Cracking the code of place value: The relationship between *place* and *value* takes years to
master

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Author note

A pre-registration of this study can be found at <https://osf.io/pfuje> and all data and analysis code are available at <https://osf.io/4n7fz/>.

Abstract

Place value, which underlies the meanings of multi-digits, encompasses the principle of position and base-10 rules. To understand “65,” one needs to know that the digits “6” and “5” occupy different positions and thus represent ordered values of different magnitudes (the principle of position), and that the value of each position is determined by base-10 rules (e.g., the rightmost position is 10^0 , followed by 10^1 , 10^2 , etc.). Without the principle of position, children cannot construct meanings for multi-digits. Previous studies show that children do not know the exact value of digit positions until early elementary school years, but less is known about the acquisition of positional knowledge for multi-digits. To study when and how children construct a relationship between position and value, this study explored when children begin to know that the leftmost digit represents the largest value, and whether such knowledge relates to learning number names. Four to 7-year-olds from primarily Caucasian, middle-class families were asked to compare different pairs of multi-digits. Some comparisons (e.g., 12 vs. 21) required knowledge of positional property, and some did not (e.g., 35 vs. 36). We found that as a group, 6-year-olds could recruit positional knowledge to compare multi-digits. We also found that children who knew the number names of both of the multi-digits in a comparison pair were above chance on multi-digit comparison. Our results shed light on the developmental steps towards acquiring place-value notation, and highlight a role of learning number names for learning positional property of the place-value notation.

Keywords: place-value understanding, positional property, multi-digits, digit comparison, digit reading, base-10

Multi-digit numbers in the Arabic numeral system, such as “65” and “314,” are some of the essential ingredients of mathematics. Also known as multi-digits, they took centuries to attain their current form (Ifrah, 1981; Menninger, 1969; see also Zhang & Norman, 1995, for discussions). Without them, we cannot do formal arithmetic efficiently or advance much in civilization. With just 10 digits (0 to 9) and a positional structure, we can easily represent numbers of increasing magnitude with multi-digits. For example, in the Arabic numeral system, we represent numbers in the decades such as “sixty-five” as double-digit numbers (“65”), and to represent numbers in the millions, we simply include additional positions to the left, e.g., “31,434,578.” Representing the same 8-digit number in spoken numerals requires 13 words in English (i.e., “thirty-one million four hundred thirty-four thousand five hundred and seventy-eight”). The ingenious feature of the Arabic numeral system is that each position represents a distinct value, eliminating the need to use symbols to represent values such as “hundred,” “thousand,” or “million.” The positions of the digits themselves have numerical meanings. Previous studies have shown that children do not fully grasp meanings of multi-digits in early elementary school years (e.g., Fuson & Briars, 1990; Kamii, 1986; Ross, 1989). The current study examined when and how children begin to acquire a relationship between position and value in multi-digit numbers.

The dominant view in numerical cognition is that culturally constructed symbols for representing quantity are provided by evolutionarily ancient representations of numbers (e.g., Carey, 2009; Dehaene, 1997; Feigenson, Dehaene, & Spelke, 2004; Núñez, 2017; Szudlarek & Brannon, 2017). The Approximate Number System (ANS), which we share with pre-verbal human infants and nonhuman animals, is often evoked as the candidate system that supports the

learning of symbolic numbers and mathematical thought (e.g., Dehaene, 1997; Siegler & Booth, 2005; Szklarek & Brannon, 2017). The ANS account posits that numerical symbols for collections of objects are represented as approximate magnitudes—e.g., “65” is mapped onto magnitudes that may range from 60 to 70 elements, and not onto discriminably different magnitudes such as 400 or 1,000 elements (e.g., Lipton & Spelke, 2005). However, it is unclear how the ANS could explain the acquisition of symbols for large quantities such as “a thousand” or “1,000.” Our experience with the physical referents of “a million” or “1,000,000” is rare and limited. Developmental studies suggest that children do not understand multi-digits in the form of ANS representations (Yuan, Prather, Mix, & Smith, 2019; see also Le Corre & Carey, 2007; Sullivan & Barner, 2014). Even university students have been shown to be poor at estimating the relative magnitudes of one thousand, one million, and one billion (Landy, Silbert, & Goldin, 2013). These data suggest that the ANS is likely not foundational for the acquisition of multi-digit numbers. How, then, do children begin to acquire their meanings?

To acquire the meaning of multi-digit numbers so that one can compute with them, one needs to understand place value.¹ Place value is a notational system for representing written numerals in Arabic digits that originated in India and was brought to Europe by Arabic mathematicians.² In an Arabic digit, each position is associated with a value determined by base-

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¹ Here, we refer to multi-digits as a category that includes all possible multi-digit numbers. While it is possible that semi-numerate individuals may deal with currency notes and coins without place-value understanding, different denominations are often printed in different colours and sizes, and therefore it is not known whether semi-numerate individuals draw on non-numerical cues (e.g., colour, size, shape of symbol) when dealing with currency notes and coins.

² The Hindu-Arabic numeral system is not the first positional numeral system. The Babylonian numeral system was developed even earlier (Ifrah, 1981; Powell, 1976).

10. For example, the second position from the right represents tens and the third represents hundreds. Thus, in the Arabic numeral system, positional property and base-10 rules dictate the meaning of multi-digit numbers. We proposed a conceptual framework for studying place-value acquisition, and hypothesized three major steps children take to acquire the meaning of multi-digit numbers (see also Fuson, 1990; Fuson & Briars, 1990; Fuson et al., 1997; Ross, 1989; Sinclair, Garin, & Tièche-Christinat, 1992; Sinclair & Scheuer, 1993):

1. Individuate and identify a multi-digit number as a number. For example, “65” is not merely “a six and a five that are close together,” but a whole number.
2. Understand that a multi-digit number with N number of digits has N unique positions, or places, of values, and that the magnitudes of the values each position represents increase in ascending order from right to left.
3. Understand that the precise value of each position is determined by the base-10 system. In a base-10 system, there are 10 possible digits (0 to 9) that can fill each position, and each position represents powers of 10, in increasing order from right to left, from 10^0 to 10^1 , 10^2 , 10^3 , and so on.

At the core of place-value notation is the principle of position—namely that each position represents a distinct value (see also Ifrah, 1981; Ross, 1989; Sinclair & Scheuer, 1993), as outlined in Step 2.³ The principle of position underscores the relationship between place (or position) and value. In Mari, an ancient region of modern-day Syria and Iraq, written numerals for large numbers such as “16,740” used seven symbols, including symbols for 1, 6, 7, and 40, in

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³ It is important to note that positional property can integrate with other base systems.

addition to symbols for 10,000, 1,000, and 100 (see Ifrah, 1981). The people of Mari therefore wrote large numbers similar to how we use words such as “thousand” or “hundred” in spoken numerals. But these words or symbols are not necessary when using Arabic digits because the different values are stated implicitly according to their positions in place-value notation (e.g., the third position from the right in a multi-digit number represents hundreds, and the fourth position represents thousands).

The notion that different positions represent different values is the foundation of place-value understanding, and following Ross (1989), we use the term “positional property” or positional understanding throughout this paper to indicate that positional structure is a property of place-value notation. Without knowledge of positional property, we cannot construct precise meanings for multi-digit numbers. Acquiring knowledge of positional property is thus crucial to acquiring place value.

Despite the importance of positional property, few studies have directly examined its acquisition. Rather, previous studies on place value have focused on the mapping between digit positions and their values (e.g., the tens-digit position represents values of tens) and have not teased apart children’s understanding of positional property and base-10 rules. For example, in one paradigm, children were asked to represent a multi-digit number (e.g., “35”) using familiar objects such as sticks in a bag or manipulatives such as tens blocks and ones blocks (Miura, 1987; Miura, Okamoto, Kim, Chungsoon, Steere, & Fayol, 1993; Ross, 1989; Saxton & Towse, 1998; Towse & Hitch, 1997; Vasilyeva et al., 2014). In another paradigm, children were shown different combinations of tens blocks and ones blocks (e.g., 4 tens blocks and 4 ones blocks arranged in either a canonical or non-canonical order) and were asked to name or write down the

total quantity the blocks represented (e.g., “44”; Chan, Au, & Tang, 2014; Miura et al., 1993). These studies have generally shown that first- and second-graders are capable of representing “35” with 3 tens blocks and 5 ones blocks, or rearranging the tens and ones blocks to derive an answer in the form of a multi-digit (Chan, et al., 2014; Chan, Au, Lau, & Tang, 2017; Miura, 1987; Miura & Okamoto, 1993).

Studies that test the correspondence relationship between digit position and digit value using manipulatives attempt to assess knowledge about the *precise* meanings of digits in a multi-digit number—or how digit positions represent base-10—but leave open the question about when children acquire knowledge of positional property for multi-digits. Having knowledge of positional property and understanding the mapping between digit position and base-10 rules are two different, though closely related, aspects of acquiring place-value notation. Studies on the acquisition of place value must tease them apart for a full understanding of the acquisition process. For example, a child may know that the digit “8” in “81” represents a larger value than that in “18” because the former occupies the leftmost position, which represents the largest value in a multi-digit number (Ross, 1989). But this child may not necessarily know that the digit “8” in “81” occupies the tens position and can be represented by 8 tens blocks, and the digit “8” in “18” occupies the ones position and can be represented by 8 ones blocks. Understanding that the leftmost digit represents the largest quantity (and thus “81” > “18”) draws on knowledge of the principle of position, whereas knowing how to represent “81” or “18” with tens and ones blocks taps into knowledge of both positional property and base-10 rules. Knowing that digit positions represent different magnitudes of values is different from, and a prerequisite for, knowing the precise value of digit in a multi-digit number.

In addition, paradigms that use tens and ones blocks may obscure children's understanding of place value as a notational system for written numerals (Kamii, 1986; Mix, Prather, Smith, & Stockton, 2014). First, a child's failure to use tens blocks to represent a double-digit number may reflect a preference for strategy (e.g., counting out individual blocks is similar to counting objects). For example, modifying the task instructions to include practice trials with tens blocks, rather than only ones blocks (as in Miura et al.'s original studies), has prompted children as young as 5 and 6 to use tens blocks on new and larger numbers (Towse & Saxton, 1997; Saxton & Towse, 1998; Vasilyeva et al., 2014). These results suggest that children can use different strategies on manipulatives tasks when provided with different instructions. Second, any difficulty children have with manipulatives could be due to lack of understanding of how manipulatives represent quantities (Kamii, 1986). Although manipulatives are often touted as effective tools for teaching math, factors such as the type of manipulatives and student characteristics may affect their effectiveness in the classroom (Baroody, 1990; Kamii, Lewis, & Kirkland, 2001; Mix et al., 2014; see Carbonneau, Marley, & Selig, 2013, for a review). A child's failure to use manipulatives may represent an incomplete understanding of the relationship between manipulatives and quantities. Therefore, tasks that use manipulatives may not be ideal for assessing children's knowledge of positional property.

To investigate when and how children acquire place-value notation as a representation of multi-digits, it is important to examine positional property separately from children's knowledge of base-10 rules. Recent studies probing preschool children's knowledge of multi-digits have examined what children understand before acquiring base-10 rules, and researchers have suggested that younger children possess basic, though incomplete, knowledge of multi-digits

(Byrge, Smith, & Mix, 2014; Mix et al., 2014; Mix, Smith, Stockton, Cheng, & Barterian, 2017; Yuan et al., 2019). Mix and colleagues (2014) found that by around age 4, children can identify and compare multi-digit numbers. Specifically, when shown two multi-digit numbers (e.g., 807 vs. 78, or 305 vs. 350) and asked which is N (e.g., “seventy-eight” or “three hundred and five”), or when asked to choose the number that is more (e.g., 30 vs. 60, or 123 vs. 321), preschoolers performed above chance. Based on these findings, the researchers argued that children “must know—at the very least—that place matters” (p. 1316).

Nevertheless, children may succeed on the two tasks described above without recruiting positional knowledge. For example, children who map “seventy-eight” to “78” rather than to “807” may think that a longer number should have a large-value word such as “hundred” in its name, and thus rule out “807” as the referent for “seventy-eight.” Furthermore, correctly mapping number words to multi-digits reflects an understanding of left-to-right reading of multi-digits, but does not necessarily show that children know different positions have different *values*.

Children’s demonstrated understanding that the leftmost digit represents the largest value would serve as evidence that they have constructed a relationship between place and value. Studies have shown that by around ages 4 to 5, children can compare multi-digits (Mix et al., 2014; Yuan et al., 2019). However, not all multi-digit comparisons in previous studies required an understanding of the relationship between place and value. For example, a child who judges that “807” is more than “78” may rely on the length of a number or the number of digits—a non-positional-based strategy. When considering pairs of multi-digits with the same number of digits (e.g., a double-digit vs. another double-digit), children can draw on face-value comparisons. For example, a child may judge that “60” is more than “30” because one of the digits is the same,

and 6 is more than 3. This child can thus adopt a digit-by-digit comparison strategy and is able to ignore the position of the digits, making this comparison inconclusive in demonstrating whether the child recognizes positional property. To explore how children develop an understanding of the relationship between position and value, researchers must include different types of multi-digit comparisons and analyze those that require knowledge of positional property separately from those that do not.

In an exploratory analysis, Yuan et al. (2019) did just that using a Digit Comparison Task and a Digit Identification Task, similar to Mix et al. (2014). Collapsing across three different experiments, they analyzed five pairs of multi-digits that required knowledge of positional property (e.g., 14 vs. 41, and 206 vs. 260) and found that children could identify them by their names at age 5 and compare them at age 6. Based on these results, Yuan et al. concluded that children have made an induction about the structural principles underlying multi-digits by age 5; specifically, children know number words are mapped onto multi-digits from left to right, and that each position's value decreases from left to right. The researchers interpreted their findings to mean that children have acquired the relationship between place and value by 5 years of age.

However, as discussed, the Digit Identification Task provides insights into reading direction, but it does not address whether children understand digit positions have distinct values, and thus does not address the acquisition of positional property. More importantly, Yuan et al.'s study leaves open the question about whether learning number names has a direct impact on learning positional property. For example, for critical positional pairs with reverse multi-digits (14 vs. 41 and 27 vs. 72) or multi-digits with different positions for zero (206 vs. 260), they found that children could identify the names of multi-digits one year earlier than when they were

able to compare them. Although the researchers suggested that the alignment between spoken and written numerals may support the learning of place value, their findings do not provide strong support for this claim. The difference between Digit Identification and Digit Comparison tasks for critical positional pairs appears minimal in Yuan et al.'s data, despite the fact that they reported an age-related difference in above-chance performance in digit comparison and digit identification of critical pairs. Analyzing data by age group can obscure the relationship between knowing number names and comparing multi-digits, because the children who perform above chance on positional comparisons may or may not be the same children who identify the correct multi-digit when provided with a spoken numeral. This is a crucial consideration for study design as within-subjects comparison addresses the direct relationship between learning number names and learning positional property.

The present study

In an effort to address the outstanding questions about when and how children acquire positional property in place-value notation, we designed this study with two goals in mind. First, we included different types of multi-digit comparisons to directly examine the acquisition of positional property. Specifically, we tested whether children make relative judgments about multi-digit numbers by drawing on the fact that the leftmost digit represents the largest value.

Comparisons between multi-digit numbers that are mirror images of one another, such as 23 vs. 32, provide a strong test of positional understanding. On these reverse-digit comparisons, children cannot ignore position and rely on face-value comparisons the way they may when comparing numbers that differ by one digit (e.g., 35 vs. 36). Children who succeed on unit-

difference comparisons such as 35 vs. 36 but not on reverse-digit comparisons such as 23 vs. 32 may not yet possess knowledge of the positional property of multi-digit numbers. We thus included pairs of comparisons that required children to recruit knowledge of positional property (e.g., 23 vs. 32), as well as comparisons that did not (e.g., 35 vs. 36). As an additional test of positional understanding, we included cross-decade comparisons (e.g., 47 vs. 62, where the decade and the ones digits lead to different answers if a participant focuses on their face values). Previous research with first-graders and adults has shown that these comparisons are slower and more error-prone than within-decade comparisons or congruent trials (e.g., 42 vs. 57, where both the decade and ones digits lead to the same answer (Nuerk, Weger, & Willmes, 2001; Sinclair & Scheuer, 1993). Cross-decade comparisons have not previously been examined in children younger than ages 6 to 7. Similar to reverse-digit comparisons, cross-decade comparisons cannot be answered based on face-value comparisons, and they are a good test of whether children understand that the leftmost digit represents the largest value.

Thus, to test when children acquire positional property, we compared children's performance on trials that require positional knowledge (reverse-digit comparisons, cross-decade comparisons) and those that do not (unit-difference and decade-difference comparisons). We then asked whether children's failure to compare trials that require positional knowledge is due to a procedural error of failing to compare the decade digit, and whether their success can be explained by formal schooling. We also examined other strategies that children may adopt when comparing multi-digit numbers, and these include a length-based approach or a digit-by-digit approach to multi-digit comparisons.

Second, we examined the mechanism that underlies children's success on multi-digit comparisons. We asked whether their success in comparing reverse digits could be explained by their knowledge of number names. We therefore included a Digit Reading Task to explore whether children's performance on the Digit Comparison task was driven by their ability to read the digits. Critically, we conducted a within-child analysis to examine the impact of learning number names on multi-digit comparisons.

Method

Participants

We tested 120 4- and 7-year-old children ($M = 5;8$, $SD = 11.3$ months, range = 4;2 to 6;11; 52 males and 68 females) recruited from preschools and museums in Southwestern Ontario, Canada. No information about race, ethnicity or socio-economic background was collected, but the sample was drawn from a primarily Caucasian population, with 3.6% identified as visible minority (Statistics Canada, 2017). 11.1% of the sample population was classified as low income, which was slightly lower than the provincial rate of 14.4% (Statistics Canada, 2017). The sample size was determined based on a power analysis prior to the beginning of the study, and was based on an estimation that 28 children from each age group were needed for the analyses.⁴ To fully counterbalance the four item orders, we planned to test a minimum of 32 children per age group. A pre-registration of this study can be found online at:

<https://osf.io/pfuje/>. Ethics approval was obtained from the Western University Non-Medical Research Ethics Board (project title: "The acquisition and processing of large numbers, file

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⁴ With 80% power, we estimated our sample size based on a predicted interaction effect (partial eta squared = .1; an estimate based on results reported in Mix, Prather, Smith, & Stockton, 2014, Experiment 1) and the main between-subject and within-subject effect of an ANOVA analysis. Details about the power analysis can be found in the pre-registration document.

number: 109387). At the beginning of the study, we found that 6- and 7-year-olds were bored by the tasks and performed at ceiling; we thus decided not to actively recruit 7-year-olds. An additional two children were tested but were not included in the analyses because they did not want to continue with the study. One 5-year-old did not complete the Digit Reading Task, but he completed the Digit Comparison task and his data were included. Table 1 presents the number of children and their average ages.

Table 1

Mean Ages in Months and Number of Children in Each Age Group

Age Group	N	Mean (SD)	Range in months
4-year-olds	37	56.4 (2.3)	49.7–60.0
5-year-olds	34	65.7 (3.5)	60.1–71.5
6-year-olds	32	76.4 (3.1)	72.0–82.8
7-year-olds	17	87.9 (2.8)	84.4–95.0

Stimuli and Procedure

Digit Comparison Task

Children were shown pairs of digits on a computer screen and asked to point to the larger number (“Which one is more?”). The experimenter pressed the appropriate keys on a keyboard to record the children’s responses. Children were allowed to correct their responses, and the last response was recorded as the final answer; however, self-corrections were infrequent.

At the beginning of the task, there were three practice trials: a single-digit comparison (2 vs. 8), a single- vs. double-digit comparison (1 vs. 17), and a double-digit comparison (31 vs. 88), always in this same order. After the practice trials, children were shown 44 test trials. There

were two blocks of trials, and each block consisted of the same comparison pairs, but the left-right position of the correct choice was counterbalanced between the two blocks. No feedback was provided in this task.

Trial-Type Manipulation. The test trials included four main trial types. The first two trial types were critical positional trials that tested whether children understood that the leftmost digit in a multi-digit number represents the largest value. In the first trial type, we used the same digits to create a pair of mirror-image double-digit numbers (e.g., 12 vs. 21, and 36 vs. 63). We called this the reverse-digit trial. To succeed on this critical trial, children could not simply compare the two double-digits on face value (i.e., compare the values of the individual digits and ignoring their positions). Rather, they needed to recruit knowledge about the positions of digits and understand that the leftmost digit represents the largest value. To ensure that failure on the reverse-digit trial was not due to a child's inability to compare the decade digit, we included six single-digit trials consisting of comparisons that were almost identical to the decade digit of the reverse-digit comparisons. For example, for the reverse-digit comparison 67 vs. 76, the corresponding single-digit trial compared 6 vs. 7.

In the second trial type, we included six cross-decade incongruent comparisons, such as 19 vs. 22 and 31 vs. 29, to test children's understanding of positional property. Previous studies with adults and older children have shown that they tend to respond slower on similar trials when the unit and decade of the multi-digits are incompatible with each other (Nuerk et al., 2001). But such comparisons have not been tested systematically in younger children. Including the cross-decade comparisons allowed us to assess whether and when children could overcome the decade–unit incompatibility.

The two other trial types examined whether children could perform face-value

comparisons. In the decade-difference trial, each pair of digits had the same unit digit and different decade digits (e.g., 36 vs. 26). In the unit-difference trial, each pair of digits had the same decade digit and different unit digits (e.g., 36 vs. 35). We predicted that if children relied on face-value comparisons, they should fail on the critical positional trials but succeed on both the decade-difference and the unit-difference trials.

In addition to the four main trial types, we included different-length trials in which the two numbers in each pair differed in their number of digits. Three trials were single- vs. double-digit comparisons (e.g., 7 vs. 26) and three trials were double- vs. triple-digit comparisons (e.g., 15 vs. 123).

To ensure that children stayed engaged throughout the study, we included two congruent trials as fillers. In these trials, both the unit and decade digits in one number were larger than those digits in the other number (e.g., 33 vs. 66, 82 vs. 99). We predicted that children should have little to no difficulty with these trials because they could compare the numbers at face value.

Finally, we examined whether children had generalized knowledge of place-value understanding by including reverse-digit trials with triple-digits. If the children had a rule-based understanding that the leftmost digit represents the largest value, we should observe success on reverse-digit comparisons regardless of the number of digits. We conducted six reverse-digit trials with triple-digits. See Table 2 for a list of digit comparisons used in this study.

Table 2

List of Pairs of Digit Comparisons Separated by Trial Type

Trial Type	Comparisons

Congruent	33 vs. 66
	82 vs. 99
Cross-Decade	19 vs. 22
	31 vs. 29
	37 vs. 42
	59 vs. 61
	77 vs. 83
	87 vs. 92
Decade-Difference	13 vs. 23
	21 vs. 31
	36 vs. 26
	63 vs. 73
	76 vs. 86
	88 vs. 98
Different-Length	7 vs. 26
	8 vs. 13
	9 vs. 11
	73 vs. 123
	97 vs. 101
	98 vs. 200
Reverse-Digit	12 vs. 21
	18 vs. 81
	23 vs. 32
	36 vs. 63
	67 vs. 76
	89 vs. 98
	123 vs. 321
	223 vs. 322
	458 vs. 854
	572 vs. 275
Single	614 vs. 416
	768 vs. 867
	1 vs. 2
	1 vs. 6
	2 vs. 3
	3 vs. 7
	6 vs. 7
	8 vs. 9

Unit-Difference	11 vs. 12
	17 vs. 18
	32 vs. 33
	67 vs. 66
	81 vs. 82
	89 vs. 86

Numerical Size Manipulation. In constructing the stimuli for double-digits, we manipulated the numerical size of the digits. That is, we created double-digits using three digit-size combinations: (a) exclusively small digits (1–3; e.g., 12, 22), (b) exclusively large digits (5–9; e.g., 78, 67), and (c) a mix of small and large digits (e.g., 27, 63). This manipulation helped control for familiarity effects (see Brysbaert, 1995, on the role of digit frequency in number processing). For example, double-digit numbers composed of exclusively small numbers may be more familiar to children and may thus be easier to compare. We predicted that if there were a familiarity effect for small numbers, we would observe a main effect of digit-size combination favouring combinations with exclusively small numbers. On the other hand, a digit-size combination favouring combinations with a mix of small and large digits would provide support for the ANS because of their favourable ratios relative to combinations of exclusively small and exclusively large digits.

For each of the three digit-size combinations, there were two trials of reverse-digit comparisons, two trials of decade-difference comparisons, and two trials of unit-difference comparisons.

Counterbalancing. All double-digits used in the reverse-digit, decade-difference, and unit-difference trials occurred twice in the study, once as the larger number and once as the smaller number (e.g., for the double-digit 18: 18 vs. 81 and 17 vs. 18). The exceptions were 31 and 98, which both appeared twice as the larger number and once as the smaller number. The

order of the trials was pseudo-randomized, and we created four item orders, which were counterbalanced across children.

Digit Reading Task

Children were asked to read aloud a digit shown on a computer screen. These digits were taken from the single-digit, reverse-double-digit, and reverse-triple-digit trials on the Digit Comparison task. A total of 31 trials included seven single-digits, 12 double-digits, and 12 triple-digits. Items were pseudo-randomized, and the two item orders were counterbalanced across children. The experimenter recorded the children's responses on paper.

Apparatus

Pairs of digits were shown on a 13-inch computer screen using PsychoPy2 (Peirce et al., 2019). Stimuli for single-digit, double-digit, and triple-digit judgments were presented in white Arial font on a grey background. Digits were 0.6 inches tall. A black line down the centre of the screen divided the left and right sides.

Coding

Digit Comparison Task

If children selected both numbers, the experimenter recorded the response on the keyboard and it was coded as incorrect. These responses were infrequent (2.3% of all responses), and 90.0% of them occurred on reverse-digit comparisons.

Digit Reading Task

Responses were coded as correct or incorrect. We pre-registered a coding scheme to document the frequency of left-to-right reading, which is reported in the Supplementary Materials.

Analysis Plan

Our primary questions were when and how children acquire knowledge of positional property. Our data analyses proceeded in five steps, with the first four steps addressing the question of ‘when’ and the last step addressing the question of ‘how’:

1. We examined age-related patterns in the four main trial types (reverse-digit, cross-decade, decade-difference, and unit-difference trials), looking for developmental differences in children’s ability to compare multi-digits based on positional property.
2. We addressed the possibility that some children knew positional property but simply failed to compare the leftmost digits.
3. We then addressed whether children’s success on positional property trials was due to explicit instruction at school.
4. We asked what other strategies children may adopt to compare multi-digits in the absence of knowledge of positional property.
5. Finally, we asked what may predict children’s ability to compare multi-digits based on digit positions. We examined two possibilities: the role of the ANS and the role of multi-digit reading.

All data and analysis code are available at <https://osf.io/4n7fz/>. Analyses that were not pre-registered were noted as exploratory and post-hoc.

Preliminary analyses showed no difference between the two task orders and the two block orders. We thus collapsed across these variables in our analyses. Multiple comparisons were corrected using Holm-Bonferroni adjustments.

Results

When Do Children Demonstrate Knowledge of Positional Property?

Figure 1 shows children's performance on the Digit Comparison Task by age, collapsing across all trial types. Overall, children appear to reach ceiling on this task at around age 7, which is consistent with our impressions from initial testing.

Figure 1. Children's overall performance on the Multi-digit Comparison Task by age in months.

We asked whether children younger than age 7 understand that the leftmost digit represents the largest value. We included a within-subjects factor with the four main Trial Types as levels (reverse double-digits, cross-decade, decade-difference, unit-difference) and conducted a mixed ANOVA on proportion correct, with Age Group (4-year-olds, 5-year-olds, 6-year-olds) as a between-subjects factor. All main effects and interactions were significant. There was a main effect of Age Group, $F(2, 100) = 12.26$, $MSE = 1.63$, $p < .001$, $\eta p^2 = .20$, and a main effect of Trial Type, $F(3, 300) = 8.94$, $MSE = .021$, $p < .001$, $\eta p^2 = .082$. Importantly, there was an Age Group x Trial Type interaction, $F(6, 300) = 3.39$, $MSE = 0.078$, $p = .0030$, $\eta p^2 = .063$. Table 3 shows the proportion correct on the four main trial types of multi-digit comparisons by age group.

Table 3

Mean Performance Scores and Standard Deviations on Multi-Digit Comparisons by Age Group

	Reverse (double-digits)	Cross- Decade	Decade- Difference	Unit- Difference	Different- Length	Congruent
4-year-olds	.50 (.20)	.54 (.19)	.52 (.16)	.58 (.18)	.81 (.20)	.65 (.32)
5-year-olds	.54 (.29)	.58 (.25)	.69 (.23)	.72 (.21)	.90 (.20)	.77 (.29)
6-year-olds	.76 (.27)	.69 (.26)	.80 (.22)	.76 (.21)	.95 (.09)	.91 (.19)
7-year-olds	.99 (.03)	.93 (.08)	.97 (.06)	.99 (.04)	1.0 (0)	.97 (.08)

We conducted planned comparisons to unpack the interaction effect and to identify at what age children understand that the leftmost digit represents the largest value. We collapsed across reverse-digit and cross-decade comparisons because these trials were critical for evaluating knowledge of positional property. Paired t-tests revealed no difference between these two trial types, $t(102) < 1$, $p = .83$, $d_{av} = .067$.⁵ We also collapsed across decade-difference and unit-difference comparisons ($t(102) < 1$, $p = .37$, $d_{av} = .019$), because success on these trials but failure on the critical positional trials suggests that children compared multi-digits based on their face values. We compared against the two combined trial types and found no difference for 4-year-olds, $t(36) = -1.41$, $p = .17$, $d_{av} = .22$, who scored at chance for both combined trial types ($M_{\text{critical positional}} = .52$, $SD = .16$; $M_{\text{decade- / unit-difference}} = .55$, $SD = .14$). We found a significant difference for 5-year-olds, $t(33) = -3.95$, $p = .00039$, $d_{av} = .63$, $M_{\text{critical positional}} = .56$, $SD = .24$;

⁵ Effect sizes for paired-samples t-tests (Cohen's d_{av}) were computed using Lakens's (2013) Supplementary Materials.

$M_{\text{decade- / unit-difference}} = .70$, $SD = .21$, suggesting that 5-year-olds can compare multi-digits digit by digit, ignoring position, and that they fail at trials that require knowledge of positional property. There was no significant difference for 6-year-olds, $t(31) = -2.10$, $p = .044$, Holm-adjusted $\alpha = .025$, $d_{av} = .25$, $M_{\text{critical positional}} = .73$, $SD = .24$; $M_{\text{decade- / unit-difference}} = .78$, $SD = .20$. Unlike 4-year-olds, this non-significant finding of 6-year-olds is likely due to the presence of positional knowledge.

Consistent with this result, one-sample t-tests also revealed that only 6-year-olds were capable of comparing reverse-digit and cross-decade pairs (combined), $t(31) = 5.31$, $p < .001$, $d = .94$. Both 4- and 5-year-olds failed to do so (both t 's < 1.36 , p 's $> .19$, d 's $< .24$).

Why Did Younger Children Fail to Use the Leftmost Digit to Compare Multi-Digits?

One possibility is that 4- and 5-year-olds understood that the leftmost digit represents the largest value, but made errors when comparing the leftmost digits in a pair of numbers. Given that the single-digit comparisons were similar to the decade digits from the reverse-double-digit comparisons, we asked whether 4- and 5-year-olds performed significantly above chance on single-digit comparisons. We found that they did, t 's > 9.50 , p 's $< .001$, d 's > 1.56 , $M_{4\text{yos}} = .81$, $M_{5\text{yos}} = .94$, which suggests that their failure on reverse-digit comparisons was not due to procedural errors in comparing the decade digits in a pair.

Effect of Explicit Learning at School

Our results showed that 6-year-olds succeeded on comparing trials that require positional knowledge. Was this due to explicit learning at school? To test this idea, we conducted two analyses: one pre-planned and one exploratory. As part of the study design, we included triple-digits in reverse-digit trials to investigate whether success on double-digits would generalize to other digit types. According to the Ontario Mathematics curriculum, children are expected to

“represent, compare, and order whole numbers to 50” before the end of Grade 1 (Ministry of Education, Ontario, 2005). Triple-digits are likely unfamiliar numbers for most preschool children and first-graders, and their names are infrequent in caregiver speech (Willits, Jones, & Landy, 2016). In our study, some 6-year-olds were in Grade 1 ($M_{\text{age}} = 78.4$ months) and some were in Senior Kindergarten ($M_{\text{age}} = 74.4$ months). It is thus highly unlikely that they had learned the meaning of triple-digits at school. Nevertheless, we found that 6-year-olds performed above chance on comparisons of reverse triple-digits, $t(31) = 4.73, p < .001 (M = .74), d = .43$, and that there was no difference in performance between reverse-double- and reverse-triple-digit trials, $t(31) = .63, p = .53, d = .060$.

However, some 6-year-olds were already in Grade 1 and might have been exposed to multi-digits at school. In an exploratory analysis, we focused on younger 6-year-olds. Grade-level information was missing for some children ($N = 10$), but age is generally correlated with grade level. We used the maximum age of kindergarten children as the cut-off (76 months), and asked whether 6-year-olds who were younger than 76 months could compare pairs of reverse-digits. We found that they performed above chance on reverse-digit trials (including double- and triple-digits), $M = .70, t(15) = 2.88, p = .011$, Holm-adjusted $\alpha = .025$, and cross-decade trials, $M = .66, t(15) = 2.35, p = .033$, Holm-adjusted $\alpha = .05$. We also found that their performance did not differ from that of children older than 76 months (t 's $< -1.13, p$'s $> .27$).

Non-Positional-Based Strategies to Compare Multi-Digits

Next, we examined what strategies younger children may adopt in comparing multi-digits. First, we found that children younger than 6 were able to compare different-length trials. Four-year-olds were almost at ceiling, $M = .81$, and average accuracy continued to increase for 5-

and 6-year-olds (see Table 3). These findings show that starting at a young age, children can recruit non-positional-based strategies for comparing multi-digits.

Second, we examined their ability to compare multi-digits one digit at a time while ignoring position. Our results showed that 5-year-olds, but not 4-year-olds, were able to compare decade-difference and unit-difference trials. This is somewhat puzzling, given that both groups of children were able to compare individual digits on single-digit trials, suggesting that they can compare one digit at a time. We explored whether 4-year-olds' difficulty with face-value comparisons was specific to decade- and unit-difference trials. To find out, we examined their performance on congruent trials where both the decade and unit digits were larger in one number than the other.⁶ For this exploratory analysis, we included the two congruent pairs (82 vs. 99, and 33 vs. 66), and one of the practice trials that matched the congruent trial criterion (e.g., 31 vs. 88). Surprisingly, we found that 4-year-olds performed above chance on this comparison type, $t(36) = 3.58, p = .001$, Holm-adjusted $\alpha = .017, d = .59, M_{\text{cong}} = .67, SD = .28$, and their performance was significantly better than on decade- and unit-difference trials (combined), $t(36) = -2.40, p = .022$, Holm-adjusted $\alpha = .05, d_{\text{av}} = .51$. Five-year-olds also demonstrated the same benefit on Congruent trials relative to Decade- and Unit-difference trials, $t(33) = -2.47, p = .019$, Holm-adjusted $\alpha = .025, d_{\text{av}} = .30, M_{\text{cong}} = .77 (SD = .27)$. An inspection of the congruent stimuli suggests that numbers with a duplicated digit (e.g., 66, 88, 99) might be easier for children to compare because duplication in natural language, or the doubling of a word, often

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⁶ In the pre-registered analyses, we planned to minimize the number of multiple comparisons by combining different-length trials and congruent trials as a manipulation check to test whether children would succeed on comparison types that did not require positional-based strategies. Nevertheless, we decided to analyze the two trial types separately, as children may recruit different strategies to compare different-length trials and congruent trials.

intensifies meaning (e.g., “Steven has many many books” means he has a very large number of books; see Dalrymple & Mofu, 2012; Regier, 1994).

In the next set of analyses, we examined factors that may explain children’s success on multi-digit comparisons.

The Role of the ANS

One possible factor is that children draw on the ANS in multi-digit comparisons. We examined reverse-digit trials because numerical size was manipulated in these critical comparisons. The ratios for the six reverse-double-digit trials were as follows, in ascending order: 18 vs. 81 (0.22), 36 vs. 63 (0.50), 12 vs. 21 (0.57), 23 vs. 32 (0.71), 67 vs. 76 (0.88), and 89 vs. 98 (0.90). To test for a ratio effect, we asked whether children’s performance differed as a function of digit-size combinations. Better performance on small–large digit combinations (18 vs. 81, and 36 vs. 63), which had more favourable ratios, than small–small digit combinations (12 vs. 21, 23 vs. 32) and large–large digit combinations (67 vs. 76, 89 vs. 98), would constitute evidence of a ratio effect. We focused on 6-year-olds, since they were able to compare reverse double-digits, and because a ratio effect would only be expected in children who can process multi-digit numbers. ANOVA revealed no effect of Digit Size, $F(2, 62) = 1.16, p = .32, \eta p^2 = .036$. Six-year-olds performed similarly on small–large digit combinations ($M = .73, SD = .31$), large–large digit combinations ($M = .75, SD = .29$), and small–small digit combinations ($M = .80, SD = .31$).⁷

In addition, the lack of a Digit Size effect indicates that children did not favour combinations with exclusively small numbers, suggesting that in the context of multi-digit

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⁷ We also conducted a post-hoc analysis treating ratio as a continuous variable in a mixed-effects logistic regression, allowing ratio to vary within subjects. We found no effect of ratio ($\beta = -.39, z = -.36, p = .72$).

comparisons, there is no familiarity effect for small numbers.

We further examined the lack of a ratio effect for multi-digits by asking whether there was a ratio effect for single digits: 1 vs. 6 (0.17), 3 vs. 7 (0.43), 1 vs. 2 (0.5), 2 vs. 3 (0.67), 6 vs. 7 (0.86), and 8 vs. 9 (0.89). We divided them into easy (≤ 0.5) and hard (> 0.5) in an ANOVA, and found a main effect of Ratio ($F(1, 31) = 16.24, p < .001, \eta p^2 = .34$).⁸

The Role of Multi-Digit Reading

In our last set of analyses, we examined whether children's ability to read multi-digits predicted their ability to compare them. Figure 2 shows children's accuracy on digit reading by age in months. A visual inspection of the data suggests that children knew how to read single digits at around age 4 to 5, double-digits at around age 6, and triple-digits at 6 ½ to 7.

Figure 2. Scatterplot of average scores on single-, double-, and triple-digit reading by age in months.

⁸ We ran two further post-hoc analyses. First, we entered ratio as a continuous variable in a mixed effects analysis. Second, we included the 4- and 5-year-olds. In both of these analyses, we found a main effect of ratio, and there was no interaction with age groups.

We used linear regression to predict children's performance on the Digit Comparison Task (averaging across all double- and triple-digit trials), with the predictor variables of age in months and average multi-digit reading score on double- and triple-digit trials of the Digit Reading Task. We found that digit reading was predictive of children's ability to compare multi-digits, $\beta = .26, p < .001$, while age was not a significant predictor, $\beta = .0030, p = .13$.

In a more critical analysis, we asked whether children's success on a particular trial of the Digit Comparison Task could be explained by their ability to read those same digits. All the numbers from the Digit Comparison Task's reverse-digit pairs were included in the Digit Reading Task. We thus conducted post-hoc analyses at the item level within-child, and examined the relationship between children's ability to read one or both of the multi-digits on the Digit Reading Task and their ability to compare them. If multi-digit reading is necessary for multi-digit comparison, we should find that children performed above chance on comparing multi-digits only when they could read both of the multi-digits, but not when they could read one of the multi-digits or none of them. However, if we also find above-chance performance on multi-digit comparison when children could read one or none of the multi-digits, this would suggest that digit reading is not necessary for digit comparison.

For each child and for each pair of reverse-digits, we coded whether a child can read both of the multi-digits, one of the multi-digits, or none. We then computed, for each child, proportion correct on comparing pairs of multi-digits as a function of their digit reading performance. We found that children performed above chance on comparing pairs of multi-digits when they could read both of the multi-digits, $t(62) = 5.18, p < .001, d = .66$ ($M = .71, SD = .32$), but not when

they could read one, $t(65) = -0.53$, $p = .60$, $d = -.06$ ($M = .48$, $SD = .34$), or neither of the multi-digits, $t(80) = 0.85$, $p = .40$, $d = .08$ ($M = .52$, $SD = .24$).⁹

Figure 3. A box plot showing children's performance on comparing pairs of reverse-digits as a function of whether they read both of the multi-digits, one of the multi-digits or neither of them.

We ended our analyses with an important observation. Figure 3 reveals that despite an above-chance performance on digit comparison among children who could read both of the multi-digits, some children performed poorly. In addition, some children were able to reliably compare pairs of reverse-digits despite not being able to read both of the multi-digits correctly. Such individual variability in multi-digit processing might hold the key to individual differences in early math and is worthy of future investigation.

⁹ The degrees of freedom differed because on any given trial, not all children had data on trials in which they could read both of the digits ($n = 39$) or one of the digits ($n = 36$). And some children did not have item-level data on which they read none of the digits ($n = 21$) because they were able to read either one or both of the digits.

Discussion

Learning about place-value notation, which underlies the meaning of multi-digit numbers such as “65” or “314,” involves constructing a positional structure that carries numerical meaning. This structure establishes a relationship between position and value, and it forms a basis for the acquisition of multi-digit meanings. Once children understand that each digit position represents different values, they can begin to fill in each “positional slot” with a precise value such as 1, 10, 100, 1,000, and so on. Understanding the role of position is thus the foundation for learning place-value notation. Despite its importance, previous studies have focused on the acquisition of base-10 rules and have not directly examined the acquisition of the role of position in multi-digit understanding. In a recent study, Yuan et al. (2019) provided data that bear on children’s positional understanding, but their analysis was exploratory and post-hoc in nature. We addressed this gap by designing a task with different trial types that directly assessed positional understanding based on a conceptual framework on place-value acquisition. In addition, we examined the direct impact of learning number names on the acquisition of positional property of the place-value notation.

We found that 6-year-olds as a group were able to perform multi-digit comparisons that drew on positional knowledge and demonstrated an understanding about the relationship between place and value. For example, they knew that “76” is more than “67,” which suggests they understand that digit positions have numerical meanings, because children cannot rely on the digits’ face value to compare pairs of reverse-digits. We also found that 6-year-olds demonstrated success on reverse triple-digits, numbers they were unlikely to have encountered at school. This observation suggests that the understanding that the leftmost digit represents the largest value might be a general principle that children develop prior to entering or at the start of

elementary school. In addition, children who could compare reverse-digit pairs could also compare cross-decade pairs (e.g., 37 vs. 42). In a previous study, Nuerk and colleagues (2001) found that adults were slower and made more errors on cross-decade comparisons than on pairs of multi-digits in which the tens and ones digits in the correct answer were larger than those in the incorrect answer (e.g., 42 vs. 57, where $5 > 4$ and $7 > 2$). The current study built on the results of previous research. We found that children were able to compare cross-decade pairs at a younger age than previously attested (Moeller, Pixner, Zuber, Kaufmann, & Nuerk, 2011; Pixner, Moeller, Hermanova, Nuerk, & Kaufmann, 2011), and that this ability signals the presence of knowledge of positional property. Specifically, children who did not know that the leftmost digit represents the largest value performed at chance on cross-decade comparisons, whereas those who understood the importance of digit position performed above chance. This finding is consistent with the view that success in comparing pairs of digits in which prioritizing the tens or ones digits would lead to different answers (e.g., cross-decade comparisons in this study) indicates that children have interpreted the tens and ones digits separately (Nuerk & Willmes, 2005; Nuerk et al., 2015).

Importantly, this study's results shed light on the potential mechanism underlying children's success on multi-digit comparisons. First, we did not find a ratio effect with pairs of reverse digits, despite the presence of a ratio effect with pairs of single digits. These results suggest that comparing pairs of numbers that require positional knowledge was likely not due to mapping multi-digits onto the ANS, which was consistent with a previous study (Yuan et al., 2019). Nevertheless, future studies should examine this in-depth with a larger number of trials. Second, we found that children's ability to make relative judgments based on digit positions can be explained by their knowledge of spoken numerals. Specifically, we found that children who

knew the number names of both of the multi-digits were successful at comparing pairs of reverse-digits, but those who knew only one or neither of the multi-digits were at chance. We discuss possible pathways about how learning number names may help children acquire positional property later in this section.

Developmental Changes at the Group Level Over the Preschool Years

Our study showed clear age-related developments in multi-digit understanding. Although young children have not yet acquired knowledge of positional property for multi-digits, we found evidence that as early as age 4—the youngest age we tested—children understood that a multi-digit number is one whole number. Their success in comparing multi-digits of different lengths demonstrated this understanding. While we do not know whether children were drawing on the length of a number or the number of digits when comparing different-length pairs, both interpretations suggest that children treat numbers such as “123” or “73” as whole numbers. Yuan and colleagues (2019) reported even earlier evidence, and show that by age 3, children know a double-digit number such as “65” is not merely “a six and a five that are close together.” Children may rely on spacing between words and numbers to individuate multi-digit numbers (e.g., “13 giraffes”) in ways similar to how they use spatial cues to identify words in printed text.

Being able to individuate and identify “65” as one number, however, does not guarantee that children know multi-digits are read from left to right. Indeed, the fact that some children responded that numbers that were mirror images of one another were “the same” suggests that some may think multi-digits can be read flexibly from left to right and right to left. Such flexible digit reading direction may explain why the younger children failed at comparisons that drew on positional knowledge.

Nevertheless, flexible digit reading does not explain the somewhat intriguing finding that 5-year-olds, but not 4-year-olds, were able to compare decade- and unit-difference pairs (e.g., 35 vs. 36). Both groups of children could compare single digits, and thus had the ability to compare these two types of comparisons digit-by-digit. We suspect that 5-year-olds may know that each position in a multi-digit number represents a fixed value, despite not knowing the order of values of digit positions. Specifically, 5-year-olds may know that multi-digits of the same length can be aligned digit by digit. For example, they may understand that in a pair of double-digits, the leftmost digit of one number aligns with the leftmost digit of the other in terms of its value, but they may not know that the leftmost digit represents the largest value. This partial understanding would explain why they failed at comparing pairs of reverse-digits and cross-decade numbers, but succeeded on decade-difference and unit-difference trials. It would also explain why the 5-year-olds performed better than the 4-year-olds, but worse than the 6-year-olds. As a group, 5-year-olds may have begun to fill in positional slots with numerical meaning in multi-digits, but they lack a systematic understanding of how the *order* of values maps onto different positions.

Individual Differences in Multi-Digit Understanding

Although our data revealed clear age trends, we also found a range of individual differences. For example, not all 6-year-olds were able to compare multi-digits. Also, despite a clear relationship between digit reading and digit comparison, some children who could name both of the multi-digits were not able to compare them (Figure 3). These individual differences in understanding multi-digits might hold the key to understanding individual differences in later math achievement. Indeed, to do arithmetic, and to achieve arithmetic fluency, one must be able to understand multi-digits. Previous studies have shown that multi-digit understanding is related to arithmetic performance (Göbel, Watson, Lervåg, & Hulme, 2014; Moeller et al., 2011). For

example, Moeller et al. (2011) found that multi-digit comparison in Grade 1 predicts children's performance on addition problems in Grade 3. They also examined specific numerical effects that may predict arithmetic ability, and found that children who made more errors in cross-decade comparisons (or what they called incompatible trials; e.g., 47 vs. 62) than congruent comparisons (or what they called compatible trials; e.g., 42 vs. 57) also made more errors on addition problems 2 years later. Our results show that individual differences in multi-digit understanding emerge as early as age 4, highlighting the importance of studying multi-digit understanding in a larger sample of preschool children.

Acquiring the Leftmost Digit Represents the Largest Value Principle

How do children learn that the leftmost digit represents the largest value? Our item-level analysis within-child suggests that reading multi-digits plays an important role in allowing children to compare multi-digits based on digit positions. Here, we discuss two ways that learning number names may help children gain that understanding.

In one view, children can draw on the later-greater principle of counting—i.e., numbers that appear later in the count list represent a larger quantity. For example, children who know the names of “72” and “27” may judge that “72” is more than “27” because they recognize that the former appears later in the count list than the latter. Future studies on multi-digit understanding can examine this by testing children's verbal number comparisons. In another view, the critical piece of knowledge is an understanding of the numerical order of number words (e.g., “five,” “sixty”) that form a complex numeral (e.g., “sixty-five”), and specifically knowing that the largest number in a complex numeral is uttered first in spoken numerals in English (e.g., “thirty-six,” * “six-thirty”; Hurford, 1975). With this particular structure of complex numerals in mind, once children identify a multi-digit number as a whole number, they are in a position to make an

analogy between spoken and written numerals and to infer that the leftmost digit represents the largest value (see also Yuan et al., 2019).

It is important to note, however, not all numbers provide the right input for this type of inference. For example, teen numbers in English have an irregular structure and thus do not support an analogical mapping between spoken and written numerals. In addition, not all languages provide the same input because languages differ in their numeral structure (e.g., Cheung, Dale, & Le Corre, 2015; Schneider et al., 2020). Our proposal predicts cross-linguistic differences in the rate at which children discover that the leftmost digit represents the largest quantity that warrant further investigation. For example, children learning German, which reverses the decade and ones terms (e.g., “65” in German is “fünf-und-sechzig,” which literally means “five and sixty”), may learn the “leftmost digit represents the largest value” principle later than children learning English. Future studies should investigate this account using within- and between-language comparisons.

Nevertheless, children need to learn not only that the largest value is uttered or written first, but also that in written numerals, the value is not expressed explicitly as in spoken numerals. Previous studies have found that when asked to write multi-digit numbers, half of the 5-year-olds included all of the values (Byrge et al., 2014; see also Seron & Fayol, 1994). For example, they may write “600402” upon hearing “six hundred and forty-two.” This type of error suggests that children know the left-to-right order of numerical expressions but don’t yet understand the positional property of written numerals. In addition to explicit instruction, exposure to pairs of written and spoken numerals (e.g., “642” vs. “six hundred and forty-two,” “587” vs. “five hundred and eighty-seven”) may allow children to infer that value words such as “hundred” or “thousand” are not written out in multi-digit numbers.

Acquisition of Place-Value Notation

The bulk of work on the acquisition of numbers has focused on characterizing how counting—a cultural construction that represents a symbolic number via number words—transcends cross-culturally universal number representations (e.g., Carey, 2009; Le Corre & Carey, 2007; Nunez, 2017). But number words are not the only symbolic system humans have developed over the past thousands of years to represent numbers. The Hindu-Arabic written numeral system, or multi-digits, is another important symbolic system for number representation, and one that has perhaps received more attention in the education literature than in psychology (e.g., Chan et al., 2014; Fuson, 1990; Fuson et al., 1997; Hiebert & Wearne, 1992; Ho & Cheng, 1997; Ross, 1989; Sinclair & Scheuer, 1993). Despite earlier reports that concluded that preschool children have acquired structural principles underlying multi-digits by age 5 (Mix et al., 2014; Yuan et al., 2019), we found that as a group, children may not construct the necessary relationship between place and value for written Arabic numerals until a year later than previously expected. Furthermore, some 6-year-olds appear to lack the building blocks of understanding for place value, which may explain why some children struggle with arithmetic in the elementary classroom (e.g., Fuson & Briars, 1990; Chan, et al., 2014). Children who do not yet understand that different positions represent different quantities in a multi-digit number may not be ready to learn the precise value of each position or how to do arithmetic with multi-digit numbers.

How might acquiring an understanding of positional property relate to children's ability to learn the precise value of digit positions? Current results suggest a close temporal relationship between the acquisition of the two properties of place-value notation. We found that children understand that the leftmost digit represents the largest value to entering first grade (see also Mix

et al., 2014; Yuan et al., 2019). Previous studies testing the developmental trajectory on base-10 have shown that children acquire the precise value of digit positions during the early elementary years. By second grade, children can accurately identify the base-10 value of digit positions (e.g., Boulton-Lewis, 1998; Cotter, 2000; Fuson & Briars, 1990; Miura & Okamoto, 1993; Ross, 1989). These studies have also shown that when children can name the value of digit positions, they do so in an orderly fashion, first identifying the tens position before the hundreds position. This contrasts with our finding that children who show positional knowledge do so for both double-digit pairs and triple-digit pairs. These results suggest that knowledge of positional property is likely acquired prior to the acquisition of base-10, and that such positional knowledge can, for a brief moment in development, exist in children in the absence of base-10 knowledge. Future studies should test the extent to which positional property applies to larger multi-digits, as well as the temporal relationship in acquiring positional property and the base-10 rules of place-value notation.

The current study also highlights the importance of studying positional property as part of developing theories of place-value understanding. Existing theories of place-value understanding often focus on children who are already in elementary school, and they posit that children begin by identifying and labelling digit positions as hundreds, tens, and ones. For example, Nuerk and colleagues (2015) have proposed that to understand the meaning of multi-digit numbers, children must first identify the hundred, decade, and ones positions of individual digits in a multi-digit number. After identifying the digit positions, children can attach distinct values to each position, and finally, they can learn to compute with multi-digit numbers. Fuson (1990) and Ross (1989) have proposed similar accounts, though both have noted that children first interpret a multi-digit number as a whole number, then identify tens and ones, and decompose the tens into smaller

parts. However, these existing theories are often based on observations that are post-hoc in nature. The current study proposed a conceptual framework and designed tasks that directly examined the early acquisition of place-value notation. Our findings suggest that children begin by knowing that a multi-digit number is a whole number (similar to Ross's Stage 1), and then decompose a multi-digit number into positional parts that carry numerical meaning. That is, they learn that each position has a distinct value, which decreases from left to right, but they likely cannot identify which position corresponds to which value (i.e., they do not know the second position represents tens and the third position represents hundreds). Only after they have constructed a relationship between place and value will they be able to learn the precise value associated with each position according to base-10 rules.

Summary

Place-value notation is an ingenious way to represent numbers (see Ifrah, 1981, and Zhang & Norman, 1995, for discussions). Unlike other positional systems that exist in historical time, place-value notation represents numbers efficiently and allows for easy calculation. Compared to the abacus, which also draws on positions to represent numerical meaning and is more efficient for arithmetic calculation (Barner, Alvarez, Sullivan, Brooks, Srinivasan, & Frank, 2016; Frank & Barner, 2012), place-value notation can easily be extended to represent and support arithmetic operations with very large numbers. In this study, we systematically tested when children begin to construct a relationship between place and value, and found that as a group, children acquire knowledge of positional property at around age 6. We also discovered a range of individual variation that calls for more attention in future studies. These results highlight the importance of studying positional property in multi-digits in young children and its

relationship to acquiring base-10 rules. Studies on children's numerical understanding must look beyond the first few numbers to examine how children acquire meanings for large numbers.

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