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**Modeling response granularity with mixture models:
a case of severity ratings in child maltreatment**

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Type of Paper: Short Notes

Abstract

Heaping, where responses are concentrated in round numbers, such as 5 or 10, is one of the response behaviors observed in surveys. The heaping phenomenon in subjective quantities may be explained by response granularity, which expresses the uncertainty of subjective quantities. We conducted an exploratory study using mixture models with a response granularity assumption, and used the severity ratings of child maltreatment measured by slider scales with numerical feedback as a case of heaped subjective quantity (14 401 records from 487 respondents). The results of the model comparison showed that it was reasonable to assume a response granularity for severity ratings. We classified the respondents into five groups based on the uncertainty of response granularity because individual differences were observed. This study proposes the usefulness of an approach that utilizes mixture models to examine individual response granularity.

Keywords

Heaped data, Response granularity, Mixture model, Slider scale, Response behavior, Child maltreatment

Acknowledgments, disclosures, or funding information

On behalf of all authors, the corresponding author states that there is no conflict of interest.

We are grateful to Dr. Kensuke Okada for helpful discussions and comments on the manuscript.

Author contributions

Yui Furukawa wrote the original draft, and conceptualized the study, methodology, and software. Daiki Hojo wrote, reviewed and edited the paper, and conceptualized the study, methodology, and software. Jiro Sakamoto wrote, reviewed and edited the paper, and conceptualized the study and methodology. Kota Takaoka wrote, reviewed and edited the paper and supervised the study.

Introduction

Heaping, where responses are coarsened at some levels, is one of the response behaviors observed in surveys. Data can be coarse because responses are concentrated in round numbers, such as in multiples of 5 or 10 (known as digit preference; Vaske et al. 1996), or in the case of children's ages, in integer multiples of 6 or 12 months (Heitjan and Rubin 1991). The heaping phenomenon is often explained by satisficing theory (Krosnick 1991), in which respondents choose suboptimal but satisfactory answers to lower the cognitive effort of responding. There is a growing awareness of the problem of heaped data representing poor quality in various quantity measures (e.g., Vaske and Beaman 2006; Gideon et al. 2017). In contrast, according to Holbrook et al. (2014), the process of heaping differs depending on whether it is an objective or subjective quantity to be measured; for subjective quantity measurements, heaping does not always reflect poor data quality.

It has been reported that heaping is likely to occur when a slider scale, which is used to measure subjective quantities in web surveys, is used with a numerical feedback function (Maineri et al. 2019). With slider scales, respondents rate by moving a bar on a line segment (DeCastellarnau 2018). Survey designers can design a scale that ranges from 0 to 100, similar to the visual analogue scale often used in pain research (Chyung et al. 2018). The nuance of subjective quantity is expressed in 101 levels (or finer). It is used in health care and welfare (e.g., Stinson et al. 2013; Hyland et al. 2020) as well as in marketing and psychological research (e.g., Treiblmaier and Pollach 2006; Smith et al. 2019). Survey designers using slider scales expect to obtain measurements in 101 levels; however, numerical feedback may result in coarse data.

We expect not only satisficing via digit preference, but also that the uncertainty of the subjective quantity being measured may cause heaping in the slider scale with numerical feedback. Krifka (2007) noted that people use round numbers to coarsen their representations when expressing highly uncertain events, and named this observation the round numbers round interpretation (RNRI) principle. Based on the RNRI principle and the satisficing theory, the heaping phenomenon may be an expression of the respondents' uncertainty regarding the subjective quantity to be measured. In other words, respondents may have some response granularity that becomes coarser as their uncertainty increases. Heaped ratings can then be generated by satisficing via response granularity. Ackerman and Goldsmith (2008), for example, examined the cognitive processes of response granularity.

Variation in response granularity among respondents is problematic, not only for survey designers who measure subjective quantities. Studies on risk assessment approaches for child maltreatment have focused on reliability (the extent to which different users of a tool make the same assessment in the same situation compared to other tools; Hughes and Rycusa 2006) and validity (the accuracy in classifying children at risk of harm). The evaluation of the severity of maltreatment cases often differs among evaluators (i.e., staff of child welfare institutions). As a result, it is difficult to obtain a common understanding of the severity of the case and ensure consistency in support. Considering the individual's response granularity in severity ratings may contribute to increasing the reliability of risk assessment tools in child maltreatment.

In this study, mixture models were used as an approach for an exploratory study of response granularity, and severity ratings data in child maltreatment taken by slider scales (0 to 100) with numerical feedback were used. This idea is similar to some studies that used mixture models for heaped count data (e.g., Cummings et al. 2015; Bermúdez et al. 2017), but differs in terms of modeling the response behavior for ratings data. By comparing the

models, we consider whether it is reasonable to assume that different individuals have different response granularities for severity ratings. We also consider individual differences in response granularity.

Models

Null model: with no response granularity assumption

First, we consider a model without the assumption that different individuals have different response granularities (Model 0). Let K be the response granularity of a respondent, which is the number of possible response levels in answering the items; $K = 100/L + 1$, where L is the rounding unit. Model 0 assumes $L = 1$ for all respondents, and that the number of possible response levels, $K = 101$. Let y_{ij} be the rating that respondent i gives for item j . A rating is generated from a normal distribution with a mean μ_j , that is different for each item, and a standard deviation σ , that is the same across all items. Because the actual responses are discrete values between 0 and 100, we assume that y_{ij} is a random sampling from a categorical distribution with probability $\theta_{j101} \in [0,1]^{1 \times 100}$, where each value between 0 and 100 is chosen. θ_{j101} is a vector that divides $\text{Normal}(\mu_j, \sigma)$ into 101 parts, and is described as follows:

$$\theta_{jkl} = \begin{cases} \text{NormalCDF}(L/2|\mu_j, \sigma) & l = 1 \\ \text{NormalCDF}(L \cdot l - L/2|\mu_j, \sigma) - \text{NormalCDF}(L(l-1) - L/2|\mu_j, \sigma) & 2 \leq l \leq K-1 \\ 1 - \text{NormalCDF}(100 - L/2|\mu_j, \sigma) & l = K \end{cases} \quad (1)$$

Here, $\text{NormalCDF}(\cdot)$ represents the cumulative distribution function (CDF) of the normal distribution. We denote the likelihood of the model as

$$P(y_{ij}|\theta_{jK}) = \text{Categorical}(y_{ij}^*|\theta_{jK}). \quad (2)$$

Here, y_{ij}^* is the transformation that makes y_{ij} a sampling from the categorical distribution with category size K , and $y_{ij}^* = \frac{y_{ij}}{L} + 1$.

Counter models: with the response granularity assumption

We construct several models assuming that each respondent has their own response granularity. We assume five response granularities: 21 levels (responses are rounded in multiples of 5), 11 levels (responses are rounded in multiples of 10), five levels (responses are rounded in multiples of 25), three levels (responses are rounded in multiples of 50), and two levels (responses are 0 or 100). We construct several mixture models (Models 1 to 5), depending on which levels were included as the coarsest response granularity (as K levels). The possible values of K and L in each model are as follows:

Model 1: $K \in \{101, 21\}$ and $L \in \{1, 5\}$,

Model 2: $K \in \{101, 21, 11\}$ and $L \in \{1, 5, 10\}$,

Model 3: $K \in \{101, 21, 11, 5\}$ and $L \in \{1, 5, 10, 25\}$,

Model 4: $K \in \{101, 21, 11, 5, 3\}$ and $L \in \{1, 5, 10, 25, 50\}$,

Model 5: $K \in \{101, 21, 11, 5, 3, 2\}$ and $L \in \{1, 5, 10, 25, 50, 100\}$.

Note that Model 1 (including 21 levels) and Model 2 (including 21 and 11 levels) are valid even though they assume only digit preference, and not response granularity.

For illustration, we consider Model 5, which is the most complex model. That is, a model containing a response granularity of $K \in \{101, 21, 11, 5, 3, 2\}$ levels with the corresponding rounding in multiples of $L \in$

$\{1,5,10,25,50,100\}$ ($K = 100/L + 1$). As in Model 0, we assume that the rating is generated from $\text{Normal}(\mu_j, \sigma)$. It differs from Model 0 in that it assumes that y_{ij} is a random sampling from a mixture of categorical distributions with K categories, weighted by the response granularity uncertainty $\mathbf{p}_i = \{p_{iK}\}$ that respondent i has at each response granularity, where $\sum_K p_{iK} = 1$. In other words, p_{iK} is the probability that each respondent's answer is sampled from K levels.

The probability $\theta_{jK} \in [0,1]^{1 \times K}$ for selecting each point at the response granularity K is determined using Equation (1). Function $f_K(y)$ for the terms of the mixture distribution is defined as

$$f_K(y_{ij}|p_{iK}, \theta_{jK}) = \begin{cases} p_{iK} \text{Categorical}(y_{ij}^*|\theta_{jK}) & \text{if } y_{ij} \text{ is a multiple of } L \\ 0 & \text{otherwise} \end{cases}. \quad (3)$$

Here, $y_{ij}^* = \frac{y_{ij}}{L} + 1$, as previously described. We denote the likelihood of the model as

$$P(y_{ij}|\mathbf{p}_i, \theta_{jK}) = \sum_K f_K(y_{ij}|p_{iK}, \theta_{jK}). \quad (4)$$

Models 1 to 4 were constructed similarly.

In addition, the initial position of the slider bar can affect the response behavior (Maineri et al. 2019). To consider whether there is an effect of the initial position of the slider bar at position 50, we add Model 3 + 50I and Model 4 + 50I, which consider the 50-inflation to Models 3 and 4. For illustration, consider Model 4 + 50I ($K = \{101,21,11,5,3\}$); we define the term that represents the 50-inflation as

$$f_{50I}(y_{ij}|p_{i50I}) = \begin{cases} \text{Bernoulli}(50|p_{i50I}) & \text{if } y_{ij} = 50 \\ 0 & \text{otherwise} \end{cases}. \quad (5)$$

Then, the likelihood of the model is defined by adding the 50-inflation term to Equation (4) as

$$P(y_{ij}|\mathbf{p}_i, \theta_{jK}) = \sum_K f_K(y_{ij}|p_{iK}, \theta_{jK}) + f_{50I}(y_{ij}|p_{i50I}). \quad (6)$$

Here, $\mathbf{p}_i = (\mathbf{p}'_{iK=\{101,21,11,5,3\}}, p_{i50I})'$ and the sum of the elements of $\mathbf{p}_i$ is 1.

We set $\mathbf{p}_i \sim \text{Dirichlet}(\mathbf{1})$, $\mu_j \sim \text{Uniform}(0,100)$, and $\sigma \sim \text{Uniform}(0.1,100)$ as non-informative prior distributions. In total, eight models (Models 0 to 5, 3 + 50I, and 4 + 50I) were considered in this study.

Materials and Methods

Data

We used data obtained from the research and development of risk assessment tools for child abuse and neglect in the Japanese child protection systems conducted by the Ministry of Health, Labor and Welfare of the Japanese government (Takaoka et al. 2020). This survey aimed to develop a risk assessment tool for child maltreatment that could be shared among the agencies involved. To collect basic information on potential risk assessment items, they conducted a web-based survey of 1894 municipalities and 215 child guidance centers in Japan from mid-November to the end of December 2019.

In this study, we used severity ratings for 420 potential risk assessment items. The researchers asked respondents to rate the severity of each of the 30 items randomly selected from 420 items on a 0 to 100 slider scale with numerical feedback. The respondents subjectively rated how serious they would feel if the presented

item was applicable when assessing a child. The actual response screen used in the web survey is shown in Fig. S1 in the Supplementary Material. Respondents were able to answer by moving the slide bar with numerical feedback to the left or right using the mouse. The left end of the slider indicated "0: Not severe at all" and the right end indicated "100: Immediate life threat suspected." The slide bar was initially positioned at position 50, and no numerical feedback was provided. The responses were optional, and could be omitted. Data from respondents with fewer than 20 answers (134 records, 0.92%) were excluded from the analysis as defective responses. Finally, 14 401 data records from 487 respondents (376 municipalities and 111 child guidance centers) were used.

Settings

For statistical analysis, we used the statistical software R (R Core Team 2020) and the probabilistic programming language Stan (Carpenter et al. 2017). Parameter estimation was conducted using the Hamiltonian Monte Carlo (HMC) method. We sampled 2000 iterations in four independent random number chains and discarded 1000 samples in each chain as a warm-up period. A total of 4000 samples were used to construct posterior distributions. The codes used for the analysis are available at https://osf.io/vg4an/?view_only=14cd0f37a6294478aa8b6451e9d2aa73 (To peer reviewers: We have temporarily placed the codes at this URL because the original URL contains author information).

Results

We assumed that all parameters converged because the HMC samples were well mixed and there were no parameters with $\hat{R}$ greater than 1.1 (Gelman and Rubin 1992).

To compare the models, the following information criteria were calculated (Table 1): widely applicable information criterion (WAIC; Watanabe 2010), Pareto smoothed importance sampling leave-one-out cross-validation (PSIS-LOO; Vehtari et al. 2017), and the widely applicable Bayesian information criterion (WBIC; Watanabe 2013). All three criteria indicate that the model is better when the value is relatively lower; WAIC and PSIS-LOO estimate the predictive accuracy for future data, whereas WBIC is based on the approximation of the marginal probability density of the fitting data. We also calculated the standard deviation of the estimates for WAIC and PSIS-LOO. Note that WAIC and PSIS-LOO are known to be similar, and the values in this study were also similar (Vehtari et al. 2017).

Both WAIC and PSIS-LOO were the best for Model 4 + 50I, followed by Models 5 and 4. For Models 0 to 3, WAIC and PSIS-LOO were notably lowered by making the model more complex, whereas those of Model 3 and subsequent models were relatively close when the standard deviation was considered. WBIC was the best for Model 3, followed by Models 4 and 2.

Fig. 1 shows the results of fitting Model 4 + 50I (the best WAIC and PSIS-LOO), Model 3 (the best WBIC), Model 4 (the best overall ranking), and Model 0 (all results are shown in Fig. S2). The blue line represents the posterior predictive distribution, which is an indicator of descriptive adequacy (Shiffrin et al. 2008). Comparing the models, the posterior predictive distribution of Model 3 appears to have the best visual fit. Overall, based on the results of the information criteria and the posterior predictive distributions, we adopted Model 3 (which includes 101, 21, 11, and 5 levels in response granularity; $K \in \{101, 21, 11, 5\}$), especially because of its good

WBIC and good visual fit of the posterior predictive distribution. We also compared the results of the information criteria and the posterior predictive distribution for Models 3 and 3 + 50I, and Models 4 and 4 + 50I, respectively, as well as among all models. As a result, it could not be said that there was much of an improvement by considering the 50-inflation. Therefore, we concluded that the initial position of the slider bar had little effect on the results. There is a large gap between Model 0, which does not assume different response granularity for different individuals, and Models 1 to 5, which do, for each information criterion. The results of the posterior predictive distribution in Fig. 1 show that Model 3 is a better fit than Model 0. Based on these results, we conducted the following analyses, assuming that the assumption of response granularity was reasonable.

Fig. 2 shows the respondents' distribution for expected a posteriori (EAP) estimates of response granularity uncertainty $\mathbf{p}_i = (p_{i101}, p_{i21}, p_{i11}, p_{i5})'$. As mentioned earlier, p_{iK} is the probability that each respondent's answer is generated from K levels. The histogram of $K = 11$ shows that the respondents are distributed throughout the range of 0 to 1, which means that the values vary from individual to individual. Conversely, the histogram of $K = 101$ shows that almost all respondents' p_{i101} is close to 0. This indicates that most respondents answered with a coarser response granularity than the 101 levels.

For a more practical interpretation, we attempted to evaluate the response granularity for each respondent based on p_{iK} . If $p_{iK} > 0.5$, then the probability that the respondent answers with the response granularity of K levels is greater than the probability that they do not answer with the response granularity of K levels. Therefore, we classified respondents into five groups: 101 levels, 21 levels, 11 levels, 5 levels, and mixed group, by rating them based on 0.5, for each p_{iK} (Table 2). Specifically, respondents with $p_{iK} > 0.5$, were classified as the K levels group, and those with $p_{iK} \leq 0.5$, for all K , were classified as the mixed group. Table 2 shows that the 11 levels group was the most common, accounting for 42.92% of the total respondents, followed by the 21 levels group (20.12%) and the mixed group (18.48%). In addition, 18.48% of the respondents (mixed group) used multiple response granularities. The right side of Table 2 shows the percentages of ratings that are not multiples of five, multiples of five but not multiples of 10 or 25, multiples of 10, and multiples of 25 in the responses of each group; the underlined values in the table indicate the highest percentage in each column. The percentage of ratings that are not multiples of 5 (generated only from 101 levels) is highest for the 101 levels group, and the percentage of ratings that are multiples of 5 but not multiples of 10 or 25 (generated only from five levels and 101 levels) is highest for the 21 levels group, and so on. These are additional indications that there is a correspondence between the classification of response granularity groups and the actual responses. A histogram of the ratings for each group is presented in Fig. S3.

Discussions

In this study, mixture models were used as one of the approaches to examine response granularity, and an exploratory study was conducted using child maltreatment severity ratings data collected by slider scales (0 to 100) with numerical feedback.

Based on the results of model comparison, Model 3, assuming a response granularity of 101, 21, 11, and 5 levels was adopted for the severity ratings data. The fit to the data was much better when the response granularity of an individual was assumed. In addition, the fit of Model 3 was better than that of Models 1 and 2, which can be explained only by digit preference. Coarse levels (five levels), which cannot be explained by digit preference

alone, were included in the adopted model. These results quantitatively support the assumption of response granularity in the present study.

The severity ratings of the respondents showed individual differences in response granularity. To examine the differences, we classified the respondents according to the uncertainty of their response granularity. The results indicated that those who were most likely to respond in 11 levels accounted for approximately 40% of the total. Conversely, approximately 10% of the respondents answered comfortably with five coarse levels.

In this study, we quantitatively evaluated the differences in response granularity for the severity of child maltreatment per respondent. As a potential problem in practical situations, a person in the 5 levels group may convert a severity of 60 expressed by a person in the 11 levels group into a severity of 50. In the future, consideration of individual differences in the response granularity of severity ratings may contribute to the improvement of the reliability of risk assessment tools, and lead to the resolution of various problems arising from differences in common perceptions of severity (e.g., correction of inconsistencies in support and strengthening of cooperation among agencies involved).

It was suggested that the mixture model approach may be useful to account for individual differences in response granularity for certain subjective quantities. It is not appropriate to compare the raw scores of ratings with the individual differences in response granularity. For example, two pieces of data with a rating of 50 may be different in nature, or two pieces of data with ratings of 50 and 60 may be the same in nature. Researchers may use this approach to confirm differences in response granularity when heaped ratings of subjective quantities are available.

The approach provides more information to the researchers than simple aggregation of heaped ratings because it allows them to quantitatively evaluate how easy it is for a respondent to answer with a particular response granularity. The classification method in Table 2 was a hard classification in which each respondent always belonged to one group. By directly comparing the values of p_{iK} among the respondents, a soft classification can also be conducted. For example, among the 11 levels group, we can quantitatively identify those who are particularly likely to answer in 11 levels and those who are likely to answer in other response granularities by comparing p_{i11} . This would allow more flexibility for researchers working with heaped data.

The mixture model approach contributes to the understanding of the psychological process of heaping response behavior. It can provide suggestions for future research that considers which scales should be used when measuring subjective quantities (whether to use a slider scale or not) and how to present options when designing survey screens (such as whether to present numerical feedback on the slider scales). Furthermore, by extending the mixture models, it may be possible to construct statistical models to estimate the latent variables of the subjective quantity, such as severity, by controlling the effect of response granularity.

This is a case study that positively evaluates the psychological process by which the uncertainty of a subjective quantity provides each respondent with a response granularity, and satisficing via response granularity results in heaped data. However, this study involved only one case. It is not within the scope of this study to generalize the results to the overall severity of child maltreatment cases or to subjective quantities in general. It would be useful to examine whether a similar response granularity exists for other severity ratings and other subjective quantities, such as pain. In the future, these studies may contribute to solving problems caused by

differences in response granularity and reducing the burden on survey designers who measure subjective quantities.

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Table 1 WAIC, PSIS-LOO, and WBIC for the eight models (standard deviation in parentheses and rank represent the ranking of goodness based on each information criterion)

Model	WAIC			PSIS-LOO			WBIC	
		(se)	rank		(se)	rank		rank
Model 0	119517.35	(220.69)	8	119518.86	(220.69)	8	61559.66	8
Model 1	82787.71	(254.79)	7	82792.95	(254.92)	7	45117.69	7
Model 2	76339.76	(283.66)	6	76348.45	(283.83)	6	42415.71	3
Model 3	73050.02	(298.85)	5	73060.42	(299.03)	5	41712.64	1
Model 3 + 50I	72912.98	(312.12)	4	72923.65	(312.28)	4	42508.16	4
Model 4	72421.81	(304.97)	3	72432.72	(305.15)	3	42223.07	2
Model 4 + 50I	71347.02	(330.66)	1	71358.15	(330.82)	1	42948.28	5
Model 5	71930.78	(309.14)	2	71941.29	(309.30)	2	43145.71	6

Table 2 Breakdown of the estimated response granularity groups and the percentages of ratings that are not multiples of 5, multiples of 5 but not multiples of 10 or 25, multiples of 10, and multiples of 25 in the responses of each group (The underlined values in the table indicate the highest percentage in each column.)

Group of response granularity	Number of respondents		Percentage of responses with ratings in ^a				(%)
			not multiples	multiples of 5	multiples of	multiples of	
		(%)	of 5	but not 10 or 25	10	25	
101 levels	44	(9.03)	<u>69.95</u>	8.09	18.95	9.94	
21 levels	98	(20.12)	0.65	<u>33.72</u>	54.94	20.50	
11 levels	209	(42.92)	0.44	3.39	<u>94.24</u>	24.34	
5 levels	46	(9.45)	1.09	0.94	62.67	<u>91.79</u>	
Mixed	90	(18.48)	6.63	13.18	68.63	34.54	

a Note that being a multiple of 10 and being a multiple of 25 are not disjoint events; therefore, the sum of each group of percentages would not be 100%.

Fig. 1 Results of fitting Model 0, Model 3, Model 4, and Model 4 + 50I. The gray histogram represents the actual child maltreatment severity ratings, and the blue line represents the posterior predictive distribution (14 401 records from 487 respondents in all panels)

Fig. 2 Respondents' distribution of response granularity uncertainty $\mathbf{p}_i = (p_{i101}, p_{i21}, p_{i11}, p_{i5})'$ (487 respondents in all panels)