studies were deemed to pass all the above criteria. Several of these contained more than one comparison between control and treatment conditions appropriate for inclusion in the meta analysis, such as when results were reported separately for males and females and by grade. They gave rise to 23 post-test contrasts at grade and gender level and 8 pre-post contrasts, each contrast representing an independent and distinct population of students. In each study we selected an outcome measure that best captured the construct of arithmetic fluency and best approximated the Woodcock-Johnson Math Fluency subscale. Five studies reported the Metropolitan Readiness or Achievement Test, two studies the Science Research Associates Arithmetic test and other studies measured proficiency with fractions and missing number sentences (See Table 1). Brownell reported his results at a test item level. We used the items below to construct a measure of arithmetic proficiency from his Common test missing number sentences that we could compare with our study and the other studies in our meta-analysis (Brownell, 1967b, p. 84 & Appendix):

$$
\begin{aligned}
2 \times \square &= 12 \\
12 - \square &= 7 \\
6 + \square &= 14 \\
9 - \square &= 0 \\
\square + 7 &= 10 \\
\square - 5 &= 7 \\
\square \div 2 &= 7 \\
\square + 8 &= 8
\end{aligned}
$$

Studies can be distinguished by the experience of teachers with the Cui approach, the number of final sample subjects (n), grade level, gender, duration (teaching days, assuming a 180-day school year), design (Experiment (EX), Quasi-experiment (QEX) or Observational (OB)), pre-test, post-test, within and between subjects analysis, statistical tools and fidelity to the Cui and to the traditional approach.

Effect sizes were computed directly from the means and standard deviation values obtained from the manuscripts without regard for statistical significance reported in the source materials. For example, in one case Haynes (1963) a contrast originally reported as a null result appears in Table 1 as a small effect.

3.2 Quantifying fidelity to central Cui scholarship, curriculum and pedagogy

The Cui approach was transmitted to the world through specific artifacts: an original curriculum and text books intended for children, scholarly books and papers, secondary literature that related Cui to main currents of mathematics education research and accounts of adoption. We explored an hypothesis that transmission became less effective the further a study drifted away from these benchmarks and that this might

Table 1 *Experiments included in the post-test meta-analysis ranked in order of fidelity (Peer reviewed findings are marked *)*

Study	n	Grade	Days	Effect(d) C.I.	Metric
*Brownell (1967b)	304	3	540	1.66 (1.40, 1.92)	Missing number sentences
Wallace (1974)	154	4–6	15	0.99 (.66, 1.33)	Area model for fractions (Wallace, 1974, p. 85-9)
Steiner (1964)	102	4	180	0.53 (.12, .93)	Metropolitan Achievement Test, Arithmetic Computation
Aurich (1963)	90	1	180	1.38 (.92, 1.84)	Science Research Associates Arithmetic
Robinson (1978)	119	3, 4	5	0.10 (−.29, .48)	Decimal fractions (Robinson, 1978, p. 95-114)
Haynes (1963)	63	3	30	0.37 (−.16, .90)	Metropolitan Achievement Test, Arithmetic Computation
Crowder (1965)	425	1	143	0.25 (.06, .45)	Science Research Associates Arithmetic
Egan (1990)	81	2	180	−0.30 (−.74, .14)	Missouri Mastery Achievement Test (Mathematics)
Dairy (1969)	53	K	540	0.85 (.29, 1.42)	Metropolitan Readiness Test
*Nasca (1966)	45	2	180	−0.09 (−.68, .49)	Metropolitan Achievement Test, Mathematics
Romero (1977)	240	1–6	160	0.44 (.19, .70)	Metropolitan Achievement Test, Mathematics
Keagle & Brummett (1993)	38	4	4	−0.56 (−1.12, .09)	Custom Fraction Test
*Lucow (1962)	254	3	30	0.65 (.40, .90)	Growth in × and ÷

account for a significant element of the heterogeneity in the true effects/outcomes in the meta-analysis.

We quantified these aspects of the studies in four dimensions: the curriculum experienced by the learner ($rank_{learn}$), the teacher's experience with Cui ($rank_{teach}$), the teachers' Cui training ($rank_{train}$) and the preparation of the research team ($rank_{research}$). The 13 studies were compared against one another in each dimension and ranked in order from most (1) to least (13) faithful. In the event that all 13 studies were distinctive they would be ranked 1 to 13. In other dimensions there were fewer distinctions: some rankings were duplicated or not assigned.

The relative weights for these dimensions were chosen to reflect Brownell's account of his studies (Brownell, 1967a, p. 14). We gave the highest weighting (4) to the curriculum and pedagogy as this is what the learners experience moment by moment. Then we weigh teacher experience (3) and preparation to deliver the curriculum with fidelity (2) and finally we weigh the evidence of researcher awareness of the debate on 'number first' versus 'algebra first' progression (1). The overall metric for fidelity for a study was computed with the formula

$$fidelity = 4 * rank_{learn} + 3 * rank_{teach} + 2 * rank_{train} + rank_{research}$$

In the learn dimension the highest ranking was given to reports that exhibited evidence that they used Gattegno's curriculum in the classroom. Credit was given if the study reproduced a précis of the Cuisenaire-Gattegno approach and cited the seminal text-books for pupils (Gattegno, 1957, 1963a). Brownell, for example, devoted seven pages to a description of 'computation in the Cuisenaire program' written by the teacher who coordinated teacher training for his study (Brownell, 1967a, p. 14). The lowest ranking studies have only a rudimentary account of Cui. They do not cite the seminal books.

In the teaching experience dimension the highest rankings were given to studies that reported more than one year's prior teaching experience with the approach.

In the teacher training dimension we looked for citations of Gattegno's seminal teacher training books and his writing on educational research. These influential works are listed in the online supplement. This was taken to be evidence of the quality of teacher training.

In the research dimension we assessed the preparation of the research team by examining the extent to which the study's bibliography and discussion sections covered related literature on early algebra and manipulatives.

3.3 Statistical Analysis

Meta-analysis was performed using the open-source statistical software package R, and employing the `metafor` package. Analyses were carried out using the standardized mean difference (effect size) as the outcome measure. A random-effects model was fitted to the data. The amount of heterogeneity (i.e., τ^2), was estimated using the restricted maximum-likelihood estimator (Viechtbauer, 2005). In addition to the estimate of τ^2, the Q-test for heterogeneity Cochran (1954) and the I^2 statistic are reported (Higgins & Thompson, 2002). In case some amount of heterogeneity is detected (i.e.,$\tau^2 > 0$, regardless of the results of the Q-test), a prediction interval for the true outcomes is also provided and shown at the bottom of the forest plot. It is centred at the summary estimate, and its width accounts for the uncertainty of the summary estimate, the estimate of between study standard deviation in the true treatment effects (τ), and the uncertainty in the between study standard deviation estimate itself. It indicates the possible treatment effect in an individual setting (Riley et al., 2011). Studentized residuals and Cook's distances are used to examine whether studies may be outliers and/or influential in the context of the model (Viechtbauer & Cheung, 2010). Studies with a studentized residual larger than the $100 \times (1 - 0.05/(2 \times k))$ th percentile of a standard normal distribution are considered potential outliers (i.e., using a Bonferroni correction with two-sided $\alpha = 0.05$ for k studies included in the meta-analysis). Studies with a Cook's distance larger than the median plus six times the interquartile range of the Cook's distances are considered to be influential. The rank correlation test Begg & Mazumdar (1994) and the regression test Sterne & Eggar (2005), using the standard error of the observed outcomes as predictor, are used to check for funnel plot asymmetry.

4 Results of the Meta-analysis

The analysis was carried out using R (version 4.0.4) (R Core Team, 2020) and the metafor package (version 2.5.82) (Viechtbauer, 2010). Analysis was carried out using two different approaches: a random effects model for three analysis of arithmetic proficiency (k=8, 13, 23), and a mixed effects model for the analysis of fidelity as a moderator (k=13). Several of the 13 studies in Table 1 presented results from two or more independent samples (each with a control group) that received the same intervention. They were coded as distinct assessments in our analysis, giving an assessment count of k=23 (n=1968) for the post-test meta-analysis and k=8 (n=465) for the pre-post meta-analysis.

Metafor takes pooled standard deviation from the samples at T1 and T2. This assumes that the subjects are different at the two time points - which they are not in general. As a result the pooled standard deviation is an overestimate and the effect size is an underestimate.

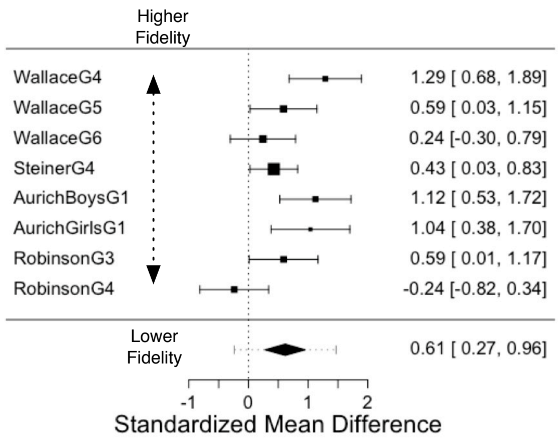


Fig. 2 *Pre-post-test effect size (d), Confidence Intervals (C.I.) for the influence of Cui on arithmetic proficiency outcomes by order of fidelity. Prediction interval and summary 'diamond' for C.I. for estimate.*

In the first k=13 analysis we used a single measure per study as shown in Table 1. The weighted average effect size was d = 0.5 (95% C.I. 0.16, 0.84) with the majority of estimates being positive (77%). Therefore, the average outcome differed significantly from zero ($z = 2.8969$, $p = 0.0038$). Cohen suggested that d = 0.2 be considered a 'small' effect size, 0.5 represents a 'medium' effect size and 0.8 a 'large' effect size (Cohen, 1988). That is, if two groups' means do not differ by 0.2 standard deviations or more, the difference is trivial, even if it is statistically significant. We analysed subgroups of studies according to the measure chosen. For the nine independent studies using the Metropolitan Achievement Test (n = 450) there was a small effect size of 0.34 (95% C.I. 0.10, 0.59) and for the 3 Science Research Associates arithmetic tests (n = 515) there was a large effect size 0.94 (95% C.I. 0.16, 1.72).

We calculated the prediction interval for the k=13 analysis (-0.70, 1.71) with the metafor `predict` function. This indicates that the average effect does not tell us much

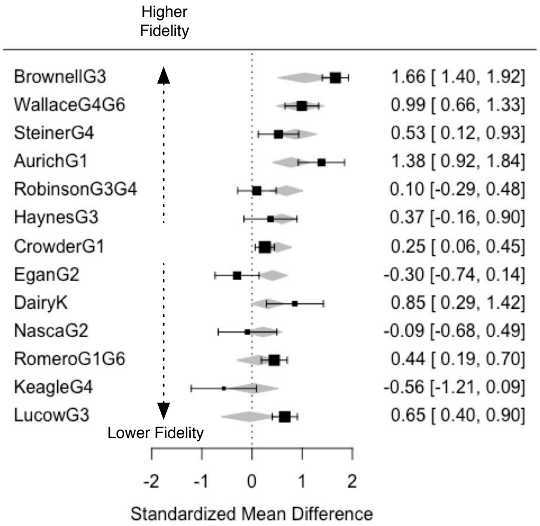


Fig. 3 *Post-test effect size (d) showing predicted (diamond) and observed (bar) proficiency outcome effect sizes by experiment in order of fidelity*

about what happens in any particular study as there is a great deal of heterogeneity, that is between study variance. In section 4.1 we explore how we might account for this variation.

The weighted effect size for the k=23 post-test experiments was d = 0.55 (experimental sample size nX = 1040, control sample nC = 928). The Confidence interval was (0.3, 0.8) and prediction interval (-0.56, 1.66) The pre-post meta-analysis is shown in Figure 2. These assessments used the same metrics as those in Table 1. The forest plot shows the prediction interval of (-0.24, 1.47) and a weighted effect size of d = 0.61 (nX = 244, nC = 221) with a summary 'diamond' at the bottom of the plot. The center of the polygon corresponding to the estimate and the left/right edges indicating the confidence interval limits. Figure 3 shows the observed outcome effects for the 13 studies in Table 1. The three random effects models confirm that our findings are broadly robust to treating each study as one observation rather than treating independent samples within each study as separate assessments.

4.1 Assessing the effect of Fidelity

We built a mixed effects model to study the extent to which arithmetic proficiency was influenced by fidelity to the Cui approach. The 13 experiments were ordered within each dimension by an external adjudicator. A weighted average ranking was calculated for each experiment and the results entered as a moderator in the meta-analysis.

Figure 3 shows the observed proficiency outcomes and a prediction based on the mixed effects model by experiment in order of fidelity. The grey diamonds show the predicted effects and their CI limits. The model shows that when fidelity changes by 1 on the 1 to 8 scale we used, the estimated effect size decreases by 0.19. The effect size for fidelity 1 was 1.2 which reduced to effect size -0.06 for fidelity 8. We checked to see if the effect of fidelity was non-linear but the model showed no sign of that and so our final model assumes the effect of fidelity is linear.

According to the Q-test, the true outcomes appear to be heterogeneous ($Q(12) = 135.7691, p< 0.0001, \tau^2 = 0.3461, I^2 = 91.8758\%$). A 95% prediction interval for the true outcomes is given by -0.6990 to 1.7054. Hence, although the average outcome is estimated to be positive, in some studies the true outcome may in fact be negative.

An examination of the studentized residuals revealed that none of the studies had a value larger than 2.8905 and hence there was no indication of outliers in the context of this model. According to the Cook's distances, none of the studies could be considered to be overly influential. Neither the rank correlation nor the regression test indicated any funnel plot asymmetry ($p = 0.6754$ and $p = 0.1617$, respectively).

A statistically significant relationship between treatment effect size and the rank order of fidelity to Gattegno's curriculum/pedagogy was revealed by a QM test of moderators (Q_M(df = 1) = 5.8416, $p = 0.0157$) (Viechtbauer, 2021)). As evident in 3 studies with the highest fidelity rankings produced effect sizes > 1, while effects fell off systematically as evidence of fidelity to the original work waned. In fact, rank order of fidelity to the seminal work accounted for 32% of the heterogeneity of outcomes (R^2).

5 Intervention study with longitudinal follow-up

In drawing on the design of the Berkeley study we asked whether Brownell's findings might be reproduced over the first four terms of schooling, and if so whether they persisted over the subsequent two terms in which Cui was replaced by traditional instruction directed towards proficiency in national assessments. We used a battery of academic performance tests drawn from the Stanford Educational Assessment (SEA) collection, in place of the Common test and the standard verbal reasoning test (Project-ILead, 2019). We also used a variant of Browell's CUI test. The SEA tests were administered as a set of five-minute modules on an iPad.

SEA observations were taken at the end of UK Year 1 (at the end of the third term of schooling, growth point $g1$) and the end of each term of Year 2 ($g2-g4$). Our CUI test was administered at the end of the second year. Initially 60 participants were selected from a population of 90 children in each of two schools. Not all children were present at each observation and we imputed missing data when one observation was missing. The resulting data set for the SEA scholastic aptitude and factual fluency analysis consisted of 56 experimental pupils and 58 control, and for the algebraic reasoning (CUI) analysis consisted of 52 experimental and 56 control.

Table 2 Study Design (□ indicates no observation)

Phase Number	Phase Name	Growth Interval	X assignment	C assignment	Duration
1	Initiation	(□,g1)	Cui training	Traditional	9 months
2	Reflection	$(g1, g2)$	Cui training	Traditional	3 months
3	Proficiency	$(g2, g3)$	Some small programs	Traditional	3 months
4	Follow-up	$(g3, g4)$	Traditional	Traditional	3 months

We adopted a between-schools design as used in studies reported in the meta-analysis. As we have discussed Brownell took account of a non-experimental treatment and control by matching students to construct a balanced design. In the present study demographic and school quality data suggest that any bias in our findings would be in favor of the control school. As a check on this bias we matched subjects in the treated (X) and control (C) samples according to their RM results at $g4$ using the R `MatchIt` package.

In the experimental school pupils closely followed Gattegno's Mathematics textbooks (GM) for four terms (Gattegno, 1963a,b). Lessons were designed by a teacher with 3 years experience with GM. She worked alongside a newly qualified teacher who taught the parallel second class of 30 children. The study was divided into four phases as shown in Table 2. The intensity of the teaching averaged 40-50 minutes per day for 12 months. This was similar to the average intensity achieved in Brownell's study (45 minutes/day).

Teaching in England is moderated in Year 2 by an external review against expectations for children's written work. In addition, schools are assessed between $g3$ and $g4$ by two nationally standardized tests of arithmetic and mathematical reasoning (the Key Stage 1 assessments). For these reasons teaching in the Proficiency Phase in the experimental school moved away from GM. It was replaced by practising for these proficiency tests and the external review, and with lessons to gain familiarity with conventional diagram notations (such as the abstract number line). The Cui approach was augmented with traditional resources and with computer science lessons.

Several factors distinguished the intervention in Phases 1 and 2 from traditional classrooms:

1. the promotion of well-designed mathematical exercises that empower students to reason about equivalence and generalise with little or no mental energy
2. the challenging, revising and changing of these generalisations by the students making them
3. the roles of listening, careful attention to the use of language and even silence in learning and teaching
4. the role of free writing. This allows the student to discover regularities for themselves and gives the teacher the opportunity to see which concepts they have mastered, and elsewhere, where the student is still working at an 'empirical' level (Goutard, 2017).

5.1 Demographics and School Quality

We selected schools for the present study who were rated ‘good’ by the Office for Standards in Education (Ofsted). Within this classification schools differ in age profile, levels of deprivation and socio-economic context. The matching between the schools was less than ideal. At $g3$ the mean age of the pupils in the experimental school was 6.1 years (0.258 sd) and in the control school it was 6.23 years (0.258 sd). They also differed in socio-economic context and school quality measures.

In England state schools recruit from their local neighborhood or ‘catchment’ area. For popular schools, such as the schools in this study, these areas can be quite small. This means that we were able to use demographic data collected for the neighborhood as a proxy for parental socio-economic status. The UK government classifies school catchment areas into bands of relative deprivation, using information on income level and the educational level of parents. Using the measure for parental education and skills the experimental school catchment was allocated to the fifth of ten intervals and the control school to the least deprived, tenth, interval. On the income measure the experimental school was allocated to the ninth and the control school to the tenth interval. On the overall multiple deprivation index, taking into account employment, income, health etc. the experimental school catchment area was assigned to the ninth interval and the control to least deprived tenth interval.

The results of the Key Stage 2 (KS2) national assessments can also be taken as a measure of school quality. Four statistics are of note.

1. Overall KS2 Maths attainment was 110 Control (C), 106 Experiment (X). 100 is the national average.
2. The % of disadvantaged children taking the test was 4% C 3% X. There is reason to believe that the X figure is understated.
3. The measure of maths progress - a value added measure - was 2.2 (C.I. 0.8, 3.7) for C and -0.5 (C.I. -1.9, 0.9) for X. This implies that the C school was in a national cohort four years earlier whose KS1 math attainment was 107.8, whereas the X school at that time was 106.5.
4. The proportion of learners with a high score in reading was 46% C, 33% X
5. The percentage of learners with medium prior attainment achieving the expected level in maths was 98% C, 75% X.

It might be expected therefore that any bias arising from parental educational level, socio-economic status, age or school quality would be in favor of our control school.

5.2 Growth Point Assessments

We used SEA modules for mathematics and reasoning to collect statistics on accuracy and speed of work done. These measures are summarized in Table 3. A number of other observations were also made but are not reported further since they are outside the scope of the present study.

Table 3 *Assessment tests and measures*

Test	Definition
RM	Relational matching. Subjects are asked to indicate whether two pairs of objects which can differ in 2 properties are alike in the same way. There are 23 trials. Measure is total accuracy.
CUI	A 5 minute paper and pencil post test derived from Brownell (1967b). The subject evaluates ten expressions in all four operations and fractions as operators. Measure is total accuracy.
AF	Arithmetic Fluency. A 3 minute test evaluating as many expressions as possible involving either single digit addition summing to 9 or subtraction of a single digit from a number less than or equal to 10. Measures are trial accuracy and response time.

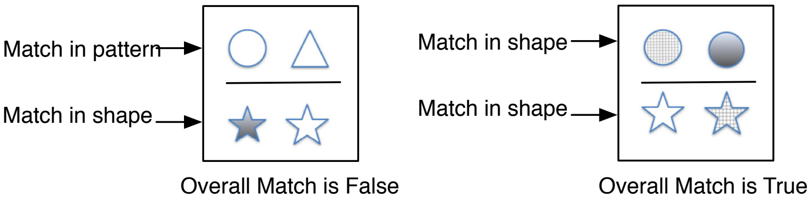


Fig. 4 Relational Matching (RM) logical reasoning test

The AF and RM modules were administered at each growth point. At $g4$ the module CUI was also assessed. Brownell found it was important to test for differences in general scholastic aptitude outside the domain of mathematics and used a verbal reasoning test. Meta-analyses of the impact of manipulatives on mathematics learning expect that older students who have developed the ability to reason abstractly will benefit most from instruction that consists exclusively of symbolic representations (Carbonneau et al., 2013). Increasingly executive function tests such as working memory or task switching and tests of global fluid intelligence are being used to assess domain-independent reasoning skills. Our choice of Relational Matching (RM) is motivated by executive function approaches as a non-verbal reasoning test for scholastic aptitude that can capture the aspect of general reasoning while de-emphasizing potential language differences between subjects. RM is designed to test similar operations to tests in the Raven's Progressive Matrix family. The more complicated Raven's tests, closer to problems found in standard scholastic aptitude tests, require analysis as well as visuo-spatial skills. SEA RM is such a logic test. It requires two steps of analysis (i) to determine how a pair of objects which are characterised by two properties (shape and fill pattern) differ from one another and (ii) to distinguish whether two such pairs differ in the same way ('True') or not ('False') (Figure 4). There were 23 trials at each time point. A high score is indicative of a good performance.

CUI was a 15-minute pen and pencil test in two parts. During the first 10 minute practice session the invigilator led the class in a discussion about how they might approach 20 questions drawn from Brownell's CUI test protocol (Brownell, 1967b, p. 250). Attention was drawn to the multi-step nature of these calculations. The control school was not familiar with this kind of questions. The discussion was a way to

Table 4 Summary Statistics and C.I. for Difference in Means (C-X) AF

Growth Point (GP)	X sample (n=56) Mean	X sample (n=56) SD	C sample (n=58) Mean	C sample (n=58) SD	99.5% C-X Confidence	Interval
g1	20.2	8.63	21.7	8.48	-1.68	4.68
g2	25.1	8.44	26.8	8.35	-1.42	4.82
g3	31.4	7.90	30.7	9.44	-3.94	2.54
g4	32.9	9.42	32.1	8.62	-4.15	2.55

Table 5 Summary Statistics and C.I. for Difference in Means (C-X) RM
Significance codes 0‘***’ 0.001‘**’ 0.01‘*’ 0.05‘.’ 0.1‘ ’

Growth Point (GP)	X sample (n=56) Mean	X sample (n=56) SD	C sample (n=58) Mean	C sample (n=58) SD	99.5% C-X Confidence	Interval	
g1	12.2	3.24	13.1	3.68	-0.39	2.19	
g2	13.3	2.94	14.1	3.59	-0.42	2.02	
g3	12.5	2.97	14.9	3.76	1.14	3.66	***
g4	14.4	4.17	16.8	3.80	0.92	3.88	**

familiarize them with what was required, while it refreshed the experimental subjects' experience with these questions. 10 further questions from Brownell's CUI test were posed in written form as set out in Figure 7 and completed in silence. In this way CUI assessed the degree to which traditional learning in the control school prepares pupils for algebraic reasoning. For the experimental school, whose recent experience was with single step questions as required by the national assessments, CUI assessed the extent to which they had retained and could build on what they had learnt in Phases 1 and 2.

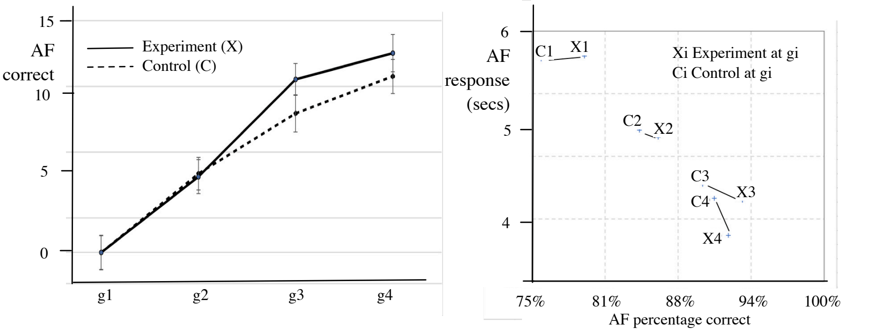
6 Results of the longitudinal study

We conducted an analysis to compare the arithmetic fluency accuracy and response time and relational matching accuracy of the two groups. We found as expected that there was a correlation between relational matching skill and math performance. However when this was controlled for in a regression model it could not account for all the gains in performance in arithmetic fluency achieved by the experimental group.

We matched the subjects in the two samples according to their RM measure and found that there was an interaction between treatment and domain general reasoning skill.

6.1 Accuracy and Response Time

Tables 4-5 show the difference of the means for AF and RM observations between the X and C samples for each term together with the 99.5% confidence intervals that there is a significant difference between the means. It shows that there is a significant



(A) AF mean accuracy gains by growth point (B) AF mean response time (secs) by proportion correct

Fig. 5 SEA AF mean accuracy gains, mean proportion correct and mean response time by growth point

C-X difference between the mean RM scores at $g3$ and $g4$. Otherwise the results are comparable.

Figure 5(A) shows the mean increment in AF accuracy for the experimental and the control groups by term over the course of the study. Figure 5(B) shows the relationship between the response time and proportion correct. In general the faster response is related to increased accuracy, but there appears to be an upper limit to this process beyond which increased speed impairs accuracy.

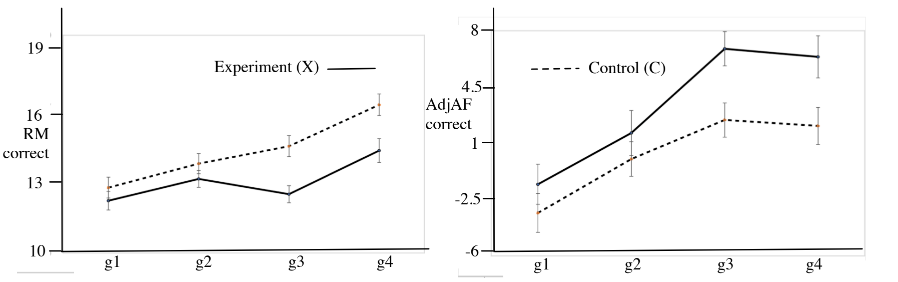
Figure 5 shows the intervention ($g1$, $g2$) prepares pupils for future learning, in that gains in speed and accuracy relative to the control group appear in Phase 3 ($g2$,$g3$) and the Follow-up Phase ($g3$,$g4$), after the Cui intervention has ended.

6.2 Scholastic aptitude

Figure 6(A) shows the mean RM accuracy for the experimental and the control groups by growth point. There is a sustained gap in relational matching skill in favor of the C sample as measured by this executive function test.

6.3 Growth in efficiency of learning

We propose a measure $adjAF$ (adjusted arithmetic fluency) of the efficiency of learning. This quantifies the extent to which the AF accuracy for a subject exceeds a prediction based on their RM measure at each growth point. It is calculated by first averaging AF and RM over all four growth points (GP) for each subject. We then built a linear regression model $avAF \sim avRM$ and extracted the intercept (8.0323) and slope (1.4059). These coefficients were used to predict a value for AF for each subject at each growth point using the formula $predAF = 8.0323 + 1.4059 * RM$. The adjusted AF accuracy was computed as $adjAF = (actual)AF - predAF$. We then investigated the model $adjAF \sim Treatment * GP$ and found that Treatment gave rise to statistically significant excess in AF over that predicted from RM alone. The ANOVA summary is shown in Table 6.



(A) Relational Matching (RM) mean accuracy

(B) Mean correct AF attempts exceeding prediction based on RM (AdjAF)

Fig. 6 RM and efficiency of learning (adjAF) by sample and growth point with Standard Error bars

Table 6 ANOVA for linear model for adjAF predicted by Treatment and Growth Point (GP) n=114. Significance codes 0‘***’ 0.001‘**’ 0.01‘*’ 0.05‘.’ 0.1‘ ’1

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
Treatment	1	404	403.51	5.6992	0.01739	*
GP	3	5076	1691.88	23.8962	2.311e-14	***
Treatment:GP	3	608	202.75	2.8636	0.03642	*
Residuals	448	31719	70.80			

Figure 6(B) shows the increment in mean adjAF accuracy for the experimental and the control groups by growth point. The acceleration in adjAF between $g2$ and $g4$ for the X sample in comparison with the C sample is consistent with a delayed ‘sleeper effect’ from the intervention, which ceased at $g2$. AdjAF has a Cohen’s d at $g4$ of 0.497, a medium effect size.

6.4 Algebraic reasoning

Figure 7 shows percentage cumulative accuracy and attempt scores for the CUI test questions. The students with experience of Cui were more willing to engage with challenging problems instead of skipping them. The mean CUI score for the X sample was 3.52 (sd 2.37) and for the C sample was 2.57 (sd 2.61).We conducted a Welch two-sample t-test comparing these statistics which confirmed that the difference between the means was likely to be significant. The Cohen’s d effect size was 1, which is a large effect size.

We asked how CUI performance in algebraic reasoning was predicted by the growth in AF accuracy. We found that using $g1$ and $g3$ was best with the model generating similar coefficients. The ANOVA is shown in Table 7. It shows that growth at $g1$ is significant at the $p<0.001$ level and growth at $g3$ at the $p<0.05$ level.

We interpreted the coefficients as +0.15 for $adjAF1$ and +0.08 for $(adjAF3 - adjAF1)$. This suggested that where a subject ended Year 1 is the main effect but subsequent growth (after allowing for RM) between $g1$ and $g3$ was also predictive of

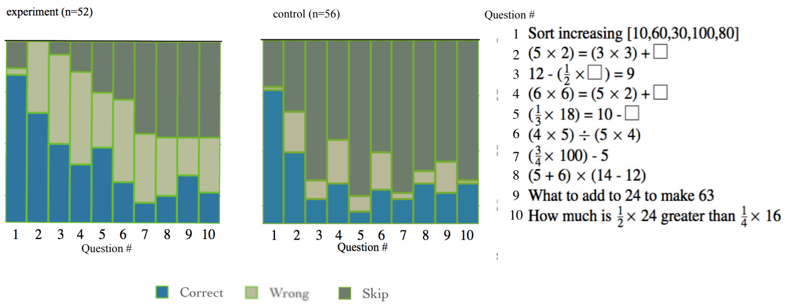


Fig. 7 Percentage cumulative accuracy and attempt data for CUI test by Question ($g4$)

Table 7 ANOVA for linear model for CUI predicted by Adjusted AF showing relationship between treatment effect on AF at $g1$ and $g3$ and performance in the CUI test at $g4$. Significance codes 0'***' 0.001'**' 0.01'*' 0.05'.' 0.1' '1

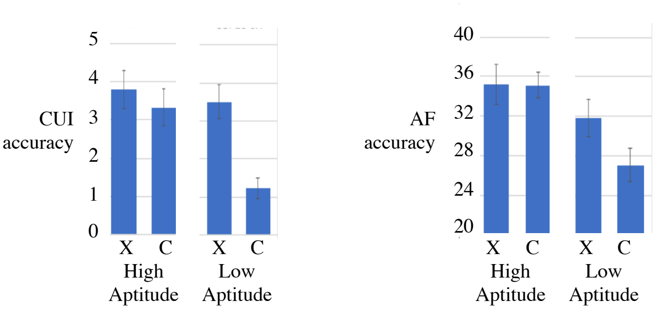
	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
adjAF1	1	96.81	96.811	18.2482	4.272e-05	***
adjAF3	1	31.06	31.059	5.8544	0.01726	*
Residuals	105	557.05	5.305			

CUI. In other words there are substantial benefits in laying the foundation for algebraic reasoning in Phase 1. These are reinforced during the Phase 3 and the Follow-up phase when students were preparing for a common standardized assessment and teacher appraisal.

6.5 Balanced Appraisal

As a check on our analysis, and any bias introduced by the non-experimental design, we constructed a matched model using the R package `MatchIt`. This was used to divide the experimental sample into two same size subsamples - labelled (H)igh SA and (L)ow SA using the non-verbal reasoning task RM at $g4$ as a proxy to correspond to Brownell's scholastic aptitude (SA) verbal reasoning test which was undertaken at the end of his study. The experimental sample was divided by `Matchit` into High and Low SA in the ratio 2:3, the control sample in the ration 3:2. `Matchit` created weights for input to the R core function `lm` to check how AF accuracy is related to the Growth Point, Treatment and SA using the formula $AF \sim GP + SA + Treatment*SA$. The ANOVA shown in Table 8 shows a Treatment:SA interaction with $p<0.01$. This is statistically significant. Figure 8 shows the mean accuracy in the common and CUI tests by subsample in this weighted model. These results are consistent with Brownell's finding of a treatment-interaction in his high intensity studies (Brownell, 1967b, p. 49). As a check that this effect was not evident at the outset we ran the model using the dataset for AF at $g1$ and found no significant interaction.

We also used the weighted model to check how CUI accuracy is related to Treatment and scholastic aptitude (SA) using the formula $CUI \sim Treatment*SA$. The ANOVA



(A) Mean CUI accuracy by scholastic aptitude (High, Low) and sample (Treatment(X), Contol(C)) with standard error bars

(B) Mean AF accuracy by scholastic aptitude (High, Low) and sample (Treatment(X), Contol(C)) with standard error bars

Fig. 8 Aptitude-treatment interaction effects CUI and AF accuracy present study (matched data) at *g*4

Table 8 ANOVA for linear model for AF predicted by Growth Point (GP), Treatment and Scholastic Aptitude (SA). Significance codes 0‘***’ 0.001‘**’ 0.01‘*’ 0.05‘.’ 0.1‘ ’1

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
GP	3	8234.0	2744.68	41.1619	<2.2e-16	***
SA	1	2988.3	2988.33	44.8159	7.008e-11	***
Treatment	1	373.3	373.28	5.5981	0.018436	*
Treatment:SA	1	638.2	638.23	9.5715	0.002109	**
Residuals	417	27805.6	66.68			

Table 9 ANOVA for linear model for CUI predicted by Treatment and SA Significance codes 0‘***’ 0.001‘**’ 0.01‘*’ 0.05‘.’ 0.1‘ ’1

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
Treatment	1	57.69	57.693	10.9994	0.001256	**
SA	2	31.41	115.707	2.9946	0.054391	.
Residuals	104	545.49	5.245			

shown in Table 9 shows a Cui treatment effect in favour of the experimental sample (F factor 10.9994, p=0.001256 < 0.01) which is statistically significant. We also carried out a 2-sample t-test $CUI \sim Treatment$ comparing high SA and low SA groups separately. The Bonferroni corrected welch tests for both High SA ($p = .01 < 0.25$) and Low SA (p = 8.134e-.09 $< .25$) were statistically significant. Finally an analysis of Cohen’s d showed a large effect size (d = 1.18) for the Low SA sample and a small effect size (0.18) for the High SA sample. The ratio of 6.5:1 (1.18:0.18) in the comparison of the two Cohen’s d effect sizes confirmed a more substantial effect for the Low SA group.

The ANOVA, t-test and effect size evidence together warrant our central claim that treatment interaction was significant and also that the effects are strongest for children demonstrating lower scholastic aptitude.

7 Discussion

In this paper we have brought together three pieces of scholarship that interact and combine to form a new view of Cuisenaire-Gattegno. We have reappraised Brownell (1967b) one of the most rigorous previous studies and been guided both by Brownell's observations on the need for fidelity and by his hypothesis that the algebraic understanding gained by following the Cui approach underpins later arithmetic and algebraic proficiency. We have performed a meta-analysis of the literature on the Cui approach which confirms Brownell's remarks on fidelity, and replicated his study of the efficacy of the Cui method using the Stanford Educational Assessment tool (SEA). The longitudinal transfer study was really important because the gain in arithmetic proficiency is bigger than the effect of Cui training at the time. That is, Cui training is having a bigger effect on learning after the training is completed.

Brownell held that "one cannot 'play around' with the Cui program.... expertness of the teachers is a prime requisite to success. Otherwise, classroom activities with the Cuisenaire rods may amount to no more than the haphazard manipulation of colored sticks" (Brownell, 1967a, p. 195). Our meta-analysis also suggested that fidelity of transmission is a moderator in arithmetic proficiency. The lead teacher in the present study had three years prior experience with the approach. She was able to integrate Gattegno's program as set out in an open resource that accompanies this article into the school's medium-term lesson plans for Key Stage 1. This was comparable to the average teaching intensity in Brownell.

Our findings reproduce Brownell's main conclusions. In particular they support his hypothesis that "prior attention to the conceptual aspects of arithmetic (will) pay large dividends in increased proficiency in the end and there is reason to believe that, if proficiency were stressed later on, the hypothesis would be established" (Brownell, 1967a, p. 117).

In replicating the earlier experiment design we used modern psychometric and statistical techniques to study learning and scholastic aptitude. Schwartz et al. (2005) propose an expanded definition of transfer of learning to encompass an enriched notion of education. They contrast assessments of "preparation for future learning" (PFL) with the sequential problem solving encountered in standardized assessments. Their studies suggest that early innovation leads to better adaption to new challenges in the short run and better efficiency in the long run in transfer situations. We have followed their suggestion that PFL is assessed through two independent measures of *efficiency* and *innovation*.

There was a difference in scholastic aptitude between the two schools as measured with the SEA RM test which we attribute to different demographics. The p-value was less than 0.001 at $g3$ and less than 0.01 at $g4$. These effects are statistically significant. We took SEA measures of RM to be equivalent to Brownell's verbal reasoning measure of scholastic aptitude (SA). We divided the samples into two groups SA High or Low by matching and found a two-way interaction between Treatment and SA in predicting arithmetic fluency (AF). This interaction had a p-values of $p < 0.05$ which is statistically significant. The magnitude and direction of the effects for our study were similar to Brownell's.

We proposed a measure for *efficiency in future learning* as the increment in growth in AF accuracy not predicted by SA. We found a medium effect size (d = 0.457) in favor of the school that received the Cui treatment at the end of Year 2. Since the Cui intervention terminated six months earlier this was a measure of the degree to which Cui prepared pupils for efficiency in arithmetic computation in the national assessments. We observed a sleeper effect in that gains in efficiency accelerated after the conclusion of the intervention.

In addition to the benefits of the Cui intervention in the AF score there was an educationally significant difference in performance and a large effect size (d = 1.0) in favor of the experimental school in the CUI test. This measure was devised by Brownell. We use it to assess the impact of the *innovation* on future learning and teaching.

Attribute-treatment interactions are increasingly studied in mathematics education research. This is because individual differences in children's cognitive resources are associated with mathematics learning, even when individual differences in elementary mathematics knowledge are statistically controlled. This indicates that mathematics intervention should be designed to help students with poor foundational mathematics skills compensate for limitations in the cognitive resources associated with poor learning. Gilmore et al. (2017) explored the procedural skill, conceptual understanding and working memory capacity of 75 children aged 5 to 6 years as well as their overall mathematical achievement. They found that, not only were all three skills independently associated with mathematics achievement, but there was also a significant interaction between them. In fact levels of conceptual understanding moderate the relationship between procedural skill and mathematics achievement. Fuchs et al. (2014) conducted a controlled experiment with fourth grade at risk students with interventions in fraction learning, emphasising fluency and conceptual knowledge. Results revealed a significant aptitude-treatment interaction, in which students with very weak working memory learned better with conceptual activities but children with more adequate (but still low) working memory learned better with fluency activities. Our effects are somewhat similar to Fuchs, except that our single treatment is beneficial for lower aptitude students, whereas she shows her alternative treatment was actually relatively better for higher aptitude students.

Sleeper effects are seldom reported in the literature, and they have not in general been studied in relation to transfer and preparation for future learning. The present study's Cui sleeper effect, evidence for Brownell's treatment interaction, presents an opportunity to relate pedagogy to our growing understanding of the neurophysiological basis of mathematical understanding.

Vandell et al. (2010) describe how the benefits of pre-school programs boost later academic performance. Barrera et al. (2002) in a study of preventative interventions on aggression highlighted the importance of measuring long term effects. Bailey et al. (2017) review the nature of interventions that lead to persistence or even later emerging effects compared to those that quickly fade out. They identify three distinct processes that might account for these effects: skill building, foot-in-the-door capacity building and sustaining environments. All three are evidenced in the Cui intervention: learners gain skill in multi-step expression evaluation, they are introduced from the outset to reasoning about equivalence which is an essential underpinning of school mathematics and their first encounters with mathematics are in creative and playful environments, which encourage a highly abstract and systematic form of learning.

The emerging corpus of studies of brain images has determined that arithmetic involves a wide network of interconnected areas, including prefrontal, posterior parietal, occipito-temporal and hippocampal areas. As the brain develops this network undergoes changes to its pattern of connections, their function and structure (Peters & DeSmedt, 2018). Foisy et al. (2020) discuss how different kinds of teaching practices shape learners' brains. They highlight how the approach that students are taught to take to arithmetic problems is reflected in distinct neural activation following learning.

Mathematical performance has been shown to be associated with activity in particular brain regions. In the network associated with mathematical performance, areas around the intraparietal sulcus have been particularly prominent (Dehaene et al., 2003; Holloway & Ansari, 2010), an area associated with the dorsal stream of visual spatial processing. Braddick et al. (2016) found that children's global visual motion performance was associated with a larger relative cortical area in the region of the intraparietal sulcus, and that performance on this visual task was correlated with mathematical and visuomotor skills that have been linked to parietal lobe function. Sohn et al. (2004) compared patterns of brain activation when students solved algebra problems presented in either word or symbolic equation form, and found that posterior parietal activation was greater when solving equations.

These insights from neural imaging have found that symbolic operations in algebra engage areas involved in visual-spatial processing and non-symbolic magnitude comparison. We hypothesise that when the experimental sample process symbolic expressions they experience a greater activation in these non-symbolic areas and in areas involved in reaching and grasping, number construction and deconstruction. Given the role of these neural circuits in mathematical ability, the contribution of Cui training may lie in strengthening the pathways between these areas enabling a closer coordination of activity and facilitating the acquisition of further expertise even after the end of the specific training. The effects we have reported may reflect an enduring enhancement of the circuits by education in the Cui approach.

Cui teachers educate learners' sensitivity to common patterns of mathematical relations by coordinating sight, hearing, touch, fine motion (writing and construction) and introspection. The integers are introduced as the names for a sequence of patterns constructed by partitioning rods. The sequence exhibits a kind of 'perceptual productivity,' by using combinatorial and recursive functions to construct limitless complex diagrams (Benson, 2015; Barsalou, 1999, p. 592). In this way we postulate that 'Cui training' activates and reinforces a more extensive sub-network than drill – one that privileges reasoning and verification over memorisation and recall (ATM, 2018). It proceeds through three stages as a preparation for future learning.

In Phase 1 experimental students are given an opportunity to experience the algebraic structure of the number system by constructing complete patterns and through free writing of equivalent expressions in all four operations and fractions as operators. The intervention strengthens their awareness of the structure of the whole number system which enhances subsequent factual fluency when efficiency is emphasized.

Phase 2 consists of exercises that employ what has been learned about addition facts to 10 and extends their Cui knowledge to reason about multiplicative relationships, factors and division through new rod constructions - crosses and towers.

Phase 3 and the Follow-up Phase are occasions for future learning to meet the common (external) requirements of the arithmetic and reasoning standardized tests and teacher moderation. The experimental sample is distinguished from the control sample by their familiarity with reasoning about the equivalent value of integer expressions. We also found that they were more willing to engage with challenging multi-step CUI problems. While they had encountered such expressions in Phase 1, this facility was not required in their practicing for the national assessments.

7.1 Limitations

A central limitation of this study is consistent with limitations of other quasi-experimental designs in that they do not involve randomization at the class or student level. Although our design was based on high fidelity replication of Brownell's, our methods departed in several noteworthy ways.

His Common arithmetic test was designed by teachers to cover material present in both X and C schools. We replaced this with an arithmetic facts fluency test based on the Woodcock-Johnson Maths Fluency subscale. Fluency with single digit arithmetic is a standard schools are expected to reach by $g1$ yet performance on this metric continues to grow well beyond this point. Brownell's CUI test examined material that was covered by the X schools, but not the C schools. We adapted his test to reflect the shorter, two-year, duration of our study. We did not replicate his TRA test of material covered by the C schools but not the X schools.

We did not match the treatment and control samples in the same way as Brownell. He recruited students from 24 schools and created a balanced quasi-experiment by matching their scholastic aptitude obtained with a standard verbal reasoning test, removing the middle 20% from the distribution and mimicking a balanced experimental design. We performed a statistical analysis using the R `MatchIt` library to match subjects according to their RM observations. This resulted in similar findings to those reported by Brownell although our sample size was too small to produce his aptitude-treatment interaction in the case of the CUI test.

7.2 Conclusions

Gattegno's work promoting Cuisenaire's invention and developing the Cui curriculum was seen by Brownell and his colleagues as a promising direction for mathematics education research. Their appraisal was endorsed by teachers' associations across the francophone and anglophone worlds. Our reappraisal has highlighted that Cuisenaire rods can have a large effect on arithmetic proficiency and algebraic understanding if rigorous attention is given to the appropriate curriculum and pedagogy.

The meta-analysis showed that the average outcome is estimated to be of medium effect size, yet the efficacy of this approach is remarkably heterogeneous. Rather than attributable to noise, efficacy results appear to follow a pattern of diffusion, in which strong effects associated with the seminal curriculum materials and pedagogical practices dissipated as the teaching aides were adapted and the curriculum materials that inspired them were left behind. A high fidelity to the Cui approach was associated with a large effect size (1.2). This impact was reduced by 16% for each of eight levels of divergence from a benchmark we based on Brownell. We found that codified lesson scripting coupled with technology, enhances communication between researchers, teachers and learners and increases fidelity.

The policy implications are significant. As with all pedagogical interventions we have asked the key questions, who does it benefit? and, in what contexts? Our findings endorse Brownell's conclusions that learners falling below expected levels of academic performance may benefit most from gains in arithmetic fluency while leaners of all aptitudes will gain in algebraic reasoning. While this study can be readily adapted by researchers and teachers as a successful intervention in early years algebra these results suggest that adoption of the Cuisenaire rods alone may be insufficient, and that careful consideration of how to effectively adopt the original curriculum and pedagogy is advisable.

Acknowledgements

Conflict of interest

References

ATM (2018). *On teaching and learning mathematics with awareness*. Association of Teachers of Mathematics.

Aurich, S. M. R. (1963). A comparative study to determine the effectiveness of the Cuisenaire method of arithmetic instruction with children at first grade level. Master's thesis, Catholic University of America.

Bailey, D., Duncan, G. J., Odgers, C. L., & Yu, W. (2017). Persistence and fade-out in the impacts of child and adolescent interventions. *Journal of Research on Educational Effectiveness*, *10*(1), 7–39.

Barrera, M., Biglan, A., Taylor, T. K., Gunn, B. K., Smolkowski, K., Black, C., Ary, D. V., & Fowler, R. C. (2002). Early elementary school intervention to reduce conduct problems: A randomized trial with Hispanic and non-Hispanic children. *Prevention Science*, *3*(2), 83–94.

Barsalou, L. W. (1999). Perceptual symbol systems. *Behavioural and Brain Sciences*, *22*, 577–660.

Begg, C. & Mazumdar, M. (1994). Operating characteristics of a rank correlation test for publication bias. *Biometrics*, *50*(4), 1088–1101.

Benson, I. (2011). *The Primary Mathematics: Lessons from the Gattegno School*. Lambert Academic.

Benson, I. (2015). Functional relationships between patterns of Cuisenaire rods. *Mathematics Teaching*, (245), 39–40.

Benson, I. & Cane, J. (2017). Using Haskell with 5- to 7- year olds. *Hello World*, (2), 60–61.

Braddick, O., Atkinson, J., Newman, E., Akshoomoff, N., Kuperman, J. M., Bartsch, H., Chen, C.-H., Dale, A. M., & Jernigan, T. L. (2016). Global visual motion sensitivity: Associations with parietal area and children's mathematical cognition. *Journal of Cognitive Neuroscience*, *28*(12), 1897–1908.

Brownell, W. A. (1928). *The Development of Children's Number Ideas in the Primary Grades*. University of Chicago.

Brownell, W. A. (1935). Psychological considerations in the learning and the teaching of arithmetic. In *The Teaching of Arithmetic*, volume 10th Yearbook. NCTM.

Brownell, W. A. (1960). Observations of instruction in lower-grade arithmetic in english and scottish schools. *The Arithmetic Teacher*, *7*(4), 165–177.

Brownell, W. A. (1967a). *Arithmetical Abstractions: The movement towards conceptual maturity under differing systems of instruction*. Number 17 in UC Publications in Education. University of California, Berkeley.

Brownell, W. A. (1967b). Arithmetical computation: Competence after three years of learning under differering instructional programmes. https://eric.ed.gov/?id=ED022703.

Cai, J. & Knuth, E. J. (2011). *Early Algebraization*. Springer.

Campbell, D. T. & Stanley, J. (1963). Experimental and quasi-experimental designs for research on teaching. In N. L. Gage (Ed.), *Handbook of Research on Teaching.* Rand McNally.

Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology*, *105*(2), 380–400.

Carraher, D. W., Martinez, M. V., & Schliemann, A. D. (2008). Early algebra and mathematical generalization. *ZDM: International Journal of Mathematics education*, *40*(1), 3–22.

Cochran, W. G. (1954). The combination of estimates from different experiments. *Biometrics*, *10*(1), 101–129.

Cohen, J. (1988). *Statistical Power Analysis for the Behavioral Sciences*. Routledge.

Crowder, A. B. (1965). *A Comparative study of two methods of teaching arithmetic in the first grade*. PhD thesis, North Texas State University.

Cuisenaire, G. & Gattegno, C. (1962). *Initiation a la méthode, Les nombres en colours en couleurs*. Delachaux et Niestlé.

Dairy, L. (1969). Does the use of Cuisenaire rods in Kindergarten, First and Second grades upgrade arithmetic achievement? https://eric.ed.gov/?id=ED032128.

Dehaene, S. (2020). *How we learn. The new science of education and the brain*. Allen Lane.

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology*, *20*, 487–506.

DfE (2013). National curriculum in England: mathematics programmes of study.

DfE (2020). Mathematics guidance: key stages 1 and 2. Technical report, Department for Education.

Egan, D. L. (1990). The effects of using Cuisenaire rods on the math achievement of second grade students. Master's thesis, Central Missouri State University.

Ellis, E. N. (1964). The use of coloured rods in teaching primary number work in Vancouver public schools.

Fedon, J. P. (1966). A study of the Cuisenaire-Gattegno method as opposed to an eclectic approach for promoting growth in operational technique and concept maturity with first grade children. Master's thesis, Temple University.

Foisy, L.-M. B., Matejko, A. A., Ansari, D., & Masson, S. (2020). Teachers as orchestrators of neuronal plasticity: Effects of teaching practices on the brain. *Mind, Brain and Education*, *14*(4), pp 415–428.

Fuchs, L. S., Schumacher, R. F., Sterba, S. K., Long, J., Namkung, J., Malone, A., Hamlett, C. L., Gersten, R., Jordan, N. C., Siegler, R. S., & Changas, P. (2014). Does working memory moderate the effects of fraction intervention? An aptitude–treatment interaction. *Journal of Educational Psychology*, *106*(2), 499–514.

Gattegno, C. (1956). New Developments in Arithmetic Teaching in Britain: Introducing the Concept of 'Set'. *Arithmetic Teacher*, *3*(3), 85–89.

Gattegno, C. (1957). *Arithmetic with Numbers in Colour*, volume I. William Heinemann.

Gattegno, C. (1963a). *Mathematics with Numbers in Colour: Numbers from 1 to 20*, volume I. Educational Explorers.

Gattegno, C. (1963b). *Mathematics with Numbers in Colour: Numbers to 1000 and the four operations*, volume II. Educational Explorers.

Gattegno, C. (1974, 2010a). *Common Sense of Teaching Mathematics*. Educational Solutions.

Gattegno, C. (1987). *Science of Education: Part I Theoretical Considerations*. Educational Solutions.

Gattegno, C. (1988, 2010b). *Science of Education: Part 2B Awareness of Mathematisation*. Educational Solutions.

Gilmore, C., Keeblea, S., Richardson, S., & Cragg, L. (2017). The interaction of procedural skill, conceptual understanding and working memory in early mathematics achievement. *Journal of Numerical Cognition*, *3*(2), 400–416.

Goutard, M. (1964, 2017). *Mathematics and Children*. Educational Explorers.

Haynes, J. O. (1963). *Cuisenaire rods and the teaching of multiplication to third-grade children*. PhD thesis, Florida State University.

Higgins, J. P. T. & Thompson, S. G. (2002). Quantifying heterogeneity in a meta-analysis. *Statistics in Medicine*, *21*, 1539–1558.

Holloway, I. D. & Ansari, D. (2010). Developmental specialization in the right intraparietal sulcus for the abstract representation of numerical magnitude. *Journal of Cognitive Neuroscience*, *22*, 2627–2637.

Howard, C. F. (1957). British teachers' reactions to the Cuisenaire-Gattegno materials. *Arithmetic Teacher*, *4*, 191–195.

Kaput, J. J. & Blanton, M. L. (2000). *Algebraic Reasoning in the Context of Elementary Mathematics: Making It Implementable on a Massive Scale*. National Center for Improving Student Learning and Achievement in Mathematics and Science.

Keagle, M. A. & Brummett, A. J. (1993). Manipulative versus traditional teaching for mathematics concepts: Instruction-testing match. Master's thesis, Ball State University.

Kilpatrick, J. & Weaver, J. F. (1977). Place of William A. Brownell in mathematics education. *Journal for Research in Mathematics Education*, *8*(5), pp. 382–384.

Lucow, W. H. (1962). Cuisenaire method compared with the current methods of teaching multiplication and division. *Manitoba Teacher*.

Nasca, D. (1966). Comparative merits of a manipulative approach to second grade arithmetic. *The Arithmetic Teacher*, *13*(3), pp. 221–226.

NCTM (2000). *Principles and standards for school mathematics*. National Council of Teachers of Mathematics.

Osburn, W. J. (1928). The development of children's number ideas in the primary grades: Review. *The Elementary School Journal*, *29*(4), 307–308.

Peters, L. & DeSmedt, B. (2018). Arithmetic in the developing brain: A review of brain imaging studies. *Development Cognitive Neuroscience*, *30*, 265–279.

Project-ILead (2019). In-school, Longitudinal, Executive Function and Academics Database. https://sites.google.com/view/projectileadnsf/home.

Riley, R. D., Higgins, J. P. T., & Deeks, J. J. (2011). Interpretation of random effects meta-analyses. *British Medical Journal*, *342*, 964–967.

Robinson, E. B. (1978). *The Effects of a Concrete Manipulative on Attitude Toward Mathematics and Levels of Achievement and Retention of a Mathematical Concept Among Elementary Students*. PhD thesis, East Texas State University.

Robinson, F. G. (1964). A note on the quantity and quality of Canadian research on the Cuisenaire method. In *Canadian experience with the Cuisenaire method*. Canadian Council for Research in Education.

Romero, R. C. (1977). *Student achievement in a pilot Cureton reading, Cuisenaire mathematics program, and a bilingual program of an elementary school*. PhD thesis, Northern Arizona University.

Schliemann, A., Carraher, D., & Brizuela, B. (2007). *Bringing out the algebraic character of arithmetic*. Lawrence Erlbaum Associates.

Schwartz, D. L., Bransford, J. D., & Sears, D. (2005). *Transfer of learning from a modern multidisciplinary perspective*, chapter Efficiency and Innovation in Transfer, (pp. 1–51). Information Age Publishing.

Sohn, M. H., Goode, A., Koedinger, K. R., Stenger, V. A., Fissell, K., Carter, C. S., & Anderson, J. R. (2004). Behavioral equivalence, but not neural equivalence - neural evidence of alternative strategies in mathematical thinking. *Nature Neuroscience*, *7*(11), 1193–1194.

Steiner, K. E. (1964). A comparison of the Cuisenaire method of teaching arithmetic with a conventional method. Master's thesis, North Texas State University.

Sterne, J. A. C. & Eggar, M. (2005). *Publication bias in meta-analysis: Prevention, assessment and adjustment*, chapter Regression methods to detect publication and other bias in meta-analysis, (pp. 99–110). Wiley.

Vandell, D. L., Belsky, J., Burchinal, M., Vandergrift, N., & Steinberg, L. (2010). Do effects of early child care extend to age 15 years? Results from the NICHD study of early child care and youth development. *Child Development*, *81*(3), 737–756.

Viechtbauer, W. (2005). Bias and efficiency of meta-analytic variance estimators in the random-effects model. *Journal of Educational and Behavioral Statistics*, *30*(3), 261–293.

Viechtbauer, W. (2021). *Handbook of meta-analysis*, chapter Model checking in meta-analysis. CRC Press.

Viechtbauer, W. & Cheung, M. W.-L. (2010). Outlier and influence diagnostics for meta-analysis. *Research Synthesis Methods*, *1*(2), 112–125.

Wallace, P. (1974). *An investigation of the relative effects of teaching a mathematical concept via multisensory models in elementary school mathematics*. PhD thesis, Michigan State.

Young, R. & Messum, P. (2011). *How we learn and how we should be taught: An introduction to the work of Caleb Gattegno*. London: Duo Flumina.