Modeling the effects of perisaccadic attention on gaze statistics during scene viewing

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April 6, 2020

Abstract

How we perceive a visual scene depends critically on the selection of gaze positions by a series of saccadic eye movements (scan path). In this selection process, visual attention is known to play a key role in two ways. First, models of image-based visual attention are able to predict the time-independent eye-fixation density. Second, millisecond-level attentional dynamics around the time of saccade drives our gaze from one position to the next. These two related research areas on attention are typically perceived as separate, both theoretically and experimentally. Here we link the two research areas by demonstrating that perisaccadic attentional dynamics improve predictions on scan path statistics in scene viewing. In a dynamical mathematical model, we identified two new theoretical principles to be essential, (i) perisaccadic covert attention for the explanation of a saccadic momentum effect and (ii) delayed inactivation of recently fixated regions for facilitation of return saccades. Our model reproduces saccade amplitude distributions, angular statistics, intersaccadic turning angles, and their impact on fixation durations as well as inter-individual differences between observers. Results are obtained from data assimilation of the mathematical model in a fully Bayesian framework that permitted rigorous statistical inference. We conclude that bridging the gap between static visual attention models and perisaccadic attentional dynamics is important for the prediction of human scene exploration behavior.

Visual perception in humans is the result of complex signal processing of visual input in the brain. Information enters the eyes at a rate of about 10^8 to 10^9 bit/s [29]. In order to handle this enormous amount of input, the visual system relies on foveation and selective attention [70]. These two mechanisms reduce the information available at any given point in time to enable the brain to efficiently process the relevant aspects of visual information. *Foveation* refers to the decrease of visual acuity from the region extending about 2° around the point of fixation (the fovea) to the periphery of the visual field. During natural viewing, regions of interest are sequentially moved into the high resolution foveal area by saccadic eye movements [15, 21]. Natural vision is therefore an active process, determined by sequential choices of fixation locations. The resulting scan path [44] is characterized by pronounced spatial correlations [67]. *Selective attention* is the second key bottleneck of visual processing with a rate of about 100 bit/s [72], prioritizing selected image regions at the cost of others. Eye movements are strongly linked to visual attention, they are normally aligned under natural viewing conditions [15].

Experimentally, however, the locus of visual attention and fixation position can diverge, a condition referred to as covert attention [46, 47]. Research on saccade dynamics in highly controlled experimental setups indicates that attention, as measured by processing benefits, precedes the fixation to the next saccade target [8, 22, 33]. Current models of eye movements and visual attention are typically based on the plausible simplification of directly equating location of attention and fixation position [27, 12, 63, 59]. Here we propose that perisaccadic covert attention shifts are an important

factor in eye movement guidance. We investigate a neurophysiologically plausible implementation of attentional dynamics and inhibitory principles which turns out to improve model performance on eye movement statistics.

Dynamical models of fixation selection

The field of modeling eye movement behavior has primarily focused on predicting where fixations are placed in an image [32, 26, 35]. The most advanced models are not far from predicting perfect fixation density maps [5]. The step from modelling static fixation densities to predicting scan paths reveals that bottom-up image information, while crucial, can not comprehensively explain the fixation selection process. Scan path dynamics also play an important role. The ability of a model to predict human-like behavior can be much improved [59] by adding basic dynamic mechanisms to the static image-based predictions [66, 38, 54, 63].

Theoretical [27, 11] and experimental work [54] agree that two essential components in explaining dynamic scan paths are attentional selection and inhibitory tagging of previously fixated locations. The former refers to the combination of foveation and the attentional field, which defines a limited area from which information can be extracted. The attentional field is often represented as a Gaussian distribution, with its peak representing the fovea. Thus, as a first-order approximation, visual input is given by a Gaussian blob defined by the fixation position in a given scene. The second component keeps track of fixation history in order to drive exploration in scan paths and prevent continuous return to the same high-saliency regions [30]. In behavioral experiments, *inhibition of return* has been widely found as a component of human visual behavior [31], electrophysiology [23], and, more recently, as a neural process in the frontal eye field [42].

Baseline model

Attentional selection and inhibitory tagging have been previously implemented in a dynamical model for scan path generation [12, 59]. The SceneWalk model serves as a platform for the current work on the analysis of the role of attention around the time of saccade. Activation maps are computed on a discrete grid i, j with current fixation position (x_f, y_f) . Both the inhibition $F_{ij}(t)$ and the activation $A_{ij}(t)$ streams evolve over time under current visual input and decay (due to limited of visual memory), i.e.,

$$\frac{dA_{ij}(t)}{dt} = \omega_A \left(\frac{S_{ij} G_A(x_i, y_j; x_f, y_f)}{\sum_{kl} S_{kl} G_A(x_k, y_l; x_f, y_f)} - A_{ij}(t) \right) \quad (1)$$

$$\frac{dF_{ij}(t)}{dt} = \omega_F \left(\frac{G_F(x_i, y_j; x_f, y_f)}{\sum_{kl} G_F(x_k, y_l; x_f, y_f)} - F_{ij}(t) \right), \quad (2)$$

where the input to the activation maps is the Gaussian-weighted local saliency $S_{kl} G_A(x_k, y_l; x_f, y_f)$ and the input to the inhibition map is a Gaussian blob at current fixation position. As illustrated in Fig. 1, activation and inhibition maps are subtractively combined to yield a priority map $u_{ij}(t)$ [3], i.e., the 2D fixation probability map for the selection of the upcoming saccade target. Mathematically, the two maps are shaped by exponent γ before subtraction, and a weight parameter C_F for inhibition is introduced, i.e.,

$$u_{ij}(t) = \frac{(A_{ij}(t))^\gamma}{\sum_{kl} (A_{kl}(t))^\gamma} - C_F \frac{(F_{ij}(t))^\gamma}{\sum_{kl} (F_{kl}(t))^\gamma}. \quad (3)$$

Saccade onsets are determined by sampling from a gamma distribution approximating experimentally observed fixation durations (see *Materials and Methods*).

Figure 1: Attentional processing streams. Visual attention and inhibitory tagging are largely independent processing streams which evolve neural activations via time-dependent input and decay. Combining these streams yields a priority map from which fixation positions can be selected.

Extended model including perisaccadic attention

Before the saccade is executed toward a target, performance benefits in accuracy and speed can be measured at the target location. This has frequently been interpreted as attention being allocated to the part of the image that is about to be fixated as part of saccadic planning. In Figure 2A (*leftmost*), we see that during a fixation, the fixation location and the center of attention are coaligned. Once the upcoming target location is selected from the priority map $u_{ij}(t)$ but before the saccade occurs (Fig. 2A, *second from left*), attention already moves to the upcoming saccade target, decoupling fixation (red 3-pointed star) and attention (green 5-pointed star). The concept that covert attention shifts precede saccadic eye movements is well-established in the literature [8, 24], with clear evidence for this *predictive attentional targeting* as early as 150 ms before saccade onset [53].

Furthermore, attention has been shown to move retinotopically with the saccade [41]. Thus, just after a saccade similar processing benefits can be found in a location along the saccade vector, which aligns with the retinotopic position of the target before the saccade [19], a phenomenon called *retinotopic attentional trace* (RAT). The pre-allocated attention peak moves with the saccade such that it lands shifted along the saccade vector away from the saccade target. Figure 2A (*third from left*) shows that immediately after a saccade, attention is shifted to the same retinotopic position as the previous pre-saccadic shift and thus spatiotopically shifted in the same direction as the saccadic movement. Experimentally, the influence of the shift lasts about 100 to 200 ms [19]. After this interval the locus of activation moves to coincide with the fixation position again (Fig. 2A, *rightmost panel*).

If we consider the added activation along the saccade vector as a component in saccade selection, this is in good agreement with the experimental finding of shorter fixation durations before forward

saccades. The post-saccadic RAT is therefore the second part of the attentional decoupling that begins before saccade onset. Behavioral evidence for attentional shifts during a saccade [8] as well as neurophysiological correlates for post-saccadic retinotopic enhancements have been found [20]. Below we suggest that attentional shifts are a likely explanation for a systematic effect on saccade statistics observed during scan path formation.

Figure 2B illustrates the influence of perisaccadic attentional shifts on the activation maps. The streams evolve over time (Eq. 5, 6). Fixation targets are selected from the priority map (Eq. 3) at time $t_{fix} - \tau_{pre}$, where t_{fix} is the duration of the fixation and τ_{pre} is the duration of the pre-saccadic shift. Once the upcoming target is selected, attention moves to its location; after saccade execution, the post-saccadic attentional shift occurs; lastly, attention and fixation position are realigned when entering the main fixation phase (for details of the implementation see *Materials and Methods*).

Results

The current work investigated the potential role of perisaccadic attention on human saccade statistics. Large-scale numerical simulations were carried out to estimate model parameters from experimental data using Bayesian data assimilation [59]. In the experiments, 35 human observers viewed 30 natural color images (see *Materials and Methods*). Model parameters were estimated independently for each participant. For model fitting, fixation sequences of 2/3 of the images were used as training data, while all subsequent analyses were carried out on the remaining test images for each participant. We will compare simulation for the baseline (SceneWalk) model [12, 59] with the extended model that includes perisaccadic attention mechanisms. The following section details some characteristic eye movement statistics found in experimental data.

Saccade amplitude distribution

The distribution of saccade amplitudes generated during a scene viewing experiments varies across participants and images. Overall, both the baseline and the extended models reproduce the qualitative shape of the saccade amplitude distribution (Fig. 3A) [1, 64, 65, 4]. The experimentally observed saccade amplitude distribution is right-skewed, reflecting that amplitudes tend to be smaller than computer-generated saccades obtained by random sampling from the static 2D fixation density [67, 12]. Previously, we suggested this drop in saccade amplitudes is caused by the foveated visual system, which preferentially selects saccade targets from within attentional span. Therefore, inter-individual differences in mean saccade amplitudes should correlate with the size of the attentional span σ_A , which is defined as the standard deviation parameter of the Gaussian-shaped attentional blob (see Eq. 9). In Figure 3B, we show the expected correlation between σ_A and mean of saccade amplitude across participants, indicating that a larger area does indeed lead to longer saccades.

With model parameters estimated for each participant using the training images, the predicted mean saccade amplitudes for test images were compared to experimentally observed mean saccade amplitudes. We found good agreement between predicted and experimentally observed mean saccade amplitudes (Fig. 4) indicated by a high correlation ($r = 0.91$). Our model is able to explain the inter-individual differences in the data via parameter variation.

Absolute and relative saccade angle distributions

Saccade angles are another important characteristic of human eye movement behavior. The absolute angle distribution reports the directions of saccades relative to the image frame. Interestingly, there is a strongly image-dependent tendency, which varies mostly with the distribution of image features.

Figure 2: Timeline for processing around the time of saccade. (A) Attention and fixation position. *Leftmost panel:* During fixations, locus of attention (green 5-pointed star) and fixation position (red 3-pointed star) are aligned. *Second panel from left:* Immediately before a saccade, the upcoming fixation location has already been selected; attention moves to the target location (green), while fixation position remains at launch-site of the saccade. *Third from left:* After saccade execution, fixation position has been updated and, simultaneously, attention has shifted along the retinotopic activation trace (RAT) of the current fixation position before the saccade. *Rightmost:* During the fixation's main phase, the locus of attention and the fixation are realigned. (B) The activation (red-orange) and inhibition (blue-green) streams evolve over time during each of three model phases in each fixation. When a new fixation location is to be selected the streams are subtracted to yield a priority map (pink-purple). The activation map consists of a Gaussian aperture around a phase dependent point in the image and image information as well as influences from past states of the model. The inhibition stream consists of Gaussian apertures around the current fixation location and past states of the model.

Figure 3: Saccade amplitude distribution and inter-individual differences. (A) The saccade amplitude distribution for experimental data (black line), the baseline SceneWalk model (blue, dashed line) and the extended model (yellow, dotted line). Shading represents the 95% confidence interval between subjects. (B) Size parameter σ_A of the attentional span and is positively correlated simulated mean saccade amplitude.

Figure 4: Correlation of experimentally observed and predicted mean saccade amplitudes. A high correlation is observed between experimental and simulated data on test images, using parameter estimates for each participant obtained for training data.

On average the distribution shows characteristic peaks in the four cardinal directions [18, 16]. Figure 5A shows that, the baseline model does not show a the pronounced pattern found in experimental data. Comparatively the extended model shows a clear improvement with distinct peaks at 0° , 90° , 180° , 270° , and 360° . The extended model implements a mechanism for an oculomotor potential, which preferentially weights the activation in the cardinal directions [14] before it is combined with the inhibition stream.

Figure 5: Saccade angle distributions. (A) Absolute angle distribution in the empirical and simulated data. Empirical data show a strong tendency for saccading in the cardinal directions. This is strongly image dependent and not specifically considered in the models. The shading shows the 95% confidence interval between subjects. (B) Saccadic turning angle distribution in the empirical and simulated data. The angle shown is the divergence from the previous saccade direction. Empirically we find a tendency to continue along the previous saccade vector or completely reverse it. The extended model partly shows this behavior. The shading shows the 95% confidence interval between subjects.

The saccade turning angle distribution characterizes the relationship of consecutive saccades. In the experimental data there is a clear bias towards forward saccades, which follow the same vector of motion, and a secondary preference for return saccades, which reverse the saccade vector. Therefore, we should expect clear peaks at 0° and 180° in the corresponding turning angle distribution [66, 61, 54, 56]. Figure 5B shows the results of the baseline and extended models in comparison to experimental data. The baseline model produces a u-shaped distribution without any indication of a forward bias. There is an increased probability of turning by about 180° , since the edges of the image represent hard constraints. This effect is large enough to overshadow the effects of return saccades that directly return to the previous fixation location (of which there are comparatively few). The extended model does develop a peak for forward saccades, showing better qualitative agreement with the experimental data, although the bias towards forward saccades is clearly weaker than in the experiment.

The statistical preference of observers to maintain current saccade direction has been referred to as saccadic momentum [61, 69, 39, 56]. Here we propose that the experimental effect is at least partially due to attentional enhancement in the current saccade direction, which generates a peak in the attention map that produced the forward bias.

Joint probability of intersaccadic angle and amplitude

More generally, we can identify potential dependencies of saccade turning angle and saccade amplitude by visualizing the corresponding joint probability (Fig. 6). As discussed above, compared to all other directions, there is a pronounced tendency for saccades to either maintain or completely reverse the direction of the previous saccade. This effect is well documented in the literature

[65, 61, 54, 66, 56] and is independent of a variety of other factors such as image content. The values on the axes in Figure 6 are relative to direction and amplitude of the previous saccade, so that $(-1, 0)$ represents a return to the previous location. Figure 6A reveals that there are two clear peaks in the experimental data, i.e., the return peak to the normalized launch site $(-1, 0)$ of the previous saccade and the forward peak that is related to the saccadic momentum effect discussed above. It is important to note that the experimental return peak is not particularly high, but it is distinct since surrounding 2D regions do not exhibit a high fixation density.

In our extended model (Fig. 6B), the mechanism responsible for the forward saccades is the attentional shift before and after a saccade. The distinctive shape of the return saccade peak, we suggest, is the result of the combination of a slow, global inhibition of return and a directed smaller facilitation of return (see *Materials and Methods*). The former is implemented as the model’s inhibition stream, while the latter is implemented as reduction in decay speed in the attention map, localized at the previous fixation location. The baseline model cannot produce the return and forward peak, since it lacks the mechanistic principles for coupling subsequent saccades.

Intersaccadic angle and fixation duration and saccadic amplitude

The next two analyses correspond to the interdependence of fixation duration and saccade amplitude, and saccadic turning angles. Both have a distinctive shape in the data, showing that forward saccades tend to be shorter and preceded by shorter fixations, while changing direction takes longer and evokes longer saccades.

The new model notably improves the fit of the dependence of fixation duration on the turning angles. While previously there was no temporal component in the model the added phases of shifted activation enable the model to dynamically respond to the duration of a fixation. In the model, each fixation begins with the post-saccadic shift phase. In terms of the attention activation map, this means that there is more activation along the previous saccade vector. After this phase the influence of the shift diminishes. Thus, when the fixation is short, there is still a lot of influence from the shift, increasing the chance of producing a forward saccade. When the fixation is long, the influence of the post saccadic shift has subsided, allowing for activation from other salient locations

Figure 6: Joint probability of saccade turning angle and saccade amplitude normalized to the previous saccade. (A) The experimental probability shows the return and forward peaks. (B) The extended model captures these characteristic properties qualitatively. (C) In the baseline model, neither the return peak nor the forward peak can be found.

Figure 7: Average saccade amplitude and fixation duration are related to the change in saccade direction (saccadic turning angle). (A) Fixation duration is shortest for saccade moving forward. Results from the extended model after in good agreement with experimental data. (B) Saccade amplitude is smallest for forward saccades and largest for return saccades. While the baseline model reproduces this effect qualitatively, the extended model produces a better fit to the experimental data.

to guide the saccade.

The central fixation tendency

An important systematic bias in eye movements is the central fixation tendency. Specifically, the first fixation in a scan path tends to be closer to center of the scene than subsequent fixations [62, 2, 55, 68]. Furthermore, the central fixation tendency is stronger when the initial saccade latency was shorter than on average [55] (see *SI Appendix: Fig. S1B*). Figure 8 shows the distance to the image center over fixations. We added a center bias to the model by initializing the model in the attention stream with a centered Gaussian activation map [55]. The characteristic dip on the second fixation (i.e., the first freely chosen fixation) is reproduced exactly by the new model.

Likelihood-based comparison

Since our approach includes the likelihood computation of the baseline and extended models, we can make use of the models’ likelihood functions for model comparison [59]. This approach entails evaluating the model likelihood given the empirical test data and computing the average log-likelihood per fixation of all scan paths. We then compare this metric to previous models [34].

The overall likelihood of the model given the data is larger for the extended model than for the original model by only a very small margin (Fig. 9). In general, improved likelihood indicates improved predictive power of a model. However, by itself, it is a fairly crude measure of model performance. As we have seen, the extended model performs much better than the baseline model at reproducing systematic tendencies of eye movement. Regardless, the improvement in performance on a general likelihood basis is negligible. The additions to the baseline model discussed in the current study, though theoretically well-founded, were extensive and considerably increased the model complexity. Conceivably adding these mechanisms could have led to improved scan path

dynamics but worsened overall likelihood predictions, or else made the model volatile or instable. Therefore, we interpret the stability of the predicted likelihood as a satisfactory result.

Figure 8: Distance-to-center effect. The distance to the image center of each fixation in the sequence. The empirical tendency to move close to the center at fixation 2 is well-replicated by the extended model.

Figure 9: General model likelihood of models fitted on the training data given the test data. Density sampling draws fixations directly from the empirical fixation distribution without dynamics. Local saliency produces scanpaths by picking from the fixation density filtered through a Gaussian window, with no dynamics. SceneWalk and extended SceneWalk are the dynamic models described in this article.

Discussion

Moving from models of static fixation probabilities to the generation of scan paths has recently begun to attract interest in the field of attention modeling [63, 59, 38, 12, 71]. The success of saliency-based visual attention modeling [27, 36, 58] over the last 30 years makes a strong case for the use of priority maps [3] as a core component in scan path generation. In addition to image and task influences biologically represented in priority maps, scan paths on scenes are also characterized by a number of statistical characteristics, e.g., saccade angles and modulations of fixation duration or saccade amplitude by saccadic turning angles. Our modeling study lends support to the fact that attentional dynamics around the time of saccade exert a fundamental influence on the behavioral statistics of scan paths.

Previous research on visual attention shows that processing resources are covertly allocated away from the current fixation location just before [53, 24, 8] and just after [53, 20, 19] a saccade is produced. In this study, we added shifts of covert attention to a dynamical model of scan path generation [12, 59] and find improved agreement with gaze statistics observed in experimental data. Most importantly, the characteristic distribution of saccadic turning angles with a clear bias towards forward and return saccades and the influences of saccadic turning angle on fixation durations and saccade amplitudes can be explained by covert attention shifts around the time of a saccade.

The first generation of computational models in scene viewing were static models which predicted fixation locations on any given image based on statistical image features. The strength of these static models lies in producing densities that resemble empirical fixation density maps. Recently, the predictive power of some models has become close to perfect and approached the gold standard [34, 35]. However, by design these models do not take temporal dynamics within a scan path and the inhomogeneity of the retinal acuity into account. From this perspective, it is not surprising that static models predict fixation density, but not sequences of fixations [17, 54, 59]. This simple fact points to the interesting observation that eye movements in scene viewing are guided in large part, but not exclusively by observer- and image-specific factors. Human eye movements are influenced by oculomotor and attention systems, producing pervasive systematic statistical tendencies in experimental data.

Previously published dynamic models outperform static models substantially [38, 59]. The most evident feature of the human visual system which indisputably influences scan path dynamics is foveation. Accordingly, even a minimal model like weighting a saliency map by the distance to a current fixation location significantly improves model performance [45]. The SceneWalk model [12], which served as a baseline for our study, incorporates foveated saliency in its activation stream. A further advance in the modelling of scan paths has been the addition of inhibitory fixation tagging [48, 30, 25]. The baseline model implements such an inhibition stream as a second component shaping the priority map [3] by difference of activation.

The fact that long fixations often occur in frequently fixated areas [9] implies that fixation duration and target selection are related. The LATEST model [63] combines the prediction of scan paths and fixation durations by interpreting scan paths as a continuous series of stay (maintain fixation) or go (saccade) decision [51, 50, 6]. Each individual location on a weighted saliency map influences two LATER units [43], i.e., one for normal and long latencies and one for short latencies. These units accumulate evidence from each location in the image until one reaches a threshold depending on the current location, triggering a saccade. The accumulation rate of each location in the image is controlled by image-content factors like image features and semantic interest, as well as by oculomotor factors like the change in saccade direction and target eccentricity. Coupling of experimental data and model is achieved by statistical linear mixed-effects modeling. Thus, the LATEST model makes little attempt at explaining the origin of the factors that influence the rate of evidence

accumulation, instead focusing on the specific selection mechanism and its relationship with fixation duration. By contrast, the extended SceneWalk model is based on mechanistic assumptions derived from neural and cognitive knowledge about the contributing factors to fixation selection. Parameters are based on statistically rigorous likelihood approach that evaluates the model assumptions given the data.

Generally, the value of a model must be quantified in terms of predictive power and explanatory value. For the models discussed here, we carried out comparisons of simulated scan paths and human eye movement data. A number of metrics have been proposed for such a comparison [28, 7, 40]. Critically, however, the choice of individual statistics has a crucial influence on the outcome and there is, in most cases, no rigorous justification for the used metric. A solution to this is to evaluate dynamical scan path models using a likelihood approach [59], which provides a statistically well-founded and reliable measure for the predictive quality of a dynamical model. In this article we relied on Bayesian data assimilation [52] as a statistically rigorous framework for testing whether the model architecture accurately represents the data generation process. This approach turned out to be particularly fruitful for strongly theory-guided models. Using general likelihood to estimate parameters of the model lends credibility to the theoretical foundations from eye movement literature implemented by the model.

In addition to better predicting human scan paths during scenes viewing, the integration of biologically-inspired attentional dynamics into models of eye guidance unifies two very disparate fields of eye movement research. The research into covert attention shifts and perisaccadic effects is typically concerned with processes that occur on a highly detailed level in very controlled experimental setups. By contrast scene viewing literature usually operates at a higher level, on which the minutia of saccade programming or covert attention are typically passed over. Thus, influences arising from the microscopic level of eye movement control can explain effects we observe at the macroscopic level.

Materials and Methods

Experiment: Stimulus material, participants, and procedures

Experimental data for this study were collected in a larger corpus study on scene viewing which is described in detail elsewhere [58, 57]. Images and fixation data from this corpus experiment can be downloaded from Open Science Foundation [57]. The corpus consists of eye movement data from 105 participants viewing 90 images of natural or urban landscapes from 6 different categories for a fixed duration (10 s). Each category contained 15 images. Images were chosen such that the most interesting image parts either fell on the left, right, upper, lower or, central image side (Fig. 10 provides some examples). The last category were images with natural patterns, minimizing the influence of particularly salient objects. During the viewing subjects were given no task except to freely view the images.

In this study we used Experiment 3 from the corpus study [58, 57], in which participants viewed color images. This subset of data contains the eye movements of 35 participants, who viewed 30 images from each category without a task. We further split the data set into test and training data by randomly choosing 1/3 of the images (10 from each category) for each participant.

For saccade detection we applied a velocity-based algorithm [13, 10]. Saccades had a minimum amplitude of 0.5° and exceeded an average velocity during a trial by six (median-based) standard deviations for at least six data samples (12 ms). The epoch between two subsequent saccades was defined as a fixation. After preparation, 312,267 fixations and saccades were detected for further analysis.

Figure 10: Six sample images from the scene viewing corpus. The first images are representative for categories of left, right, central, bottom, and top focus. The bottom right image provides an example for a natural pattern.

Baseline Model (SceneWalk)

The original SceneWalk model [11] was implemented on a 128×128 grid, where (x, y) give the physical coordinates in degrees. For each fixation in the scan path we start by computing simple 2-D Gaussians centered at current fixation position (x_f, y_f) for both the inhibition and the attention pathway, each with an appropriate standard deviation $\sigma_{A/F}$ (A denotes the attention stream, F denotes the fixation stream to generate inhibitory tagging).

$$G_{A/F}(x, y) = \frac{1}{2\pi\sigma_{A/F}^2} \exp\left(-\frac{(x - x_f)^2 + (y - y_f)^2}{2\sigma_{A/F}^2}\right) \quad (4)$$

The differential equations that determine the temporal evolutions of activation maps, Eq. 1 for the attention map and Eq. 2 for the fixation/inhibition map, can be integrated analytically to provide a closed solution for the activation changes during fixation, i.e.,

$$A(t) = \frac{G_A S}{\sum G_A S} + e^{-\omega_A(t-t_0)} \left(A_0 - \frac{G_A S}{\sum G_A S} \right), \quad (5)$$

and

$$F(t) = \frac{G_F}{\sum G_F} + e^{-\omega_F(t-t_0)} \left(F_0 - \frac{G_F}{\sum G_F} \right), \quad (6)$$

where we dropped the indices i, j for simplicity. In the equations, the term $e^{-\omega_{A/F}(t-t_0)}$ determines the speed of decay of the past states of the map.

Next, both activation maps were combined to compute the priority map $u_{ij}(t)$, Eq. 3. As this operation can cause negative activation, in the next step we take only the positive component of the map,

$$u_{ij}^*(u_{ij}) = \begin{cases} u_{ij}, & \text{if } u_{ij} > 0 \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

Phase	Start	End	F center	A center
Post-saccadic shift	0	τ_{post}	fix_n	$remap$
Main (No shift)	τ_{post}	$t_{fix} - \tau_{pre}$	fix_n	fix_n
Pre-saccadic shift	$t_{fix} - \tau_{pre}$	t_{fix}	fix_n	fix_{n+1}

Table 1: Model Phases: onset times and locations around which the Gaussians in both streams are centered. t_{fix} is the fixation’s duration, $\tau_{pre/post}$ are the the phase durations and $fix_n/n+1/remap$ are the locations.

and, finally, add noise ζ

$$\pi(i, j) = (1 - \zeta) \frac{u_{ij}^*}{\sum_{kl} u_{kl}^*} + \zeta \frac{1}{\sum_{kl} 1} \quad (8)$$

to obtain the probability map $\pi(i, j)$ for the selection of saccade targets. This process is repeated for each fixation in a sequence, where the current state information is combined with the past activation maps to produce a continuously evolving prediction of the next fixation.

The model structure reveals the following parameters: (1, 2) σ_A and σ_F , which are the standard deviations of the current fixation’s attention and inhibition Gaussians respectively, (3, 4) ω_A and ω_F , which are the speed at which past states of the model lose influence over the current, (5) γ , the shaping parameter for the Gaussians, (6) the coupling factor C_F , which is the weight of the inhibition pathway, and (7) the noise parameter ζ determining the background noise for the probability map $\pi(i, j)$.

Extended model: Pre-saccadic attentional shifts

Once a new fixation location is chosen the center of attention moves to the upcoming fixation location, while the center of of the inhibition map remains at the current fixation location (see Table 1). In the model, the pre-saccadic shift is implemented by moving the attentional Gaussian to center around the next fixation location, while the inhibition remains in the same position for a time τ_{pre} . The inhibition stream is calculated for the entire fixation duration using Eq. 6, therefore, we have

$$G_A^{pre}(x, y) = \frac{1}{2\pi\sigma_A^2} \exp\left(-\frac{(x - x_{f+1})^2 + (y - y_{f+1})^2}{2\sigma_A^2}\right), \quad (9)$$

and then continue computations using Eqs. 5-6 with G_A^{pre} instead of G_A for the duration of τ_{pre} . When the pre-saccadic phase terminates, the saccade is executed.

Extended model: Post-saccadic attentional shifts

The center of the post-saccade attention peak is determined by extending the vector of the preceding saccade by a shift amplitude η , i.e.,

$$(x_s, y_s) = (x_n, y_n) + \frac{(x_\delta, y_\delta)}{\sqrt{x_\delta^2 + y_\delta^2}} \cdot \eta, \quad (10)$$

where the saccade direction is given by the vector (x_δ, y_δ) with $x_\delta = x_n - x_{n-1}$ and $y_\delta = y_n - y_{n-1}$. Thus, the attentional Gaussian is centered at the shifted location

$$G_A^{post}(x, y) = \frac{1}{2\pi\sigma_{post}^2} \exp\left(-\frac{(x - x_s)^2 + (y - y_s)^2}{2\sigma_{post}^2}\right) \quad (11)$$

during the post-saccadic phase. Temporal evolution of activation maps continues based on Eqs. 5-6 with G_A^{post} instead of G_A for a duration of τ_{post} . Meanwhile the inhibition stream evolves with the center of inhibition in the same location as the fixation position.

After the post-saccadic shift phase, the cycle is completed and another main phase follows. The attention center moves to each of the three locations in turn via discrete steps as shown in Table 1. We have chosen this discrete approximation with constant durations of pre- and post-saccadic shifts to compute activation changes in all fixation phase efficiently. Neurophysiological support for our discrete approximation has been found [20], indicating that attention does not move smoothly over space from location n to location $n + 1$ but instead selectively starts building up at the target location $n + 1$.

Extended model: Facilitation of Return

To account for Facilitation of Return (FoR) we implement a selectively slower decay of the attention map in a spatial window centered at the previous fixation location. Different from the overall decay rate ω_A , we define a reduced decay rate ω_{FoR} for a window $x - \nu < x_{f-1} < x + \nu$ and $y - \nu < y_{f-1} < y + \nu$ around the previous fixation location (x_{f-1}, y_{f-1}) , where ν is the size of the window. Therefore, reduced decay of activation in the attention map, Eq. 5, is given by

$$A(t) = \frac{G_AS}{\sum G_AS} + e^{-\omega_{FoR}(t-t_0)} \left(A_0 - \frac{G_AS}{\sum G_AS} \right) \quad (12)$$

for the spatial window defined above.

Extended model: Center Bias

The original SceneWalk model initiates its activation maps with uniform distributions. While it is difficult accurately know the initial state of the visual system when viewing images, previous work has shown that the central fixation bias has a strong influence on the first fixation. Starting the model with a central activation improves the predictions of the model [55]. In line with this finding, we also initiated the model with central activation. The evolution equation for the first fixation is

$$A(t) = \frac{G_{fix}S}{\sum G_{fix}S} + e^{-\omega_{cb}(t-t_0)} \left(A_{0_{CB}} - \frac{G_{fix}S}{\sum G_{fix}S} \right) \quad (13)$$

Extended model: Oculomotor Potential

Research into the oculomotor system has revealed a marked preference for saccades in the cardinal directions. In order to implement this tendency in the model we introduced an additive oculomotor component. A plus-shaped oculomotor map centered on the current fixation position is generated

$$OMP = \left((x - x_f)^2 \cdot (y - y_f)^2 \right)^\chi, \quad (14)$$

where the factor χ determines the steepness of the slopes. The oculomotor map is added to the combined map u_{ij} , before the normalization and the addition of noise (Eq. 7, 8)

$$u_{OMP} = u + \left(\psi \cdot \left| -\frac{OMP}{\sum(OMP)} \right| \right). \quad (15)$$

Extended model: Additional parameters

The implementation of the extended SceneWalk model gives rise to several new parameters. To the 7 parameters of the original model we add (a) ω_{cb} , the decay speed of the center bias, (b, c) σ_{cb_x} and σ_{cb_y} , the size of the center bias, (d, e) τ_{pre} and τ_{post} , the durations of the attention shift phases, (f) η , the distance of the post-saccadic shift, (g) σ_{post} the size of the shifted Gaussian, (h, i) ω_{For} , the attention decay at the previous fixation position and ν , the size of the facilitation window, and (j, k) the steepness χ and factor ψ of the oculomotor potential.

Estimated and fixed model parameters

We implemented a fully Bayesian approach to parameter inference [59] using numerical computation of the models' likelihood functions and advanced Monte Chain Monte Carlo (MCMC) techniques. Details are given in the next section. For a discussion of the full results including marginal posterior densities see below (Section entitled *Detailed results from parameter estimation*).

In Table 2 we report point estimates for all parameters as averages over participants. The full estimates for each participant can be found in the *SI Appendix (Tables S1 and S2)*. These point estimates were computed from the posterior densities by determining the highest posterior density region for an alpha of 0.5 (i.e., the highest 50% of the density are in this region), assuming a unimodal distribution. The reported credibility intervals the lower and upper bounds of the highest density interval. The point estimate for the parameter represents the center of the highest posterior density interval.

Some of the model parameters could be constrained by the physiological literature and some of the parameters had to fixed in order to improve convergence of the parameter estimation. The latter case was checked by large-scale pilot simulations with different model versions using a separate data set. In Table 3 we list all fixed model parameters.

First, we separated the time scales of attention and inhibition stream by one order of magnitude, i.e., $\omega_F = \omega_A/10$. We assume ω_F is slower to decay by a magnitude than ω_A , to enable long term inhibition of return and fast build-up of activation for attentional capture. Second, we set $C_F = 0.3$, where the numerical value was obtained from pilot simulations indicating that the relative influence of the inhibition stream must be smaller (but not negligible) compared to the corresponding influence of the attention stream.

In the extended model, some of the additional parameters need further discussion. First, we set $\sigma_{CB} = (4, 3)$ and $\omega_{CB} = 1.5$ as described in [55], for a typically sized center bias and an attention decay that is slower than the regular ω . The center bias parameters are difficult to estimate, since

Parameter	Baseline SceneWalk		Extended model	
	Mean	CI +/-	Mean	CI +/-
ω_A	14.802	2.555	10.121	2.581
σ_A	7.482	1.165	7.315	0.986
σ_F	4.629	1.041	6.958	2.799
γ	0.935	0.095	0.960	0.100
$\log(\zeta)$	-1.132	0.131	-1.794	0.279
η	-	-	0.439	0.094
$\log(\psi)$	-	-	-0.630	0.197

Table 2: Maximum posterior density estimates of the model parameter estimations of all subjects and credibility intervals (see text).

Parameter	Baseline	SceneWalk	Extended Model
ω_A/ω_F		10	10
CF		0.3	0.3
τ_{pre}		—	0.05
τ_{post}		—	0.1
ν		—	2
σ_{post}		—	2
ω_{CB}		—	1.5
σ_{CBx}		—	4
σ_{CBy}		—	3
ω_A/ω_{FoR}		—	10
χ		—	0.10

Table 3: Fixed model parameters for baseline and extended model

their influence is mainly limited to the first fixation. Second, we fixed $\omega_{FoR} = \omega_A/10$, representing an approximate value for decay slower by a magnitude and the size of the facilitation of return window to be approximately the size of the fovea, i.e., 2° of visual angle. As before, only a relatively small amount of fixations are influenced by this mechanism, making it difficult to identify the numerical value reliably. Third, we set the times for post- and pre-saccadic attentional shifts to $\tau_{pre} = 0.1$ s and $\tau_{post} = 0.05$ s, where the numerical values are determined by pilot simulations. Due to their small magnitude, values for ζ and ψ were estimated in the log scale.

Data assimilation: Bayesian parameter inference

Parameter inference of the dynamical models discussed here was implemented in the general framework of data assimilation [52] using a fully Bayesian estimation procedure [59, 60, 49]. In this statistical inference we used the computation of the models’ likelihood functions. Given a fixation sequence $f_1 \dots f_{i-1}$, where is fixation f_i is determined by its coordinates $f_i = (x_i, y_i)$, the likelihood of the model specified by a set of parameters θ can be computed as a product of probabilities, i.e.,

$$L_M(\theta|\text{data}) = P_M(f_1) \cdot \prod_{i=2}^n P_M(f_i|f_1, \dots, f_{i-1}, \theta) , \quad (16)$$

where $P_M(f_1)$ is the probability of the initial fixation starting at time $t = 0$ and the conditional probabilities $P_M(f_i|f_1 \dots f_{i-1}, \theta)$ can be read off from the models priority map $\pi(i, j)$.

For scaling and numerical reasons the log-likelihood is usually used. Thus, the sum of the scan path’s log-likelihood per fixation for the entire data set gives one value that characterizes model performance. As suggested by [59], taking the $\log_2$ of the likelihood enables the use of the unit bit. A null model, in which the probability of choosing each point a 128×128 pixel image is the constant, would be $\log_2(1/128^2) = -14$. A hypothetical model which, unrealistically, perfectly predicts the data would have a log-likelihood of 0. It is important to note that for model comparison we can take the mean log-likelihood per fixation while for the parameter estimation the non-normalized sum log-likelihood of a scan path is the appropriate measure.

Based on the likelihood $L_M(\theta|\text{data})$ and a prior distribution $P(\theta)$, the posterior distribution is computed via Bayes’ rule as

$$P(\theta|\text{data}) = \frac{L(\theta|\text{data})P(\theta)}{\int_{\Omega} P(\theta)L(\theta|\text{data})d\theta} , \quad (17)$$

where typically a Markov Chain Monte Carlo (MCMC) approach is needed to compute the posterior density numerically. For our parameter estimations we used the implementation of the DREAM Algorithm that is published as PyDream [37]. Each estimation ran three chains of 20,000 iterations. Since the DREAM estimation procedure requires a large number of model evaluations, the computing time of the likelihood function is critical for the baseline SceneWalk model and, in particular, for the extended SceneWalk model. We therefore implemented parallel computations of the likelihood for fixation sequences. The priors, loosely based on pilot estimations on a separate dataset, were chosen to be broad and relatively uninformative.

Inter-individual differences in behavior are a main source of variance in eye movement data. Here we took advantage of these differences by testing model generalizability. We implemented individual independent model fitting for each participant by running a DREAM parameter estimation for each participant separately. The advantage of using this method is that when simulating data, we obtain an upper limit for the variance of parameters between individual participants.

Detailed results from parameter estimation

Based on the statistical methods described in the previous section, we obtained the parameter point estimates reported in Table 2 averaged across participants (see *SI Appendix: Tables S1 and S2. for by-participant estimates*)

As the parameter estimation was conducted in a fully Bayesian framework, we have also access to the full posterior likelihood distribution of each parameter. In Fig. 11 we show the marginal posterior distributions of each parameter of the model for all subjects. The marginal posterior distributions can serve as a tool to understand how well the parameters constrain the data.

For each parameter, we link the interpretation of the marginal posteriors to the function of the corresponding parameter in the model. The speed of the decay of the attention stream ω_A controls the duration of the memory of the process for allocation to past target locations. The numerical value $\omega_A = 10.12$ indicates a half life of 70 ms of the previous map’s influence (i.e., $\exp(-10.12 \cdot 0.070) = 0.49$). The Gaussian of the attention and inhibition streams are specified by parameters $\sigma_A = 7.3$ and $\sigma_F = 6.9$, resp., which are corresponding to standard deviation parameters of the Gaussian function in units of degrees of visual angle. The marginal posteriors of both parameters are largely overlapping. The exponent $\gamma \approx 1$ indicates that the weighting of the corresponding activation maps is negligible. Finally, the shift parameter η is clearly smaller than one, as expected.

There are two indicators lend support to the stability of our parameter estimations using the baseline SceneWalk model and the extended model in combination with the DREAM method. Firstly, the three chains which we ran for each subject resemble each other quite closely. Also, while the parameter estimates vary between participants, in most cases they do not differ dramatically (see Fig. 12). Secondly, we conducted recovery analyses of the estimated parameters, where a parameter estimation is run on simulated data. The dream algorithm was able to identify the parameter values from the simulated data (see *SI Appendix: Parameter recovery analysis*).

Data and Code Availability

The experimental data used in this study represent a subset of the Potsdam Corpus on Spatial Frequency Search in Natural Scenes [57], which is publicly available via the Open Science Framework (<https://osf.io/caqt2>). Source code used for numerical simulations, analyses, and plotting as well as other project-related files are also available via OSF (<https://osf.io/qsx4w/>).

Figure 11: Marginal posteriors of the estimated model parameters as calculated by fitting the model to the training data using PyDream.

Figure 12: Posterior density of individuals and chains of the model fit using DREAM. The different colors show chains belonging to one subject. The black line is the prior. The consistency of the chains within each subject indicates that we achieved a reliable fit. All chains are markedly different from the posterior, allowing us to update our beliefs.

Acknowledgments We thank Noa Malem-Shinitzki, Maximilian Rabe, Stefan A. Seelig, Silvia Makowski for valuable discussions. This work was supported by a grant from Deutsche Forschungsgemeinschaft, Germany (SFB 1294, project no. 318763901). We acknowledge a grant for computing time from Norddeutscher Verbund für Hoch- und Höchstleistungsrechnen (HLRN), Germany (grant no. bbx00001).

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Supplementary Information Appendix

This PDF file includes:

- Supplementary Text
- Table S1 (Individual point estimates, baseline model)
- Table S2 (Individual point estimates, extended model)
- Figures S1 (Model performance on further statistics)
- Figures S2 (Correlations of empirical and simulated data)
- Figure S3 (Recovery Analysis)

SI Methods

Further Systematic Tendencies

In addition to the measures of scan path statistics reported in the *Results* we also investigated model performance with respect to two further statistics. First, as a measure of how fixations spread over an image over time we investigated the *mean lag distance*, defined as the distance between two fixations, separated by x other fixations (Fig. S1A). Empirical data indicate that the distance between fixation n and $n + x$ separate quickly for 3 to 5 fixations before reaching peak distance and returning to chance-level distance. We interpret this overshoot-type behavior as an indication of inhibitory tagging as one of the key driving mechanisms during scene exploration. The general tendency is present in the baseline model. While the extended model improves the fit to experimental data, the overshoot in the distance is not present. From this result, we might conclude that the inhibitor component is currently too weak in both mathematical models.

Second, we analyzed the dependence of the central fixation bias on the initial saccade latency (Figure S1B). Note that this analysis includes the full data set (i.e., the analysis is not limited to the test data) to produce more stable results. As a result, a larger latency before the first saccade is systematically related to a reduced central fixation bias compared to shorter fixations. The changes to the model improve the dynamic dependency of this measure.

Individual parameter estimates

Each subject’s data was individually fitted to both the baseline model and the extended model. In Table S1 we report the results of parameter estimation of the baseline model (SceneWalk). Table S2 gives the estimated parameters for the extended model. For each estimated parameter we computed the point estimate and the corresponding 50% credibility interval.

Individual Differences

Using separate by-participant estimates, it is possible to examine how well the fitted model parameters capture inter-individual differences. For this analysis we compared the subject-specific experimental data to data simulated with the subject’s set of parameters. We expect that subjects with high expression of a statistic should also show a higher degree of that statistic in simulated data. the specific model parameters fitted for each participant show good agreement between experimental and simulated data, and explain inter-individual differences, which we quantified by the corresponding correlation coefficient.

In Figure 3 we show the correlation of mean saccade amplitude in experimental and simulated data. Correspondingly, Figure S2A shows the correlations for the mean lag distance metric. We chose the distance at fixation 5 as our correlation measure, which on average is the point of peak distance in the experimental data. Both saccade amplitude and inhibition show good agreement between experimental and simulated data, suggesting that the model parameters capture the inter-individual differences adequately. The correlation is particularly interesting given that the mean lag distance peak is not strongly present in the simulated data.

Mean saccade amplitude (Fig. 4) is closely related to the activation stream and lag distance to the inhibition stream. Thus, as the two most fundamental mechanisms in the model, the fact that they are well-represented in the individual model fits lends support to our model.

In Figure S2B and C we show correlations between empirical and simulated data concerning the amount of forward and return saccades. These metrics represent the two most important new model components. The values we compare are the number of fixations per square degree that fall within a defined window for each peak. While there is more variation between subjects in the case of forward saccades, the magnitude of each tendency is well-represented by the model fit. The positive correlation of these measures shows that the model extension capture the relevant aspects in the data and account for inter-individual differences.

Parameter recovery analysis

As an indicator for the reliability of our numerical simulations and, in particular, of our statistical inference, we investigated the parameter inference on simulated data with known parameters. We simulated data using parameter point estimates for one representative participant. Simulated data were then fed back to parameter inference, as outlined above (Section *Materials and Methods*). In Figure S3 we report the posteriors over parameters in comparison to the true numerical values. Thus, the combination of model simulations and the estimation procedures can recover values from the experimental data reliably. These analyses strengthen the credibility, stability, and robustness of our mathematical modeling approach.

Figure S1: Model performance on additional statistics. (A) Mean distance between a fixation and subsequent fixations in experimental and simulated data. (B) Modulation of the the first fixation's mean distance to the image center by initial saccade latency.

Figure S2: Correlations between experimental and simulated data across participants. (A) Mean lag distance at fixation 5. (B) Number of fixations per square degree that land within the area expected to contain forward saccades. (C) Number of fixations per square degree that land in the area expected to contain return saccades.

Figure S3: Model parameter recovery analysis. Panel report posteriors for all estimated model parameters obtained from simulated data. The three curves represent three estimation chains of the DREAM algorithm. The green, dotted line is the parameter value from which data was generated. The red lines show the recovered parameter values: the maximum posteriori value (solid line) and the 50% credibility interval (dotted line). Black lines show the prior.

Subject	γ	γ +/-	ω_A	ω_A +/-	σ_A	σ_A +/-	σ_F	σ_F +/-	$\log(\zeta)$	$\log(\zeta)$ +/-
0	0.927	0.025	24.368	1.702	7.551	0.141	5.566	0.264	-1.053	0.041
1	0.911	0.026	12.819	0.825	7.919	0.134	3.144	0.103	-1.172	0.057
2	1.082	0.023	17.484	1.154	6.406	0.101	5.111	0.208	-1.162	0.044
3	0.752	0.019	15.499	0.919	6.605	0.101	3.760	0.212	-1.402	0.072
4	1.409	0.030	44.364	2.301	9.069	0.183	0.061	0.014	-1.974	0.270
5	1.070	0.020	12.667	0.921	8.067	0.175	4.651	0.232	-1.343	0.058
6	0.963	0.020	14.522	0.791	5.137	0.085	3.333	0.181	-1.178	0.047
7	1.091	0.026	11.743	0.660	4.054	0.064	3.744	0.128	-1.056	0.031
8	0.741	0.014	18.658	1.272	5.595	0.090	5.228	0.178	-1.104	0.045
9	0.990	0.019	10.869	0.631	5.632	0.072	3.728	0.098	-1.697	0.077
10	0.960	0.027	12.875	0.925	6.161	0.097	5.844	0.224	-1.651	0.083
11	1.141	0.027	21.191	1.683	8.478	0.145	6.282	0.267	-1.091	0.041
12	0.935	0.021	14.712	1.112	4.923	0.090	3.760	0.228	-1.093	0.040
13	0.867	0.020	13.251	0.840	4.893	0.078	4.288	0.154	-1.366	0.050
14	0.810	0.018	15.372	0.929	7.214	0.109	4.275	0.190	-1.608	0.098
15	1.245	0.026	16.038	1.688	7.659	0.159	8.140	0.803	-1.106	0.035
16	0.982	0.029	21.626	2.352	4.775	0.077	5.497	0.234	-1.028	0.033
17	0.913	0.018	13.101	0.996	7.977	0.136	5.982	0.255	-1.896	0.136
18	0.978	0.020	16.447	1.258	8.533	0.166	6.587	0.333	-1.225	0.051
19	0.917	0.018	12.971	0.879	6.828	0.120	4.256	0.164	-1.199	0.050
20	0.958	0.033	13.985	1.574	8.016	0.177	7.987	0.502	-1.040	0.041
21	0.943	0.014	8.820	0.802	6.920	0.136	6.137	0.483	-1.496	0.062
22	1.367	0.029	15.801	0.892	8.700	0.135	3.196	0.110	-0.971	0.035
23	0.952	0.038	13.998	1.466	6.886	0.153	16.378	1.585	-1.045	0.044
24	1.167	0.021	15.325	1.163	6.360	0.104	5.206	0.277	-1.266	0.044
25	1.067	0.018	16.058	1.094	7.538	0.116	5.264	0.250	-1.384	0.054
26	1.229	0.030	8.680	0.869	9.245	0.269	5.290	0.271	-1.213	0.079
27	0.859	0.018	18.938	1.356	4.884	0.098	4.240	0.210	-1.525	0.085
28	1.196	0.019	20.110	1.568	8.180	0.142	5.410	0.199	-1.170	0.043
29	1.463	0.036	13.326	2.919	10.551	0.243	16.203	1.534	-1.076	0.036
30	0.920	0.019	15.208	0.947	7.034	0.122	3.822	0.132	-1.201	0.051
31	0.231	0.011	43.633	3.538	2.939	0.091	3.158	0.179	-0.977	0.045
32	0.862	0.023	16.398	1.213	3.920	0.070	2.775	0.122	-1.294	0.048
33	1.067	0.024	14.702	1.038	5.711	0.076	3.825	0.155	-1.261	0.045
34	0.944	0.032	24.298	2.569	9.347	0.227	6.307	0.349	-0.898	0.043

Table S1: Estimated parameter values of the baseline SceneWalk model for all participants (see Table 3 for fixed parameters).

Subject	η	η +/-	γ	γ +/-	ω_A	ω_A +/-	σ_A	σ_A +/-	σ_F	σ_F +/-	$\log(\psi)$	$\log(\psi)$ +/-	$\log(\zeta)$	$\log(\zeta)$ +/-
0	0.512	0.023	0.914	0.033	23.005	1.481	8.903	0.283	8.809	0.916	-0.603	0.061	-1.539	0.148
1	0.431	0.015	0.924	0.026	7.806	0.457	9.181	0.190	4.089	0.232	-0.742	0.074	-2.408	0.463
2	0.410	0.019	0.932	0.027	15.209	1.091	7.071	0.184	7.218	0.517	-0.749	0.102	-1.715	0.156
3	0.493	0.018	0.805	0.021	12.805	0.655	7.537	0.164	4.543	0.209	-1.036	0.093	-1.653	0.093
4	0.433	0.048	1.267	0.032	62.825	3.678	8.411	0.181	0.058	0.014	-1.569	0.183	-2.671	0.398
5	0.374	0.022	0.900	0.015	19.035	1.607	7.795	0.146	4.846	0.211	-1.967	0.348	-1.609	0.074
6	0.414	0.008	1.246	0.024	12.265	0.568	7.126	0.150	0.014	0.014	-1.341	0.113	-1.912	0.158
7	0.511	0.019	1.455	0.042	8.511	0.462	4.254	0.159	14.681	1.293	-0.347	0.023	-1.624	0.105
8	0.428	0.020	1.052	0.025	9.402	0.469	7.452	0.197	9.765	0.598	-0.474	0.024	-1.891	0.136
9	0.509	0.016	1.098	0.019	8.602	0.350	6.438	0.111	3.211	0.117	-2.069	0.303	-1.729	0.076
10	0.465	0.019	0.983	0.029	8.198	0.381	6.599	0.142	6.944	0.502	-0.844	0.054	-2.290	0.166
11	0.399	0.014	0.926	0.021	12.107	0.944	9.496	0.245	8.526	0.410	-0.889	0.051	-1.529	0.077
12	0.309	0.018	1.066	0.025	10.456	0.671	6.396	0.146	12.607	0.699	-0.461	0.017	-2.336	0.233
13	0.521	0.014	0.905	0.019	9.722	0.371	5.660	0.096	6.942	0.297	-0.794	0.037	-1.962	0.102
14	0.582	0.022	0.822	0.019	14.860	0.710	7.919	0.113	4.573	0.159	-1.330	0.155	-1.908	0.157
15	0.319	0.025	0.901	0.020	7.657	0.649	8.310	0.201	10.074	0.391	-0.693	0.026	-3.053	0.398
16	0.350	0.010	1.282	0.029	10.339	0.500	6.669	0.164	17.669	1.397	-0.519	0.025	-1.431	0.063
17	0.380	0.035	1.143	0.064	39.646	6.552	9.251	0.266	15.806	1.866	-0.598	0.064	-2.013	0.182
18	0.320	0.010	1.040	0.023	9.748	0.565	11.723	0.374	10.026	0.673	-0.654	0.036	-1.973	0.129
19	0.302	0.019	0.876	0.020	8.678	0.654	7.834	0.178	4.396	0.161	-1.362	0.204	-1.408	0.070
20	0.751	0.019	0.983	0.036	29.106	3.220	8.227	0.193	9.823	0.748	-0.738	0.051	-1.255	0.065
21	0.368	0.022	1.020	0.029	10.521	0.903	8.142	0.156	9.599	0.532	-0.812	0.047	-1.919	0.123
22	2.816	0.138	1.487	0.041	70.738	3.948	10.418	0.274	15.144	0.875	-0.445	0.024	-2.384	0.320
23	0.200	0.019	1.349	0.038	0.638	0.043	9.597	0.571	18.442	1.329	-0.455	0.020	-2.016	0.140
24	0.164	0.011	1.162	0.022	9.698	0.464	8.059	0.200	7.684	0.329	-0.747	0.033	-1.935	0.126
25	0.041	0.017	0.847	0.032	12.919	1.234	7.930	0.232	9.957	1.213	-0.731	0.071	-2.286	0.272
26	0.159	0.021	1.142	0.032	6.360	0.498	11.858	0.599	9.812	0.731	-0.738	0.039	-2.197	0.237
27	0.416	0.022	1.015	0.019	8.781	0.425	6.421	0.100	4.872	0.203	-1.506	0.193	-1.672	0.079
28	2.592	0.192	1.604	0.039	84.978	3.954	9.473	0.208	20.181	1.192	-0.388	0.011	-2.360	0.218
29	2.763	0.069	1.623	0.043	60.694	4.229	10.687	0.258	19.423	1.219	-0.497	0.019	-1.935	0.136
30	0.483	0.032	0.823	0.019	10.922	0.573	7.493	0.157	4.252	0.263	-1.074	0.094	-1.544	0.077
31	0.687	0.012	0.933	0.024	13.246	0.642	6.981	0.141	9.399	0.505	-0.539	0.023	-2.062	0.179
32	0.488	0.016	0.910	0.017	9.710	0.357	4.605	0.069	3.810	0.133	-1.122	0.094	-1.681	0.088
33	0.254	0.010	0.946	0.024	17.716	1.464	6.460	0.148	6.940	0.700	-0.777	0.062	-1.817	0.119
34	0.013	0.006	1.024	0.035	25.395	2.560	11.680	0.433	9.178	0.428	-0.480	0.029	-2.665	0.417

Table S2: Estimated parameter values of the extended SceneWalk model for all participants (see Table 3 for fixed parameters).