Interpreting Cross-Cultural Bias in Psychological Instruments: An Empirical Example

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Abstract

Cockcroft et al. (2015) fit a multigroup confirmatory factor model to Wechsler Adult Intelligence Scale III (WAIS-III) data from South Africa and the United Kingdom. They found evidence for measurement bias and suggested that bias was greatest in those subtests of the WAIS-III that showcased the largest mean group differences. They went on to state that the “[i]dentification of tests that do not favor individuals from Eurocentric and favorable SES circumstances with advantaged educational backgrounds is valuable in providing direction for the development of culture fair tests”, in effect suggesting that the results confirmed the test was biased towards culturally European groups. However, a reanalysis that included estimation of the effect sizes and directions of bias showed that, far from their conclusions, bias operated in different directions, but primarily *in favor* of the lower-scoring South African group. The need for greater qualification in measurement invariance testing is shown, discussed, and recommended.

Format Note

This paper includes the textual body of a reanalysis of Cockcroft et al. (2015) in addition to the code and data necessary to replicate all analyses contained within. This format is intended to give interested readers the opportunity to review methods and analyses while reading most of the same text that will appear in the study itself. This paper is considerably longer and more winding than the final paper will be in some parts and it is shorter in others, but it includes details that certain readers may find interesting or edifying. Additionally, it contains analyses and some pertinent discussion that will not be contained in the final paper that will be formatted without code, data, or analyses included within the paper itself; those will be presented as supplementary materials. Analyses for the final paper are marked “Primary”.

Setup

```
#Packages

library(pacman); p_load(lavaan, qpcR, ggplot2, semPlot, psych, ggpubr)

#Function for measuring test bias, from Gunn, Grimm & Edwards (2019)

SDI2.UDI2 = function(p, loading1, loading2, intercept1, intercept2, stdev2, fmean2, fsd2,
                      nodewidth = 0.01, lowerLV = -5, upperLV = 5) {
```

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```

LV = seq(lowerLV,upperLV,nodewidth)
DiffExpItem12 = matrix(NA,length(LV),p)
pdfLV2 = matrix(NA,length(LV),1)
SDI2numerator = matrix(NA,length(LV),p)
UDI2numerator = matrix(NA,length(LV),p)
SDI2 = matrix(NA,p,1)
UDI2 = matrix(NA,p,1)
for(j in 1:p){
  for(k in 1:length(LV)){
    DiffExpItem12[k,j] <- (intercept1[j]-intercept2[j]) + (loading1[j]-
                                                                loading2[j])*LV[k]

    pdfLV2[k] = dnorm(LV[k], mean=fmean2, sd=fsd2)
    SDI2numerator[k,j] = DiffExpItem12[k,j]*pdfLV2[k]*nodewidth
    UDI2numerator[k,j] = abs(SDI2numerator[k,j])}
    SDI2[j] <- sum(SDI2numerator[,j])/stdev2[j]
    UDI2[j] <- sum(UDI2numerator[,j])/stdev2[j]}
colnames(SDI2) = "SDI2"
colnames(UDI2) = "UDI2"
return(list(SDI2=round(SDI2,4), UDI2=round(UDI2,4)))}

#Naaman-adjusted p-value

NP <- function(N, S = 2) {
  NP = 1 - pnorm(qnorm(1-(N^(-6/5))/S))
  return(NP)}

#Functions to (1) compute confidence intervals from eigenvalues from a PCA
#and (2) entered manually

MAPA <- function(x, a = 0.05, s = 2, d = 3){
  LE = x$values; LECIL = LE - abs((qnorm(a/s) * sqrt((2*LE^2)/(x$n.obs)))); LECIU = LE +
  abs((qnorm(a/s) * sqrt((2*LE^2)/(x$n.obs))))
  S1 <- cat(paste("This eigenvalue was", round((LE), d), "with a confidence interval of",
                  round((LECIL), d), "to", round((LECIU), d), "\n"))}

MAPM <- function(x, a = 0.05, s = 2, d = 3, n){
  LECIL = x - abs((qnorm(a/s) * sqrt((2*x^2)/(n))))
  LECIU = x + abs((qnorm(a/s) * sqrt((2*x^2)/(n))))
  S1 <- cat(paste("With the provided eigenvalue of", round((x), d),
                  " and the provided n, \n you achieve a confidence interval of",
                  round((LECIL), d), "to", round((LECIU), d), "\n"))}

#Upper and lower bounds to eigenvalue confidence intervals

LECLOW <- function(x, a = 0.05, s = 2){
  LE <- x$values
  LECIL <- LE - abs((qnorm(a/s) * sqrt((2*LE^2)/(x$n.obs))))
  return(LECIL)}

LECHIGH <- function(x, a = 0.05, s = 2){
  LE <- x$values
  LECIU <- LE + abs((qnorm(a/s) * sqrt((2*LE^2)/(x$n.obs))))
  return(LECIU)}

```

```

#Function to derive minimum significant correlations

CRITR <- function(n, alpha = .05) {
  df <- n - 2; CRITT <- qt(alpha/2, df, lower.tail = F)
  CRITR <- sqrt((CRITT^2)/((CRITT^2) + df ))
  return(CRITR)}

#Function to provide the proportion of a variance difference due to
#bias, per Millsap & Olivera-Aguilar (2012)

MOES <- function(T1, T2, S1, S2) {
  ES = (T1 - T2) / (S1^2 - S2^2)
  return(ES)}

#Fit measures

FITM <- c("chisq", "df", "nPar", "cfi", "rmsea", "rmsea.ci.lower", "rmsea.ci.upper",
          "srmr", "bic")

#Data

##Correlation matrices

lowerUK = '
1.00
0.71    1.00
0.32    0.37    1.00
0.10    0.14    0.38    1.00
0.65    0.60    0.35    0.14    1.00
0.61    0.65    0.51    0.45    0.61    1.00
0.17    0.26    0.46    0.61    0.22    0.38    1.00
0.23    0.27    0.34    0.17    0.27    0.25    0.24    1.00
0.05    0.05    0.18    0.26    0.05    0.08    0.24    0.26    1.00
0.32    0.36    0.46    0.32    0.29    0.41    0.36    0.44    0.33    1.00
0.31    0.39    0.48    0.25    0.35    0.38    0.35    0.49    0.30    0.55    1.00
0.28    0.29    0.42    0.33    0.31    0.38    0.39    0.65    0.50    0.66    0.69
1.00
0.15    0.12    0.28    0.36    0.12    0.23    0.31    0.33    0.77    0.43    0.39
0.54    1.00'

lowerSA = '
1.00
0.52    1.00
0.43    0.41    1.00
0.21    0.18    0.32    1.00
0.62    0.56    0.52    0.15    1.00
0.60    0.53    0.41    0.31    0.56    1.00
0.23    0.20    0.39    0.45    0.18    0.19    1.00
0.19    0.25    0.20    0.09    0.25    0.19    0.19    1.00
0.09    0.16    0.13    0.16    0.03    -0.04    0.20    0.08    1.00
0.20    0.28    0.34    0.36    0.33    0.21    0.24    0.42    0.28    1.00
0.06    0.07    0.21    0.19    0.09    0.17    0.17    0.39    0.07    0.40    1.00
0.24    0.14    0.31    0.14    0.35    0.26    0.10    0.30    0.12    0.26    0.21

```

```

1.00
0.09    0.12    0.18    0.22    0.06    0.06    0.25    0.24    0.45    0.39    0.25
0.19    1.00'

##Variable names

Names = list("V", "S", "A", "DS", "I", "C", "LNS", "PC", "DSC", "BD", "MR", "PA", "SS")

##Convert to variance-covariance matrices

WAISIIIW.cor = getCov(lowerUK, names = Names)
WAISIIIB.cor = getCov(lowerSA, names = Names)
UKSDs <- c(2.66, 3.04, 3.14, 2.94, 2.72, 2.59, 2.72, 3.13, 2.68, 2.87, 2.71, 2.26, 2.62)
SASDs <- c(2.02, 1.4, 2.53, 2.26, 3.21, 2.5, 2.42, 2.11, 2.51, 2.61, 2.37, 2.82, 2.67)
WAISIIIW.cov = lavaan::cor2cov(R = WAISIIIW.cor, sds = UKSDs)
WAISIIIB.cov = lavaan::cor2cov(R = WAISIIIB.cor, sds = SASDs)

##Means

Wmeans = c(14.56, 14.22, 9.58, 9.5, 11.75, 12.48, 8.88, 10.57, 7.72, 10.66, 10.59, 10.08,
            8.25)
Bmeans = c(8.92, 8.12, 8.66, 9.35, 10.19, 8.99, 9.17, 8.05, 9.75, 8.67, 10.68, 8.33, 9.71)

##Group inputs

GCovs <- list(WAISIIIW.cov, WAISIIIB.cov)
GMeans <- list(Wmeans, Bmeans)
GNs <- list(349, 107)

```

Introduction

One way of conceptualizing psychometric bias (often just called ‘test bias’ but henceforth only referred to as ‘bias’) is as the state in which assessments must be interpreted differently in different populations. This state is usually assessed by looking for differential item functioning or by testing for differences in model parameters in multi-group confirmatory factor analyses (MGCFAs); both methods of testing for bias are theoretically comparable and the definition of a lack of bias is the same in each. That is, per Mellenbergh (1989),

$$f(X|\eta, v) = f(X|\eta)$$

where X and η are the observed and factor scores in a model and v is a grouping variable. Restated differently, for a given factor score, a person’s observed score is the same regardless of group membership when there is no bias. When the grouping variable does affect the relationship between factor and observed scores, a psychological instrument is biased and observed scores cannot be interpreted the same way in different groups.

This is as far as most people who assess bias take understanding it. However, they can go further and assess the *degree* of bias and how it affects the observed scores and their variances. Doing this allows researchers to better interpret bias. However, the typical and unfortunate conclusion of finding evidence of bias is interpretation of the bias *without* computing the degree or direction of bias. This leads to potentially harmful conclusions like that an assessment is biased against or in favor of a certain group when that may not be the case. To illustrate this problem and the need to compute effect sizes when assessing bias, I have

provided an empirical example, below, where researchers found evidence of bias and interpreted it in a way that is likely incorrect. In their case, the bias went in the opposite of the direction they thought it did.

Methodology, Analysis, and Results

Analysis One: Factor Structure in South Africa and the United Kingdom

Cockcroft et al. (2015) performed what they called an exploratory factor analysis (EFA); problematically, their EFA was actually a principal components analysis (PCA). While the two methods are related, they are not identical and cannot be interpreted identically. The difference between the two methods affects their interpretation: in an EFA, the common variance is used and the resulting factor is supposed to be interpreted as a common cause, but in a PCA, the total (i.e., specific plus common) variance is used, so the latter is more likely to contain measurement error and additional variance from local independence violations. EFA is a technique for representing latent variables whereas PCA is intended for dimensionality reduction. This was a minor issue for the present reanalysis but it needed to be clarified to avoid reader confusion.

Their PCA results were overinterpreted because they interpreted them at all without testing if the immediately presented output of a PCA could be confirmed. They wrote

The factor structure for the SA sample (factor loadings of >0.40) matched the specified WAIS-III indices for VCIs, POIs, WMIs, and PSIs. The only difference was that the Arithmetic subtest also loaded on the VCI.

In contrast, there was a different pattern for the UK sample, as eigenvalues greater than one indicated three factors (factor loadings of 0.40 or greater). The first factor included subtests from both the POI and PSI. The VCI and WMI were largely similar to those for the original WAIS-III. However, the Arithmetic subtest also loaded on the VCI and the Comprehension subtest also loaded on the WMI.

For these inferences to be supported, Cockcroft et al. would have had to do three things. Firstly and secondly, they would have had to test the discovered factor structures for the South African and British samples in confirmatory models in addition to specifically testing for whether cross-loadings improved model fit. Third, they would have had to test whether the factor structures differed, likewise using confirmatory models.

This section does each of those things to reassess how confident we can be in their conclusions. I will be using the Akaike weights function from the **R** package **qpcR** to produce normalized probabilities for different models. For nested models, I will utilize Schwartz weights (i.e., weights based on the Bayesian Information Criterion or BIC) and for nonnested models, I will utilize Akaike weights (i.e., weights based on the Akaike Information Criterion or AIC), though I understand that this may be inappropriate with models that are far more divergent in the parameters they fit. The χ^2 test will take precedence, since it is an exact fit index. When it is used, a result will be considered significant if it crosses the p value threshold for its sample size as calculated from Naaman (2016); though intended for z 's, it is workable here due to the acceptably large sample sizes being used¹. I will use approximate fit indices with current common thresholds in mind (i.e., $\Delta CFI > 0.01$ or $\Delta RMSEA > 0.015$ as evidence for measurement invariance violations), but my order of preference for model selection is exact, then information theoretic, and finally approximate fit indices. When interpretability indices are available, they can take precedence, but they are not particularly relevant for testing for measurement invariance; they are much more useful for selection of initial models for subsequent invariance testing. I am well aware that my definition of when a model is biased is considered by most practitioners to be too stringent.

¹In Cockcroft et al.'s paper, they write that the χ^2 statistic "compares the degree to which... predicted covariances in [a] model differ from observed covariances", evidently forgetting the steps of testing measurement invariance that deal with intercepts and residual variances.

Part I: Recapitulating their PCA Results

First, the accuracy of their PCA results must be shown. The viability of the extraction of three or four factors will be assessed with a parallel analysis. Cockcroft et al. utilized a cutoff of $r = 0.4$, but I will use a value more typical for this field of research, $r = 0.3$.

```
#South African
```

```
psych::fa.parallel(WAISIIIIB.cor, n.obs = 107)
```

Parallel Analysis Scree Plots

```
## Parallel analysis suggests that the number of factors = 2 and the number of components = 2
```

```
SAPCA1 <- pca(WAISIIIIB.cor, n.obs = 107, nfactors = 1)
SAPCA2 <- pca(WAISIIIIB.cor, n.obs = 107, nfactors = 2)
SAPCA3 <- pca(WAISIIIIB.cor, n.obs = 107, nfactors = 3)
SAPCA4 <- pca(WAISIIIIB.cor, n.obs = 107, nfactors = 4)
```

```
print.psych(SAPCA1, cut = 0.3, sort = F, digits = 3)
```

```
## Principal Components Analysis
```

```
## Call: principal(r = r, nfactors = nfactors, residuals = residuals,
## rotate = rotate, n.obs = n.obs, covar = covar, scores = scores,
```

```

##      missing = missing, impute = impute, oblique.scores = oblique.scores,
##      method = method, use = use, cor = cor, correct = 0.5, weight = NULL)
## Standardized loadings (pattern matrix) based upon correlation matrix
##      PC1      h2      u2 com
## V   0.679 0.4614 0.539  1
## S   0.661 0.4371 0.563  1
## A   0.706 0.4985 0.502  1
## DS  0.501 0.2506 0.749  1
## I   0.720 0.5190 0.481  1
## C   0.677 0.4577 0.542  1
## LNS 0.497 0.2471 0.753  1
## PC  0.497 0.2474 0.753  1
## DSC      0.0843 0.916  1
## BD  0.626 0.3919 0.608  1
## MR  0.400 0.1602 0.840  1
## PA  0.485 0.2354 0.765  1
## SS  0.403 0.1628 0.837  1
##
##
##      PC1
## SS loadings  4.153
## Proportion Var 0.319
##
## Mean item complexity = 1
## Test of the hypothesis that 1 component is sufficient.
##
## The root mean square of the residuals (RMSR) is  0.127
## with the empirical chi square 270.502 with prob < 6.92e-27
##
## Fit based upon off diagonal values = 0.806

```

```
print.psych(SAPCA2, cut = 0.3, sort = F, digits = 3)
```

```

## Principal Components Analysis
## Call: principal(r = r, nfactors = nfactors, residuals = residuals,
##      rotate = rotate, n.obs = n.obs, covar = covar, scores = scores,
##      missing = missing, impute = impute, oblique.scores = oblique.scores,
##      method = method, use = use, cor = cor, correct = 0.5, weight = NULL)
## Standardized loadings (pattern matrix) based upon correlation matrix
##      RC1      RC2      h2      u2 com
## V   0.809      0.658 0.342 1.01
## S   0.731      0.551 0.449 1.07
## A   0.628 0.341 0.510 0.490 1.54
## DS      0.483 0.301 0.699 1.54
## I   0.838      0.710 0.290 1.02
## C   0.804      0.650 0.350 1.01
## LNS      0.493 0.305 0.695 1.48
## PC      0.511 0.317 0.683 1.41
## DSC      0.573 0.333 0.667 1.03
## BD      0.709 0.563 0.437 1.24
## MR      0.580 0.341 0.659 1.02
## PA  0.349 0.343 0.239 0.761 2.00
## SS      0.729 0.534 0.466 1.01
##
##
##      RC1      RC2

```

```
## SS loadings          3.310 2.704
## Proportion Var      0.255 0.208
## Cumulative Var      0.255 0.463
## Proportion Explained 0.550 0.450
## Cumulative Proportion 0.550 1.000
##
## Mean item complexity = 1.3
## Test of the hypothesis that 2 components are sufficient.
##
## The root mean square of the residuals (RMSR) is 0.091
## with the empirical chi square 136.74 with prob < 2.5e-09
##
## Fit based upon off diagonal values = 0.902
```

```
print.psych(SAPCA3, cut = 0.3, sort = F, digits = 3)
```

```
## Principal Components Analysis
## Call: principal(r = r, nfactors = nfactors, residuals = residuals,
## rotate = rotate, n.obs = n.obs, covar = covar, scores = scores,
## missing = missing, impute = impute, oblique.scores = oblique.scores,
## method = method, use = use, cor = cor, correct = 0.5, weight = NULL)
## Standardized loadings (pattern matrix) based upon correlation matrix
##      RC1      RC2      RC3      h2      u2      com
## V      0.810              0.666 0.334 1.03
## S      0.727              0.558 0.442 1.11
## A      0.618 0.332          0.528 0.472 1.74
## DS      0.644          0.489 0.511 1.35
## I      0.821              0.720 0.280 1.14
## C      0.797              0.650 0.350 1.05
## LNS      0.668          0.514 0.486 1.30
## PC              0.765 0.613 0.387 1.09
## DSC      0.667          0.461 0.539 1.07
## BD      0.424 0.610 0.589 0.411 2.02
## MR              0.723 0.543 0.457 1.08
## PA              0.534 0.375 0.625 1.57
## SS      0.631 0.382 0.549 0.451 1.68
##
##
##      RC1      RC2      RC3
## SS loadings          3.182 2.044 2.030
## Proportion Var      0.245 0.157 0.156
## Cumulative Var      0.245 0.402 0.558
## Proportion Explained 0.439 0.282 0.280
## Cumulative Proportion 0.439 0.720 1.000
##
## Mean item complexity = 1.3
## Test of the hypothesis that 3 components are sufficient.
##
## The root mean square of the residuals (RMSR) is 0.086
## with the empirical chi square 124.382 with prob < 4.4e-10
##
## Fit based upon off diagonal values = 0.911
```



```
print.psych(SAPCA4, cut = 0.3, sort = F, digits = 3)
```

```
## Principal Components Analysis
## Call: principal(r = r, nfactors = nfactors, residuals = residuals,
##      rotate = rotate, n.obs = n.obs, covar = covar, scores = scores,
##      missing = missing, impute = impute, oblique.scores = oblique.scores,
##      method = method, use = use, cor = cor, correct = 0.5, weight = NULL)
## Standardized loadings (pattern matrix) based upon correlation matrix
##      RC1      RC3      RC4      RC2      h2      u2      com
## V      0.818
## S      0.762
## A      0.571          0.406      0.541 0.459 2.16
## DS          0.821      0.709 0.291 1.11
## I      0.839          0.742 0.258 1.11
## C      0.755          0.660 0.340 1.33
## LNS          0.775      0.654 0.346 1.18
## PC          0.759          0.614 0.386 1.13
## DSC          0.887 0.796 0.204 1.02
## BD          0.595          0.349 0.589 0.411 2.36
## MR          0.771          0.657 0.343 1.21
## PA      0.334 0.507          0.389 0.611 1.93
## SS          0.304          0.731 0.659 0.341 1.47
##
##
##      RC1      RC3      RC4      RC2
## SS loadings      3.080 1.995 1.680 1.557
## Proportion Var      0.237 0.153 0.129 0.120
## Cumulative Var      0.237 0.390 0.520 0.639
## Proportion Explained 0.371 0.240 0.202 0.187
## Cumulative Proportion 0.371 0.611 0.813 1.000
##
## Mean item complexity = 1.4
## Test of the hypothesis that 4 components are sufficient.
##
## The root mean square of the residuals (RMSR) is 0.075
## with the empirical chi square 93.875 with prob < 5.41e-08
##
## Fit based upon off diagonal values = 0.933
```

#British

```
psych::fa.parallel(WAISIIW.cor, n.obs = 349)
```

Parallel Analysis Scree Plots

Parallel analysis suggests that the number of factors = 4 and the number of components = 3

```
BRPCA1 <- pca(WAISIIIW.cor, n.obs = 349, nfactors = 1)
BRPCA2 <- pca(WAISIIIW.cor, n.obs = 349, nfactors = 2)
BRPCA3 <- pca(WAISIIIW.cor, n.obs = 349, nfactors = 3)
BRPCA4 <- pca(WAISIIIW.cor, n.obs = 349, nfactors = 4)
```

```
print.psych(BRPCA1, cut = 0.3, sort = F, digits = 3)
```

Principal Components Analysis

```
## Call: principal(r = r, nfactors = nfactors, residuals = residuals,
##   rotate = rotate, n.obs = n.obs, covar = covar, scores = scores,
##   missing = missing, impute = impute, oblique.scores = oblique.scores,
##   method = method, use = use, cor = cor, correct = 0.5, weight = NULL)
## Standardized loadings (pattern matrix) based upon correlation matrix
##      PC1    h2    u2 com
## V   0.595 0.354 0.646  1
## S   0.636 0.405 0.595  1
## A   0.678 0.460 0.540  1
## DS  0.526 0.277 0.723  1
## I   0.603 0.363 0.637  1
## C   0.725 0.525 0.475  1
## LNS 0.591 0.350 0.650  1
```

```

## PC  0.603 0.363 0.637  1
## DSC 0.463 0.214 0.786  1
## BD  0.731 0.534 0.466  1
## MR  0.734 0.539 0.461  1
## PA  0.789 0.622 0.378  1
## SS  0.586 0.343 0.657  1
##
##
##          PC1
## SS loadings  5.350
## Proportion Var 0.412
##
## Mean item complexity =  1
## Test of the hypothesis that 1 component is sufficient.
##
## The root mean square of the residuals (RMSR) is  0.154
## with the empirical chi square 1298.374 with prob < 1.02e-228
##
## Fit based upon off diagonal values = 0.845

```

```
print.psych(BRPCA2, cut = 0.3, sort = F, digits = 3)
```

```

## Principal Components Analysis
## Call: principal(r = r, nfactors = nfactors, residuals = residuals,
## rotate = rotate, n.obs = n.obs, covar = covar, scores = scores,
## missing = missing, impute = impute, oblique.scores = oblique.scores,
## method = method, use = use, cor = cor, correct = 0.5, weight = NULL)
## Standardized loadings (pattern matrix) based upon correlation matrix
##      RC1      RC2      h2      u2      com
## V      0.834 0.703 0.297 1.02
## S      0.844 0.729 0.271 1.04
## A 0.488 0.474 0.463 0.537 2.00
## DS 0.547      0.326 0.674 1.18
## I      0.805 0.662 0.338 1.04
## C      0.793 0.709 0.291 1.25
## LNS 0.553      0.373 0.627 1.42
## PC 0.578      0.395 0.605 1.36
## DSC 0.755      0.604 0.396 1.12
## BD 0.664 0.344 0.559 0.441 1.50
## MR 0.639 0.379 0.552 0.448 1.63
## PA 0.815      0.729 0.271 1.19
## SS 0.808      0.656 0.344 1.01
##
##
##          RC1      RC2
## SS loadings      4.029 3.431
## Proportion Var      0.310 0.264
## Cumulative Var      0.310 0.574
## Proportion Explained 0.540 0.460
## Cumulative Proportion 0.540 1.000
##
## Mean item complexity =  1.3
## Test of the hypothesis that 2 components are sufficient.
##
## The root mean square of the residuals (RMSR) is  0.092
## with the empirical chi square 463.616 with prob < 6.2e-67

```

```
##
## Fit based upon off diagonal values = 0.945

print.psych(BRPCA3, cut = 0.3, sort = F, digits = 3)

## Principal Components Analysis
## Call: principal(r = r, nfactors = nfactors, residuals = residuals,
##   rotate = rotate, n.obs = n.obs, covar = covar, scores = scores,
##   missing = missing, impute = impute, oblique.scores = oblique.scores,
##   method = method, use = use, cor = cor, correct = 0.5, weight = NULL)
## Standardized loadings (pattern matrix) based upon correlation matrix
##      RC1      RC2      RC3      h2      u2      com
## V          0.847          0.730 0.270 1.04
## S          0.844          0.738 0.262 1.07
## A  0.307  0.416  0.504  0.521  0.479  2.63
## DS          0.876  0.797  0.203  1.08
## I          0.809          0.675 0.325 1.06
## C          0.739  0.448  0.761  0.239  1.71
## LNS          0.803  0.715  0.285  1.22
## PC  0.673          0.533 0.467  1.34
## DSC 0.749          0.628 0.372  1.24
## BD  0.634  0.336          0.575 0.425  1.86
## MR  0.655  0.387          0.602 0.398  1.75
## PA  0.834          0.796 0.204  1.30
## SS  0.762          0.664 0.336  1.28
##
##              RC1      RC2      RC3
## SS loadings      3.349  3.275  2.109
## Proportion Var      0.258  0.252  0.162
## Cumulative Var      0.258  0.510  0.672
## Proportion Explained 0.383  0.375  0.241
## Cumulative Proportion 0.383  0.759  1.000
##
## Mean item complexity = 1.4
## Test of the hypothesis that 3 components are sufficient.
##
## The root mean square of the residuals (RMSR) is 0.072
## with the empirical chi square 281.413 with prob < 3.45e-37
##
## Fit based upon off diagonal values = 0.966
```

```
print.psych(BRPCA4, cut = 0.3, sort = F, digits = 3)
```

```
## Principal Components Analysis
## Call: principal(r = r, nfactors = nfactors, residuals = residuals,
##   rotate = rotate, n.obs = n.obs, covar = covar, scores = scores,
##   missing = missing, impute = impute, oblique.scores = oblique.scores,
##   method = method, use = use, cor = cor, correct = 0.5, weight = NULL)
## Standardized loadings (pattern matrix) based upon correlation matrix
##      RC2      RC1      RC3      RC4      h2      u2      com
## V  0.884          0.804 0.196  1.06
## S  0.840          0.759 0.241  1.15
## A  0.305  0.450  0.535          0.583 0.417  2.58
```

```

## DS          0.862          0.798 0.202 1.15
## I    0.812          0.701 0.299 1.12
## C    0.741          0.444          0.776 0.224 1.77
## LNS          0.811          0.728 0.272 1.22
## PC      0.811          0.679 0.321 1.07
## DSC          0.914 0.887 0.113 1.13
## BD      0.656          0.608 0.392 1.89
## MR      0.756          0.686 0.314 1.42
## PA      0.793          0.371 0.829 0.171 1.65
## SS          0.866 0.869 0.131 1.33
##
##              RC2   RC1   RC3   RC4
## SS loadings    2.965 2.780 2.090 1.870
## Proportion Var    0.228 0.214 0.161 0.144
## Cumulative Var    0.228 0.442 0.603 0.747
## Proportion Explained 0.305 0.286 0.215 0.193
## Cumulative Proportion 0.305 0.592 0.807 1.000
##
## Mean item complexity = 1.4
## Test of the hypothesis that 4 components are sufficient.
##
## The root mean square of the residuals (RMSR) is 0.052
## with the empirical chi square 149.439 with prob < 4.27e-17
##
## Fit based upon off diagonal values = 0.982

```

The PCA results arrived at by Cockcroft et al. are accurate for their selected numbers of factors in each sample in terms of the loadings reported, but when we look at the three-factor configuration in the South African group or the four-factor configuration in the British group, the results are extremely similar. This makes sense: the difference in the numbers of $\lambda > 1$ was decided based on the UK sample having their fourth marginally below 1. This variety of problem is always important to consider. Practitioners may avoid interpreting factor solutions differently like this if they (1) use multiple methods for determining the number of factors, (2) plot the λ values to make it easier to visualize their relative and absolute sizes and, (3) compute confidence intervals. I will demonstrate the latter two methods below, thus inadvertently showing the first. I have also demonstrated the first at <https://rpubs.com/JLLJ/BF>. The formula for the λ confidence intervals comes from Larsen & Warne (2010; see also Warne & Larsen, 2014). Before printing these results, note that interpretation should happen on a case-by-case basis, so even though a confidence interval exceeds 1, that does not mean it should be introduced into subsequent confirmatory models. Additional testing is required to make that decision, though the confirmatory test nonetheless ought to be informed by the intervals.

```

NUM <- seq(1, 13, 1); EVSA <- SAPCA4$values; EVSB <- BRPCA4$values
EVSAL <- LECLOW(SAPCA4); EVSAU <- LECHIGH(SAPCA4)
EVBRL <- LECLOW(BRPCA4); EVBRU <- LECHIGH(BRPCA4)
SAPDF <- data.frame("NUM" = NUM, "EVSA" = EVSA, "EVSB" = EVSB,
                    "EVSAL" = EVSAL, "EVSAU" = EVSAU, "EVBRL" = EVBRL, "EVBRU" = EVBRU)

EVPSA <- ggplot(SAPDF, aes(x = NUM, y = EVSA, fill = as.factor(NUM))) +
  geom_bar(stat = "identity", show.legend = F) + theme_bw() +
  scale_fill_hue(c = 30) +
  theme(text = element_text(size = 12, family = "serif"),
        plot.title = element_text(hjust = 0.5), axis.title.x = element_blank()) +
  geom_hline(yintercept = 1, color = "dark gray", linetype = "dashed", size = 1) +
  ggtitle("Eigenvalues in South African Sample") +
  geom_errorbar(aes(x = NUM, ymax = EVSAU, ymin = EVSAL),

```

```

    position = position_dodge(0.7), stat = "identity",
    width = 0.5, size = 0.7)

EVPBR <- ggplot(SAPDF, aes(x = NUM, y = EVSB, fill = as.factor(NUM))) +
  geom_bar(stat = "identity", show.legend = F) +
  xlab("Number of Eigenvalue") + theme_bw() +
  scale_fill_hue(c = 30) +
  theme(text = element_text(size = 12, family = "serif"),
        plot.title = element_text(hjust = 0.5)) +
  geom_hline(yintercept = 1, color = "dark gray", linetype = "dashed", size = 1) +
  ggtitle("Eigenvalues in British Sample") +
  geom_errorbar(aes(x = NUM, ymax = EVBRU, ymin = EVBRL),
               position = position_dodge(0.7), stat = "identity",
               width = 0.5, size = 0.7)

ggarrange(EVPSA, EVPBR, nrow = 2)

```

```
MAPM(SAPCA4$values[4], n = 107)
```

```
## With the provided eigenvalue of 1.056 and the provided n,  
## you achieve a confidence interval of 0.773 to 1.338
```

```
MAPM(BRPCA4$values[4], n = 349)
```

```
## With the provided eigenvalue of 0.972 and the provided n,  
## you achieve a confidence interval of 0.828 to 1.117
```

Part II: Confirmatory Tests of the Proposition that Configural Factor Structure Differed

As quoted above, Cockcroft et al. proposed that the configuration of factors and their loadings - the so-called configural structure - of the WAIS-III differed between the South African and British samples. They did not test that proposition so this section contains those tests. The three-factor Perceptual Organization (POI), Verbal Comprehension (VCI), and Working Memory Index (WMI) model - henceforth dubbed the British configuration - was fitted in both groups alongside the four-factor VCI, POI, WMI, and Processing Speed Index (PSI) model - henceforth dubbed the South African configuration. Model fits were compared.

```
#British configuration
```

```
WIIIUK.model <- '  
POI =~ PA + SS + DSC + PC + MR + BD  
VCI =~ V + S + I + C + A  
WMI =~ C + DS + LNS + A'
```

```
#South African configuration
```

```
WIIISA.model <- '  
VCI =~ I + V + S + C + A  
POI =~ MR + PC + BD + PA  
WMI =~ A + DS + LNS  
PSI =~ DSC + SS'
```

```
#British Fitting
```

```
WIIIUKBRBR.fit <- cfa(WIIIUK.model, sample.cov = WAISIIIW.cov, sample.nobs = 349,  
  std.lv = T, orthogonal = F)  
WIIIUKBRSA.fit <- cfa(WIIISA.model, sample.cov = WAISIIIW.cov, sample.nobs = 349,  
  std.lv = T, orthogonal = F)  
  
round(cbind("British Model" = fitMeasures(WIIIUKBRBR.fit, FITM),  
  "South African Model" = fitMeasures(WIIIUKBRSA.fit, FITM)), 3)
```

##	British Model	South African Model
## chisq	403.439	243.295
## df	60.000	58.000
## npar	31.000	33.000
## cfi	0.857	0.923
## rmsea	0.128	0.096
## rmsea.ci.lower	0.116	0.083

```
## rmsea.ci.upper      0.140      0.108
## srmr                0.080      0.062
## bic                 20201.428    20052.995
```

#South African Fitting

```
WIIIIKSABR.fit <- cfa(WIIIIUK.model, sample.cov = WAISIIIB.cov, sample.nobs = 107,
  std.lv = T, orthogonal = F)
WIIIIKSASA.fit <- cfa(WIIIIISA.model, sample.cov = WAISIIIB.cov, sample.nobs = 107,
  std.lv = T, orthogonal = F)

round(cbind("British Model"      = fitMeasures(WIIIIKSABR.fit, FITM),
  "South African Model" = fitMeasures(WIIIIKSASA.fit, FITM)), 3)
```

```
##          British Model South African Model
## chisq          70.896          53.773
## df            60.000          58.000
## npar          31.000          33.000
## cfi            0.969          1.000
## rmsea          0.041          0.000
## rmsea.ci.lower 0.000          0.000
## rmsea.ci.upper 0.075          0.052
## srmr           0.066          0.053
## bic           6128.893         6121.117
```

Based on the above result, the model derived from the South African PCA results fit better than the model derived from the British PCA results *in the British group* as well as the South African one. There is something problematic at foot here: a violation of local independence, the condition where manifest variables are uncorrelated net of modeled latent variables. Peering at the residual correlations, we see that Symbol Search and Digit-Symbol Coding are residually correlated to a high degree. This is also the highest residual correlation in the South African group. The numbers produced for each group below are the residual correlations, the numbers of significant residual correlations with “significant” meaning $p < 0.05$ and the numbers of significant residual correlations with significance scaled to the sample sizes of the respective groups.

```
resid(WIIIIUKBRBR.fit, "cor"); length(which(resid(WIIIIUKBRBR.fit, "cor")$cov > CRITR(349)))
```

```
## $type
## [1] "cor.bollen"
##
## $cov
##   PA    SS    DSC    PC    MR    BD    V    S    I    C
## PA  0.000
## SS -0.024  0.000
## DSC 0.000  0.441  0.000
## PC  0.035 -0.074 -0.098  0.000
## MR  0.007 -0.059 -0.098  0.001  0.000
## BD -0.004 -0.007 -0.057 -0.036  0.021  0.000
## V  -0.038 -0.059 -0.135  0.003  0.057  0.074  0.000
## S  -0.024 -0.087 -0.133  0.045  0.140  0.117  0.014  0.000
## I   0.021 -0.070 -0.118  0.063  0.120  0.067  0.011 -0.033  0.000
## C  -0.052 -0.054 -0.171 -0.059  0.037  0.076 -0.030  0.016  0.028  0.000
## A   0.063  0.046 -0.028  0.085  0.196  0.184 -0.046  0.008  0.017  0.014
```



```
## DS -0.061 0.103 0.032 -0.110 -0.061 0.017 -0.073 -0.031 -0.017 0.026
## LNS 0.017 0.065 0.023 -0.027 0.053 0.071 0.005 0.097 0.070 -0.024
##      A      DS      LNS
## PA
## SS
## DSC
## PC
## MR
## BD
## V
## S
## I
## C
## A      0.000
## DS -0.068 0.000
## LNS 0.033 0.011 0.000
```

```
## [1] 12
```

```
length(which(resid(WIIIUKBRBR.fit, "cor")$cov > CRITR(349, alpha = NP(349))))
```

```
## [1] 4
```

```
resid(WIIIUKSABR.fit, "cor"); length(which(resid(WIIIUKSABR.fit, "cor")$cov > CRITR(107)))
```

```
## $type
## [1] "cor.bollen"
##
## $cov
##      PA      SS      DSC      PC      MR      BD      V      S      I      C
## PA      0.000
## SS -0.026 0.000
## DSC -0.028 0.258 0.000
## PC  0.074 -0.052 -0.121 0.000
## MR -0.001 -0.023 -0.118 0.104 0.000
## BD -0.048 -0.009 0.006 0.003 0.010 0.000
## V   0.109 -0.079 -0.026 0.013 -0.105 -0.041 0.000
## S   0.022 -0.033 0.055 0.090 -0.080 0.061 -0.018 0.000
## I   0.213 -0.117 -0.092 0.065 -0.083 0.077 -0.001 -0.004 0.000
## C   0.125 -0.115 -0.160 0.007 -0.001 -0.040 0.041 0.023 -0.026 0.000
## A   0.148 -0.029 -0.014 -0.019 0.005 0.041 -0.024 -0.002 0.044 -0.041
## DS -0.014 0.021 0.023 -0.118 -0.005 0.076 0.016 0.004 -0.053 0.086
## LNS -0.057 0.048 0.061 -0.022 -0.028 -0.049 0.033 0.021 -0.027 -0.038
##      A      DS      LNS
## PA
## SS
## DSC
## PC
## MR
## BD
## V
## S
```

```
## I
## C
## A    0.000
## DS  -0.037  0.000
## LNS  0.026  0.003  0.000
```

```
## [1] 4
```

```
length(which(resid(WIIIUKSABR.fit, "cor")$cov > CRITR(107, alpha = NP(107))))
```

```
## [1] 0
```

#Adding a large, plausible residual covariance

```
WIIIUKR.model <- '
POI =~ PA + SS + DSC + PC + MR + BD
VCI =~ V + S + I + C + A
WMI =~ C + DS + LNS + A
```

```
DSC ~~ SS'
```

#British Fitting

```
WIIIUKBRBR.fit <- cfa(WIIIUKR.model, sample.cov = WAISIIIW.cov, sample.nobs = 349,
  std.lv = T, orthogonal = F)
WIIIUKBRSA.fit <- cfa(WIIIISA.model, sample.cov = WAISIIIW.cov, sample.nobs = 349,
  std.lv = T, orthogonal = F)

round(cbind("British Model"      = fitMeasures(WIIIUKBRBR.fit, FITM),
  "South African Model" = fitMeasures(WIIIUKBRSA.fit, FITM)), 3)
```

##	British Model	South African Model
## chisq	192.576	243.295
## df	59.000	58.000
## npar	32.000	33.000
## cfi	0.944	0.923
## rmsea	0.081	0.096
## rmsea.ci.lower	0.068	0.083
## rmsea.ci.upper	0.093	0.108
## srmr	0.062	0.062
## bic	19996.420	20052.995

#South African Fitting

```
WIIIUKSABR.fit <- cfa(WIIIUKR.model, sample.cov = WAISIIIB.cov, sample.nobs = 107,
  std.lv = T, orthogonal = F)
WIIIUKSASA.fit <- cfa(WIIIISA.model, sample.cov = WAISIIIB.cov, sample.nobs = 107,
  std.lv = T, orthogonal = F)

round(cbind("British Model"      = fitMeasures(WIIIUKSABR.fit, FITM),
  "South African Model" = fitMeasures(WIIIUKSASA.fit, FITM)), 3)
```

##	British Model	South African Model
## chisq	56.822	53.773
## df	59.000	58.000
## npar	32.000	33.000
## cfi	1.000	1.000
## rmsea	0.000	0.000
## rmsea.ci.lower	0.000	0.000
## rmsea.ci.upper	0.055	0.052
## srmr	0.058	0.053
## bic	6119.492	6121.117

#Schwarz Weights for the South African Sample

```
BICsSA <- c(fitMeasures(WIIIUKSABR.fit, "bic"), fitMeasures(WIIIUKSASA.fit, "bic"))
BICweightsSA <- akaike.weights(BICsSA)$weights
```

```
ModsSA <- data.frame(Model = factor(rep(c("British", "South African"))),
                     CRIT = factor(rep("BIC", each = 2)),
                     values = BICweightsSA)
```

```
SWSA <- ggplot(ModsSA, aes(Model, values, fill = Model)) +
  geom_bar(stat = "identity", position = "dodge", show.legend = F) +
  coord_cartesian(ylim = c(0, 1)) + theme_bw() +
  theme(axis.title.x = element_blank(),
        text = element_text(size = 12, family = "serif"),
        plot.title = element_text(hjust = 0.5)) +
  ggtitle("South African Sample") +
  ylab("Normalized Probability")
```

#Schwarz Weights for the British Sample

```
BICsBR <- c(fitMeasures(WIIIUKBRBR.fit, "bic"), fitMeasures(WIIIUKBRSA.fit, "bic"))
BICweightsBR <- akaike.weights(BICsBR)$weights
```

```
ModsBR <- data.frame(Model = factor(rep(c("British", "South African"))),
                     CRIT = factor(rep("BIC", each = 2)),
                     values = BICweightsBR)
```

```
SWBR <- ggplot(ModsBR, aes(Model, values, fill = Model)) +
  geom_bar(stat = "identity", position = "dodge", show.legend = F) +
  coord_cartesian(ylim = c(0, 1)) + theme_bw() +
  theme(text = element_text(size = 12, family = "serif"),
        plot.title = element_text(hjust = 0.5)) +
  ggtitle("British Sample") +
  ylab("Normalized Probability") +
  xlab("Model Configuration")
```

```
ggarrange(SWSA, SWBR, nrow = 2)
```

ModsSA; ModsBR

```
##           Model CRIT    values
## 1         British   BIC 0.6925842
## 2 South African   BIC 0.3074158
```

```
##           Model CRIT    values
## 1         British   BIC 1.000000e+00
## 2 South African   BIC 5.188117e-13
```

Now, it appears, the British model fits best in both groups, and it does so by a considerable margin and with greater parsimony. Though the χ^2 difference was only significant in the British group and the approximate fit indices did not differ to a meaningful degree in the South African group, the information theoretic ones, arguably, did. I have also plotted these models below.

```
BRLAT <- list(
  POI = c("PA", "SS", "DSC", "PC", "MR", "BD"),
  VCI = c("V", "S", "I", "C", "A"),
  WMI = c("C", "A", "DS", "LNS"))
```

```

SALAT <- list(
  VCI = c("I", "V", "S", "C", "A"),
  POI = c("MR", "PC", "BD", "PA"),
  WMI = c("A", "DS", "LNS"),
  PSI = c("DSC", "SS"))

suppressWarnings(semPaths(WIIIUKBRBR.fit, "model", "std", title = F, residuals = F,
  groups = "BRLAT", pastel = T, mar = c(2, 1, 3, 1)))

```

```

suppressWarnings(semPaths(WIIIUKSABR.fit, "model", "std", title = F, residuals = F,
  groups = "BRLAT", pastel = T, mar = c(2, 1, 3, 1)))

```

```
suppressWarnings(semPaths(WIIUKBRSA.fit, "model", "std", title = F, residuals = F,
  groups = "SALAT", pastel = T, mar = c(2, 1, 3, 1)))
```

```
suppressWarnings(semPaths(WIIUKSASA.fit, "model", "std", title = F, residuals = F,
  groups = "SALAT", pastel = T, mar = c(2, 1, 3, 1)))
```

Part III: Are Cross-Loadings Supported?

The British configuration came with two cross-loadings, per Cockcroft et al. The need to model these cross-loadings creates problems for testing for measurement invariance for the same reasons the bifactor model does (see Hood, 2008). Below, I compared the models with and without cross-loadings, with the weaker loadings removed for the test.

#Removing Cross-Loadings

#British Model

```
WIIUKRNCI.model <- '
POI =~ PA + SS + DSC + PC + MR + BD
```



```

VCI =~ V + S + I + C
WMI =~ DS + LNS + A

DSC ~~ SS'

#British Fitting

WIIIIUKBRUKRNCI.fit <- cfa(WIIIIUKRNCI.model, sample.cov = WAISIIIW.cov, sample.nobs = 349,
                             std.lv = T, orthogonal = F)

round(cbind("With CLs"           = fitMeasures(WIIIIUKBRER.fit, FITM),
            "Without CLs" = fitMeasures(WIIIIUKBRUKRNCI.fit, FITM)), 3)

```

##	With CLs	Without CLs
## chisq	192.576	290.273
## df	59.000	61.000
## npar	32.000	30.000
## cfi	0.944	0.904
## rmsea	0.081	0.104
## rmsea.ci.lower	0.068	0.092
## rmsea.ci.upper	0.093	0.116
## srmr	0.062	0.080
## bic	19996.420	20082.407

```

#South African Fitting

WIIIIUKSABRUKRNCI.fit <- cfa(WIIIIUKRNCI.model, sample.cov = WAISIIIB.cov, sample.nobs = 107,
                              std.lv = T, orthogonal = F)

round(cbind("With CLs"           = fitMeasures(WIIIIUKSABR.fit, FITM),
            "Without CLs" = fitMeasures(WIIIIUKSABRUKRNCI.fit, FITM)), 3)

```

##	With CLs	Without CLs
## chisq	56.822	69.214
## df	59.000	61.000
## npar	32.000	30.000
## cfi	1.000	0.977
## rmsea	0.000	0.035
## rmsea.ci.lower	0.000	0.000
## rmsea.ci.upper	0.055	0.071
## srmr	0.058	0.065
## bic	6119.492	6122.539

The British model was notably negatively affected by two plausible residual covariances when the cross-loadings were dropped. Adding these, the model fit better and it could have been considered more theoretically acceptable for bias testing. Additionally, residual covariances could be constrained to equality, meaning that invariance with respect to them could be tested which also means that the replicability of the residual covariance in the model could be tested in a separate group.

```

#Adding two plausible residual covariances

```

```

WIIIIUKRNCII.model <- '

```

```
POI =~ PA + SS + DSC + PC + MR + BD
VCI =~ V + S + I + C
WMI =~ DS + LNS + A
```

```
DSC ~~ SS
DS ~~ C + LNS'
```

#British Fitting

```
WIIIIUKBRUKRNCII.fit <- cfa(WIIIIUKRNCII.model, sample.cov = WAISIIIW.cov, sample.nobs = 349,
                             std.lv = T, orthogonal = F)
```

```
round(cbind("With CLs"          = fitMeasures(WIIIIUKBRBR.fit, FITM),
            "Without CLs" = fitMeasures(WIIIIUKBRUKRNCII.fit, FITM)), 3)
```

##	With CLs	Without CLs
## chisq	192.576	187.613
## df	59.000	59.000
## npar	32.000	32.000
## cfi	0.944	0.946
## rmsea	0.081	0.079
## rmsea.ci.lower	0.068	0.066
## rmsea.ci.upper	0.093	0.092
## srmr	0.062	0.064
## bic	19996.420	19991.457

#South African Fitting

```
WIIIIUKSABRUKRNCII.fit <- cfa(WIIIIUKRNCII.model, sample.cov = WAISIIIB.cov, sample.nobs = 107,
                                std.lv = T, orthogonal = F)
```

```
round(cbind("With CLs"          = fitMeasures(WIIIIUKSABR.fit, FITM),
            "Without CLs" = fitMeasures(WIIIIUKSABRUKRNCII.fit, FITM)), 3)
```

##	With CLs	Without CLs
## chisq	56.822	55.378
## df	59.000	59.000
## npar	32.000	32.000
## cfi	1.000	1.000
## rmsea	0.000	0.000
## rmsea.ci.lower	0.000	0.000
## rmsea.ci.upper	0.055	0.053
## srmr	0.058	0.061
## bic	6119.492	6118.048

The South African model also had a cross-loading that could be removed.

#South African Model

```
WIIISANCI.model <- '
VCI =~ I + V + S + C
POI =~ MR + PC + BD + PA
```

```

WMI =~ A + DS + LNS
PSI =~ DSC + SS'

#British Fitting

WIIIBRSANCI.fit <- cfa(WIIISANCI.model, sample.cov = WAISIIIW.cov, sample.nobs = 349,
                        std.lv = T, orthogonal = F)

round(cbind("With CLs"      = fitMeasures(WIIIUKBRSa.fit, FITM),
            "Without CLs" = fitMeasures(WIIIBRSANCI.fit, FITM)), 3)

```

##	With CLs	Without CLs
## chisq	243.295	277.826
## df	58.000	59.000
## npar	33.000	32.000
## cfi	0.923	0.909
## rmsea	0.096	0.103
## rmsea.ci.lower	0.083	0.091
## rmsea.ci.upper	0.108	0.115
## srmr	0.062	0.075
## bic	20052.995	20081.670

```

#South African Fitting

WIIISASANCI.fit <- cfa(WIIISANCI.model, sample.cov = WAISIIIB.cov, sample.nobs = 107,
                        std.lv = T, orthogonal = F)

round(cbind("With CLs"      = fitMeasures(WIIIUKSASA.fit, FITM),
            "Without CLs" = fitMeasures(WIIISASANCI.fit, FITM)), 3)

```

##	With CLs	Without CLs
## chisq	53.773	66.697
## df	58.000	59.000
## npar	33.000	32.000
## cfi	1.000	0.978
## rmsea	0.000	0.035
## rmsea.ci.lower	0.000	0.000
## rmsea.ci.upper	0.052	0.071
## srmr	0.053	0.062
## bic	6121.117	6129.368

Looking in the British group, there were two large residual covariances: one between digit span and comprehension and another between digit span and letter-number sequencing. The former made fit much better in the British group but not in the South African group in which the model fit considerably worse than the model with cross-loadings. The latter of these replicated in the South African group and when added, they both yielded evidence for the equally parsimonious model without cross-loadings, though some tests were unavailable in that comparison, like above, due to the lack of a difference in degrees of freedom. Both models worked about equally well and could thus be used for analysis. The fact that they *could* be used may not be persuasive to many researchers, but the fact that they ought to be used because it is not possible to test invariance in a coherent fashion with multidimensional models should. The doubly important fact that we could test the equality of these residual covariances should be considered reassuring. Despite these findings and this reasoning, to maintain equivalence between results, I utilized the models given by Cockcroft

et al. (2015). However, I computed interpretability indices with the British model without cross-loadings with the note that the values for these indices would not differ much at all when computed with or without those cross-loadings or if they were computed with the South African configuration², to compute them with cross-loadings would require extra steps, assumptions, and the need to modify some of the indices in what are generally considered allowable ways.

#South African Model

```
WIIISANCII.model <- '
VCI =~ I + V + S + C
POI =~ MR + PC + BD + PA
WMI =~ A + DS + LNS
PSI =~ DSC + SS

LNS ~~ DS'
```

#British Fitting

```
WIIIBRSANCII.fit <- cfa(WIIISANCII.model, sample.cov = WAISIIIW.cov, sample.nobs = 349,
                        std.lv = T, orthogonal = F)

round(cbind("With CLs"      = fitMeasures(WIIIBRSANCII.fit, FITM),
            "Without CLs"   = fitMeasures(WIIIBRSANCII.fit, FITM)), 3)
```

##	With CLs	Without CLs
## chisq	243.295	240.128
## df	58.000	58.000
## npar	33.000	33.000
## cfi	0.923	0.924
## rmsea	0.096	0.095
## rmsea.ci.lower	0.083	0.083
## rmsea.ci.upper	0.108	0.107
## srmr	0.062	0.060
## bic	20052.995	20049.827

#South African Fitting

```
WIIISASANCII.fit <- cfa(WIIISANCII.model, sample.cov = WAISIIIB.cov, sample.nobs = 107,
                        std.lv = T, orthogonal = F)

round(cbind("With CLs"      = fitMeasures(WIIISASANCII.fit, FITM),
            "Without CLs"   = fitMeasures(WIIISASANCII.fit, FITM)), 3)
```

##	With CLs	Without CLs
## chisq	53.773	58.130
## df	58.000	58.000
## npar	33.000	33.000
## cfi	1.000	1.000
## rmsea	0.000	0.005
## rmsea.ci.lower	0.000	0.000
## rmsea.ci.upper	0.052	0.060
## srmr	0.053	0.059
## bic	6121.117	6125.473

²With the exception of the Percentage of Uncontaminated Correlations.

Part IV: The Utility of the South African Configuration

The South African configuration had four groups factors. This allowed it to be used for the purposes of testing between higher-order and correlated group factor models³. The British configuration, with its three factors, would have the same number of degrees of freedom in a correlated group factor and a model with a higher-order general factor. It may be theoretically useful to test these in both groups. Additionally, interpretability indices can inform on the same thing these can, providing evidence for unidimensionality or the lack thereof. Some may argue that a bifactor model should be compared here, but I do not (see Hood, 2008). The fact that the South African configuration was worse-fitting makes its use here somewhat dubious, so the results presented here were only considered exploratory in nature. More seriously than this model comparison, interpretability indices were presented based on the British configuration; these were likely more informative about dimensionality since we lack the ability to actually test it with exact, approximate, or information theoretic fit indices given the limited number of factors in the British configuration⁴.

#Higher-order South African Configuration

```
WIIISAHOF.model <- '
VCI =~ I + V + S + C + A
POI =~ MR + PC + BD + PA
WMI =~ A + DS + LNS
PSI =~ DSC + SS

g =~ POI + VCI + WMI + PSI'

WIIIBRSAHOF.fit <- cfa(WIIISAHOF.model, sample.cov = WAISIIIW.cov, sample.nobs = 349,
                      std.lv = T, orthogonal = T)

WIIISASAHOF.fit <- cfa(WIIISAHOF.model, sample.cov = WAISIIIB.cov, sample.nobs = 107,
                      std.lv = T, orthogonal = T)

round(cbind("Correlated Group Factor Model" = fitMeasures(WIIIUUKBRSA.fit, FITM),
           "Higher-Order Model" = fitMeasures(WIIIBRSAHOF.fit, FITM)), 3)
```

##	Correlated Group Factor Model	Higher-Order Model
## chisq	243.295	259.641
## df	58.000	60.000
## npar	33.000	31.000
## cfi	0.923	0.917
## rmsea	0.096	0.098
## rmsea.ci.lower	0.083	0.086
## rmsea.ci.upper	0.108	0.110
## srmr	0.062	0.072
## bic	20052.995	20057.630

```
round(cbind("Correlated Group Factor Model" = fitMeasures(WIIIUKSASA.fit, FITM),
           "Higher-Order Model" = fitMeasures(WIIISASAHOF.fit, FITM)), 3)
```

##	Correlated Group Factor Model	Higher-Order Model
----	-------------------------------	--------------------

³These latter models are sometimes referred to as measurement models.

⁴Such a test is not without flaws regarding dimensionality detection though (see Reise et al., 2013). Indeed, it is trivial to simulate data in which exact and approximate fit indices favor correlated group factor or higher-order models, for example if one includes insignificant model parameters or very high or low correlations, among other potential problems, they might make any particular metric inaccurate.

## chisq	53.773	58.722
## df	58.000	60.000
## npar	33.000	31.000
## cfi	1.000	1.000
## rmsea	0.000	0.000
## rmsea.ci.lower	0.000	0.000
## rmsea.ci.upper	0.052	0.057
## srmr	0.053	0.061
## bic	6121.117	6116.720

```
1 - pchisq(259.641 - 243.295, 2); NP(349)
```

```
## [1] 0.0002821702
```

```
## [1] 0.000444201
```

```
1 - pchisq(58.722 - 53.773, 2); NP(107)
```

```
## [1] 0.08420508
```

```
## [1] 0.00183531
```

The χ^2 test favored the correlated group factor model in the British but not the South African group. BIC favored the correlated group factor model in the British but not the South African group. Approximate fit indices were inconclusive, but lean towards the correlated group factor model in both groups. It is unfortunate that this data was not amenable to stronger theory testing due to its selected nature⁵. Below, the British model sans cross loadings was presented as a source for data to allow the computation of interpretability indices via Schmid-Leiman transformation. The interpretability indices used were H (Hancock & Mueller, 2001), the Percentage of Uncontaminated Correlations (PUC), ω_t or total omega, ω_h or omega hierarchical and ω_{hs} or omega hierarchical-subscale, the explained common variance (ECV), and the total variance explained by a given factor (see Reise, Scheinners, Widman & Haviland, 2012; Rodriguez, Reise & Haviland, 2015, 2016; Bonifay, Reise, Scheinners & Meijer, 2015). These were given by the following formulae:

$$H = 1 / \left[1 + \frac{1}{\sum_{i=1}^k \frac{\lambda_i^2}{1 - \lambda_i^2}} \right]$$

$$PUC = \frac{\frac{k_{items} \times (k_{items} - 1)}{2} - k_{factors} \times \frac{k_{itemsperfactor} \times (k_{itemsperfactor} - 1)}{2}}{\frac{k_{items} \times (k_{items} - 1)}{2}}$$

$$\omega_h = \frac{\Sigma \lambda_g^2}{(\Sigma \lambda_g^2) + (\Sigma \lambda_{\eta_1}^2) + (\Sigma \lambda_{\eta_x}^2) + (\Sigma 1 - h^2)}$$

$$ECV = \frac{\Sigma \lambda_g^2}{(\Sigma \lambda_g^2) + (\Sigma \lambda_{\eta_1}^2) + (\Sigma \lambda_{\eta_x}^2)}$$

⁵The phenomenon whereby g is less pronounced in more intelligent samples known as Spearman's Law of Diminishing Returns or SLODR is the reason this is the case. Whether this phenomenon is substantial or artefactual is contested but its reality is not (see Murray, Dixon & Johnson, 2013).

#Higher-order British Configuration

```
WIIIBRHOF.model <- '  
POI =~ PA + SS + DSC + PC + MR + BD  
VCI =~ V + S + I + C  
WMI =~ DS + LNS + A  
  
DSC ~~ SS  
DS ~~ C + LNS  
  
g =~ POI + VCI + WMI'  
  
WIIIBRHOF.fit <- cfa(WIIIBRHOF.model, sample.cov = WAISIIW.cov, sample.nobs = 349,  
  std.lv = T, orthogonal = T)  
  
WIIISABRHOF.fit <- cfa(WIIIBRHOF.model, sample.cov = WAISIIIB.cov, sample.nobs = 107,  
  std.lv = T, orthogonal = T)  
  
summary(WIIIBRHOF.fit, stand = T, fit = T)
```

```
## lavaan 0.6-9 ended normally after 61 iterations  
##  
##      Estimator                      ML  
##      Optimization method          NLMINB  
##      Number of model parameters    32  
##  
##      Number of observations        349  
##  
## Model Test User Model:  
##  
##      Test statistic                187.613  
##      Degrees of freedom            59  
##      P-value (Chi-square)          0.000  
##  
## Model Test Baseline Model:  
##  
##      Test statistic                2475.174  
##      Degrees of freedom            78  
##      P-value                      0.000  
##  
## User Model versus Baseline Model:  
##  
##      Comparative Fit Index (CFI)    0.946  
##      Tucker-Lewis Index (TLI)      0.929  
##  
## Loglikelihood and Information Criteria:  
##  
##      Loglikelihood user model (H0)  -9902.047  
##      Loglikelihood unrestricted model (H1) -9808.241  
##  
##      Akaike (AIC)                  19868.095  
##      Bayesian (BIC)                 19991.457  
##      Sample-size adjusted Bayesian (BIC) 19889.942
```

```

##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                0.079
##   90 Percent confidence interval - lower    0.066
##   90 Percent confidence interval - upper    0.092
##   P-value RMSEA <= 0.05                    0.000
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                0.064
##
## Parameter Estimates:
##
##   Standard errors                        Standard
##   Information                          Expected
##   Information saturated (h1) model      Structured
##
## Latent Variables:
##
##           Estimate  Std.Err  z-value  P(>|z|)  Std.lv  Std.all
##   POI =~
##     PA           1.418    0.114   12.387    0.000    2.074    0.919
##     SS           1.014    0.112    9.024    0.000    1.484    0.567
##     DSC           0.890    0.112    7.947    0.000    1.301    0.486
##     PC           1.442    0.140   10.312    0.000    2.108    0.675
##     MR           1.398    0.125   11.175    0.000    2.044    0.755
##     BD           1.422    0.131   10.871    0.000    2.080    0.726
##   VCI =~
##     V            1.679    0.115   14.566    0.000    2.164    0.815
##     S            1.956    0.132   14.843    0.000    2.522    0.831
##     I            1.602    0.118   13.603    0.000    2.065    0.760
##     C            1.612    0.109   14.784    0.000    2.078    0.798
##   WMI =~
##     DS           0.384    0.224    1.715    0.086    1.207    0.428
##     LNS          0.519    0.299    1.737    0.082    1.632    0.601
##     A            0.759    0.448    1.693    0.090    2.386    0.761
##   g =~
##     POI          1.067    0.151    7.081    0.000    0.730    0.730
##     VCI          0.814    0.105    7.760    0.000    0.631    0.631
##     WMI          2.979    1.876    1.588    0.112    0.948    0.948
##
## Covariances:
##
##           Estimate  Std.Err  z-value  P(>|z|)  Std.lv  Std.all
##   .SS ~~
##   .DSC           3.460    0.344   10.071    0.000    3.460    0.687
##   .C ~~
##   .DS           1.658    0.236    7.024    0.000    1.658    0.414
##   .DS ~~
##   .LNS          2.338    0.361    6.482    0.000    2.338    0.423
##
## Variances:
##
##           Estimate  Std.Err  z-value  P(>|z|)  Std.lv  Std.all
##   .PA           0.793    0.142    5.597    0.000    0.793    0.156
##   .SS           4.643    0.370   12.539    0.000    4.643    0.678

```


##	.DSC	5.468	0.428	12.770	0.000	5.468	0.764
##	.PC	5.323	0.444	11.992	0.000	5.323	0.545
##	.MR	3.144	0.281	11.178	0.000	3.144	0.429
##	.BD	3.886	0.337	11.543	0.000	3.886	0.473
##	.V	2.373	0.241	9.858	0.000	2.373	0.336
##	.S	2.853	0.303	9.415	0.000	2.853	0.310
##	.I	3.111	0.285	10.909	0.000	3.111	0.422
##	.C	2.470	0.244	10.118	0.000	2.470	0.364
##	.DS	6.477	0.518	12.505	0.000	6.477	0.816
##	.LNS	4.714	0.445	10.598	0.000	4.714	0.639
##	.A	4.140	0.638	6.488	0.000	4.140	0.421
##	.POI	1.000				0.467	0.467
##	.VCI	1.000				0.602	0.602
##	.WMI	1.000				0.101	0.101
##	g	1.000				1.000	1.000

```
summary(WIIISABRHOF.fit, stand = T, fit = T)
```

```
## lavaan 0.6-9 ended normally after 74 iterations
##
##      Estimator                      ML
##      Optimization method          NLMINB
##      Number of model parameters          32
##
##      Number of observations          107
##
## Model Test User Model:
##
##      Test statistic          55.378
##      Degrees of freedom          59
##      P-value (Chi-square)          0.610
##
## Model Test Baseline Model:
##
##      Test statistic          430.393
##      Degrees of freedom          78
##      P-value          0.000
##
## User Model versus Baseline Model:
##
##      Comparative Fit Index (CFI)          1.000
##      Tucker-Lewis Index (TLI)          1.014
##
## Loglikelihood and Information Criteria:
##
##      Loglikelihood user model (H0)          -2984.259
##      Loglikelihood unrestricted model (H1)          -2956.570
##
##      Akaike (AIC)          6032.518
##      Bayesian (BIC)          6118.048
##      Sample-size adjusted Bayesian (BIC)          6016.943
##
## Root Mean Square Error of Approximation:
##
```

```

## RMSEA 0.000
## 90 Percent confidence interval - lower 0.000
## 90 Percent confidence interval - upper 0.053
## P-value RMSEA <= 0.05 0.936
##
## Standardized Root Mean Square Residual:
##
## SRMR 0.061
##
## Parameter Estimates:
##
## Standard errors Standard
## Information Expected
## Information saturated (h1) model Structured
##
## Latent Variables:
## Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## POI =~
## PA 0.978 0.254 3.854 0.000 1.215 0.433
## SS 1.006 0.241 4.178 0.000 1.251 0.471
## DSC 0.552 0.227 2.434 0.015 0.686 0.275
## PC 0.993 0.193 5.155 0.000 1.234 0.588
## MR 1.027 0.215 4.777 0.000 1.276 0.541
## BD 1.546 0.252 6.126 0.000 1.922 0.740
## VCI =~
## V 1.079 0.189 5.722 0.000 1.552 0.772
## S 0.676 0.125 5.390 0.000 0.972 0.698
## I 1.781 0.305 5.832 0.000 2.561 0.802
## C 1.281 0.228 5.619 0.000 1.842 0.738
## WMI =~
## DS 0.224 0.512 0.438 0.662 0.925 0.410
## LNS 0.270 0.617 0.438 0.661 1.117 0.464
## A 0.502 1.180 0.426 0.670 2.073 0.823
## g =~
## POI 0.738 0.205 3.606 0.000 0.594 0.594
## VCI 1.034 0.298 3.468 0.001 0.719 0.719
## WMI 4.005 9.740 0.411 0.681 0.970 0.970
##
## Covariances:
## Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## .SS ~~
## .DSC 2.129 0.628 3.392 0.001 2.129 0.378
## .C ~~
## .DS 0.770 0.363 2.122 0.034 0.770 0.222
## .DS ~~
## .LNS 1.432 0.483 2.965 0.003 1.432 0.326
##
## Variances:
## Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## .PA 6.401 0.949 6.745 0.000 6.401 0.813
## .SS 5.498 0.832 6.607 0.000 5.498 0.779
## .DSC 5.770 0.813 7.100 0.000 5.770 0.924
## .PC 2.888 0.483 5.977 0.000 2.888 0.655
## .MR 3.936 0.627 6.276 0.000 3.936 0.707

```

##	.BD	3.054	0.711	4.292	0.000	3.054	0.453
##	.V	1.632	0.300	5.444	0.000	1.632	0.404
##	.S	0.996	0.163	6.127	0.000	0.996	0.513
##	.I	3.647	0.724	5.037	0.000	3.647	0.357
##	.C	2.840	0.489	5.805	0.000	2.840	0.456
##	.DS	4.236	0.617	6.866	0.000	4.236	0.832
##	.LNS	4.555	0.687	6.634	0.000	4.555	0.785
##	.A	2.044	0.938	2.181	0.029	2.044	0.322
##	.POI	1.000				0.647	0.647
##	.VCI	1.000				0.483	0.483
##	.WMI	1.000				0.059	0.059
##	g	1.000				1.000	1.000

<i>Factor</i>	<i>H</i>	<i>PUC</i>	ω_t	ω_h or ω_{hs}	<i>ECV</i>	<i>Total Variance</i>
South African Group						
g	0.822	0.692	0.817	0.682	0.683	0.218
POI	0.483	-	0.641	0.348	0.189	0.060
VCI	0.379	-	0.748	0.232	0.127	0.041
WMI	0.004	-	0.576	0.002	0.001	<0.001
British Group						
g	0.845	0.692	0.871	0.748	0.695	0.278
POI	0.425	-	0.772	0.224	0.124	0.050
VCI	0.548	-	0.792	0.377	0.179	0.071
WMI	0.011	-	0.595	0.007	0.002	0.001

The values for relative ω in general were 0.835 and 0.859 in the South African and British groups, respectively. As alluded to above, these batteries likely experienced some diminution of their generality due to how selected they were and thus how range restricted their scores turned out to be. This possibility was extremely clear comparing the full scale IQ summary statistics to normative values. The British norms used for both samples had a mean of 100 and a standard deviation of 15, but in the British group, the standard deviation was only 86% as large at 12.9 while in the South African group, it was almost 60% as large at 8.94. Both samples were thus considerably restricted in range compared to the general population of the United Kingdom so, provided results from these samples could be generalized to their respective populations, the generality of performance is likely to be considerably greater in both. In any case, group factors were insufficiently reliable beyond the general factor in both datasets. Similar results were achieved with the South African configuration but, for brevity, were not presented.

Section Conclusion

The conclusions about the configural structure of the WAIS-III made by Cockcroft et al. (2015) were insufficiently justified. They made conclusions about differences without statistical tests and did not attempt robustness checks. When assessing the number of factors, they only used the Kaiser-Guttman rule and they did not attempt to confirm their derived PCA solutions were as different between groups as they thought they were. Moreover, they used the Kaiser-Guttman rule naively, neglecting that estimates have error so confidence intervals ought to have been computed; I did that and found the argument for differences in the numbers of factors based on this rule was poor. In fact, the acceptable fit of their configural model suggested they *would not* find evidence for different configural factor solutions. Nor did I here: cross-loadings were not group-specific and, for the most part, residual covariances were not either and. Both PCA-based solutions fit both groups similarly.

As a final point of contention in this section, the two other analyses conducted by Cockcroft et al. - the discriminant function analyses and MANOVA-based group comparisons - were not interpreted appropriately

or consistently. They could have had a place in the paper but it would have been without the unwarranted substantial interpretations they provided. With respect to the MANOVAs, Cockcroft et al. observed group differences in three of four of the WAIS-III index scores, each of the verbal subtests, three of the nonverbal subtests, and both of the processing speed subtests, and neither of the working memory subtests. The VCI and POI index scores favored the British group and the PSI index score favored the South African group while WMI scores did not significantly differ, all the POI subtests favored the British group except for matrix reasoning, the mean of which did not significantly differ, all the PSI subtests favored the South African group, and all of the verbal tests favored the British group. Only the last of these differences were interpreted by the authors. They wrote:

[Higher British scoring] suggests that these [verbal] subtests draw on culture-specific, acquired knowledge, and appear to be affected by these differences.

Despite making this statement, the authors neglected to make a similar one for other instances of the British group scoring higher or for instances where the South African group scored higher. I cannot detect any reason why, if their comments about verbal tests are considered valid, the same comments should not be made about every other difference. Perhaps South African culture does something tantamount to teaching how to do better in processing speed tests or, while speculation is on the table, maybe the lack of mean differences in the working memory index or the matrix reasoning test reflected countervailing but unique influences. To speculate as much is vacuous for a simple reason: in principle, a mean difference cannot tell you about its sources. There is simply no necessary connection between the gap and the cause. The MGCFA they ended up running was designed with assessing if group differences were driven by common influences; the results from using *it* allowed conclusions of this nature, although not exactly like them. The reason why an MGCFA would not allow this sort of conclusion is because it merely points to test bias rather than the cause(s) of the bias. Critically, MGCFA fits do not reflect the direction of the bias. In a comparison between a higher- and a lower-scoring group, a test could very well be biased in favor of the lower-scoring group. Additionally, this bias may not generalize, and another lower-scoring group may not display any evidence for bias at all, or in the same way. The issue of test bias requires understanding what MGCFA can inform practitioners about, and they do not - without becoming multiple-indicator multiple-cause models through the use of background variables that require considerable causal inference to be workable - elucidate what drove a discovered instance of bias. When bias is not found, MGCFA does not identify the substantial causes for differences between groups, like education and upbringing, they only identify psychometric ones. It is a potentially grave inferential error to become confused on this point. The discriminant function analysis was disappointing since it was, confusingly, interpreted differently than the results of the MANOVAs. That is, it was interpreted only in terms of scores:

This analysis established that higher VC scores typically characterize the UK students, while higher PS scores typify the SA students.

Though technically correct, if this point is interpreted in terms like the verbal differences discovered with the MANOVA result - and if Cockcroft et al. were allowed to do that, it should have been - it would suffer from the same issues. More importantly, classification accuracy and score differences may not reflect underlying differences in ability. That remains to be seen until *after* the degree (and, potentially, direction) of bias is known. Since their analysis did not test or adjust for bias, the differences were left dubious in terms of both substantial⁶ and psychometric sources

⁶A well-powered MGCFA finding no evidence for bias suggests influences on test performance are common to the groups being compared unless one proposes a pseudoscientific omitted variable that cannot be distinguished from modeled latent factors because it acts perfectly in tandem with them or directly on them to an identical degree in all members of an affected population. It is hard to imagine what this sort of variable might be and, to date, no one has been able to propose one that makes sense or whose influence can be assessed in any way. For more details on this interpretation of MGCFA see Lubke et al. (2003a, b).

Analysis Two: Multigroup Confirmatory Factor Analysis (Primary)

I reran Cockcroft et al.'s (2015) MGCFA to include the logical next steps that they did not: assessing higher stages of measurement invariance and producing partial measurement invariance models as well as estimating the degree of bias in the means and variances of manifest scores.

Part I: MGCFA

The procedure for fitting and testing an MGCFA is familiar⁷ to many psychometricians. Therefore, I did not seek to belabor discussing the steps involved in it that were covered by Cockcroft et al. except to add interpretation. In the first stage, the configural model was fit and when it fits in both groups, the same number and configuration of relationships exist among the manifest and latent variables, or outcomes and their proposed causes. In the second stage, the metric model was fit and it represented a test of whether the different groups attributed the same meanings to the tests; if loadings differed, the interpretation of the manifest variables by members of the groups differed. With the third stage, the scalar model was fit and it represented a similar test to that of the metric model, a test of whether the groups attributed the same meanings to the *levels* of responding. If both of these last two stages hold, intercepts and loadings will necessarily be collinear, through perhaps imperfectly so - even in a model without cross-loadings - because of measurement and sampling error. The fourth stage, strict factorial invariance, is crucial and involves testing if the residual variances of the manifest variables differ between groups as a test of whether the influences on manifest variables are the same between groups. The fourth stage is crucial in no small part because of what it literally represents. Where a factor model represents common causes, error variances represent specific causes, and in a well-fitting model these are all independent (i.e., the battery shows evidence for local independence), but when they are not, it is dubious whether the subsequent criteria-related use of scores - at least with respect to group comparisons and the like - is valid. The fifth stage involves testing whether factor covariances are equal, which means assessing if measurement error is equal between groups. The fourth stage is typically assumed to represent the point where measurement error is equal, and in a higher-order factor model where loadings are equal between groups, it would, but in a correlated group factor model, it does not and cannot, since the residual error variance, when constrained in one part of the model, effectively moves to the other varying part which would then be the factor covariances. Since manifest error variances are actually latent factor variances (and potentially *multiple* latent factor variances), their covariances ought to also be constrained in that phase. The sixth stage involves testing the breadth of the construct in different groups by constraining the latent factor variances. If this stage and previous ones hold, differences in criteria prediction between groups are due to *different* causes outside of the construct being examined in the particular MGCFA. If this last stage does not hold, it does not mean that there is bias since it is a test of structural rather than measurement bias and thus represents a genuine difference in variances between the samples, though this may impact the relative strengths of prediction and the intercepts of regression equations involving impacted manifest or estimated latent variables. The final stage is also a structural test and involves constraining factor means to see if, net of measurement, the groups differ in the means of the thing or things being measured. This stage is only appropriate to assess if previous measurement stages held to a satisfactory degree and, even with a test of partial invariance, the degree of bias may be too large for this to be accurate (Steenkamp & Baumgartner, 1998 and a considerable subsequent literature have touched on this). This stage also often involves comparisons between different mean models, such that, for example, one or another factor mean might be constrained versus an omnibus test to see if fit is better with selective constraints on any of them. The aggregate mean differences will always be redistributed among the factors, but it is important to see which factor means are consistent with difference. The pattern may be revelatory, for example guiding researchers to focus on specific tests or phenomena that link test results to specific psychometric or real-world phenomena.

I utilized the four-factor model from the South African group for the MGCFA since it was basically the model utilized by Cockcroft et al. (2015). Results from the British configuration were also presented. It may be worth noting that a four-factor configuration cannot be argued for on theoretical grounds alone; if the

⁷See Lubke et al. (2003b) if you are not.

scale lacked structural fidelity in that configuration and a three-factor one was preferred, it ought to have been the one that was interpreted.

```
#Measurement Model - South African Configuration

#Configural

WIIIMMC.fit <- cfa(WIIISA.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = T, orthogonal = F)

#Metric

WIIIMMM.fit <- cfa(WIIISA.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = "loadings")

#Scalar

WIIIMMS.fit <- cfa(WIIISA.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts"))

#Strict

WIIIMMF.fit <- cfa(WIIISA.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals"))

#Latent Variance Homogeneity

WIIIMMV.fit <- cfa(WIIISA.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = T, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals",
    "residual.covariances"))

#Mean Homogeneity

WIIIMME.fit <- cfa(WIIISA.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = T, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals",
    "residual.covariances", "means"))

round(cbind(CONFIGURAL = fitMeasures(WIIIMMC.fit, FITM),
  METRIC = fitMeasures(WIIIMMM.fit, FITM),
  SCALAR = fitMeasures(WIIIMMS.fit, FITM),
  STRICT = fitMeasures(WIIIMMF.fit, FITM),
  LVARs = fitMeasures(WIIIMMV.fit, FITM),
  MEANS = fitMeasures(WIIIMME.fit, FITM)), 3)
```

##	CONFIGURAL	METRIC	SCALAR	STRICT	LVARs	MEANS
## chisq	297.069	365.493	557.901	798.639	798.639	1060.132
## df	116.000	126.000	135.000	148.000	148.000	152.000
## npar	92.000	82.000	73.000	60.000	60.000	56.000
## cfi	0.934	0.913	0.846	0.763	0.763	0.670

```
## rmsea          0.083      0.091      0.117      0.139      0.139      0.162
## rmsea.ci.lower  0.071      0.080      0.107      0.129      0.129      0.153
## rmsea.ci.upper  0.094      0.102      0.127      0.148      0.148      0.171
## srmr           0.056      0.071      0.101      0.122      0.122      0.615
## bic            26389.960 26397.159 26534.464 26695.610 26695.610 26932.613
```

```
#Measurement model - British Configuration
```

```
#Configural
```

```
WIIIMMC.fit <- cfa(WIIIUKR.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = T, orthogonal = F)
```

```
#Metric
```

```
WIIIMMM.fit <- cfa(WIIIUKR.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = "loadings")
```

```
#Scalar
```

```
WIIIMMS.fit <- cfa(WIIIUKR.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts"))
```

```
#strict
```

```
WIIIMMF.fit <- cfa(WIIIUKR.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals"))
```

```
#Latent Variance Homogeneity
```

```
WIIIMMV.fit <- cfa(WIIIUKR.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = T, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals",
    "residual.covariances", "lv.variances"))
```

```
#Mean Homogeneity
```

```
WIIIMME.fit <- cfa(WIIIUKR.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = T, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals",
    "residual.covariances", "lv.variances",
    "means"))
```

```
round(cbind(CONFIGURAL = fitMeasures(WIIIMMC.fit, FITM),
  METRIC = fitMeasures(WIIIMMM.fit, FITM),
  SCALAR = fitMeasures(WIIIMMS.fit, FITM),
  STRICT = fitMeasures(WIIIMMF.fit, FITM),
  LVARs = fitMeasures(WIIIMMV.fit, FITM),
  MEANS = fitMeasures(WIIIMME.fit, FITM)), 3)
```

```
##          CONFIGURAL    METRIC    SCALAR    STRICT    LVARs    MEANS
```

## chisq	249.398	315.892	582.156	805.767	861.291	1179.715
## df	118.000	130.000	140.000	153.000	157.000	160.000
## npar	90.000	78.000	68.000	55.000	51.000	48.000
## cfi	0.952	0.932	0.839	0.763	0.744	0.629
## rmsea	0.070	0.079	0.118	0.137	0.140	0.167
## rmsea.ci.lower	0.058	0.068	0.108	0.128	0.131	0.158
## rmsea.ci.upper	0.082	0.090	0.128	0.146	0.150	0.176
## srmr	0.057	0.070	0.115	0.132	0.203	0.357
## bic	26330.044	26323.068	26528.107	26672.126	26703.159	27003.216

As discovered by Cockcroft et al., there was considerable measurement bias. Next, the location of the bias was discovered.

Part II: Partial Invariance

Freed parameters were discovered through the modification indices and they were step-specific, so if the metric model showed a large enough change in model fit, the loadings were inspected and if the scalar model needed to be refit after the metric model was, the intercepts were inspected.

#Measurement Model - South African Configuration, with Partial Models

```
NP(349 + 107)
```

```
## [1] 0.0003222643
```

#Configural

```
WIIIMMC.fit <- cfa(WIIISA.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = T, orthogonal = F)
```

#Metric

```
WIIIMMM.fit <- cfa(WIIISA.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = "loadings")
```

#Partial Metric Model

```
WIIIMMP.fit <- cfa(WIIISA.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = "loadings", group.partial = "VCI=~S")
```

#Scalar

```
WIIIMMS.fit <- cfa(WIIISA.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts"),
  group.partial = "VCI=~S")
```

#Partial Scalar Model

```
WIIIMMSP.fit <- cfa(WIIISA.model, sample.cov = GCovs, sample.mean = GMeans,
```



```

sample.nobs = GNs, std.lv = F, orthogonal = F,
group.equal = c("loadings", "intercepts"),
group.partial = c("VCI=~S", "I~1", "MR~1", "C~1", "A~1", "S~1"))

1 - pchisq(334.654-319.608, 4)

```

```
## [1] 0.004606755
```

#Strict

```

WIIIMMF.fit <- cfa(WIIISA.model, sample.cov = GCovs, sample.mean = GMeans,
sample.nobs = GNs, std.lv = F, orthogonal = F,
group.equal = c("loadings", "intercepts", "residuals"),
group.partial = c("VCI=~S", "I~1", "MR~1", "C~1", "A~1", "S~1"))

```

#Partial Strict

```

WIIIMFP.fit <- cfa(WIIISA.model, sample.cov = GCovs, sample.mean = GMeans,
sample.nobs = GNs, std.lv = F, orthogonal = F,
group.equal = c("loadings", "intercepts", "residuals"),
group.partial = c("VCI=~S", "I~1", "MR~1", "C~1", "A~1", "S~1",
"PA~~PA", "SS~~SS", "S~~S", "PC~~PC"))

```

```
1 - pchisq(365.203-334.654, 9)
```

```
## [1] 0.0003536426
```

#Latent Variance Homogeneity

```

WIIIMMV.fit <- cfa(WIIISA.model, sample.cov = GCovs, sample.mean = GMeans,
sample.nobs = GNs, std.lv = F, orthogonal = F,
group.equal = c("loadings", "intercepts", "residuals",
"residual.covariances", "lv.variances"),
group.partial = c("VCI=~S", "I~1", "MR~1", "C~1", "A~1", "S~1",
"PA~~PA", "SS~~SS", "S~~S", "PC~~PC"))

```

```
1 - pchisq(385.429-365.203, 4)
```

```
## [1] 0.0004506208
```

*#The residual covariances part was added for perfunctory reasons.
#There are no residual covariances in this model.*

#Mean Homogeneity

```

WIIIMME.fit <- cfa(WIIISA.model, sample.cov = GCovs, sample.mean = GMeans,
sample.nobs = GNs, std.lv = F, orthogonal = F,
group.equal = c("loadings", "intercepts", "residuals",
"residual.covariances", "lv.variances",
"means"),
group.partial = c("VCI=~S", "I~1", "MR~1", "C~1", "A~1", "S~1",

```

```

"PA~~PA", "SS~~SS", "S~~S", "PC~~PC"))

round(cbind(CONFIGURAL = fitMeasures(WIIIMMC.fit, FITM),
      METRIC = fitMeasures(WIIIMMM.fit, FITM),
      PMETRIC = fitMeasures(WIIIMMP.fit, FITM),
      SCALAR = fitMeasures(WIIIMMS.fit, FITM),
      PSCALAR = fitMeasures(WIIIMMSP.fit, FITM),
      STRICT = fitMeasures(WIIIMMF.fit, FITM),
      PSTRICT = fitMeasures(WIIIMMFP.fit, FITM),
      LVARs = fitMeasures(WIIIMMV.fit, FITM),
      MEANS = fitMeasures(WIIIMMME.fit, FITM)), 3)

```

	CONFIGURAL	METRIC	PMETRIC	SCALAR	PSCALAR	STRICT
## chisq	297.069	365.493	319.608	553.680	334.654	548.467
## df	116.000	126.000	125.000	134.000	129.000	142.000
## npar	92.000	82.000	83.000	74.000	79.000	66.000
## cfi	0.934	0.913	0.929	0.847	0.925	0.852
## rmsea	0.083	0.091	0.083	0.117	0.084	0.112
## rmsea.ci.lower	0.071	0.080	0.071	0.107	0.073	0.102
## rmsea.ci.upper	0.094	0.102	0.094	0.127	0.095	0.122
## srmr	0.056	0.071	0.061	0.100	0.064	0.081
## bic	26389.960	26397.159	26357.396	26536.366	26347.953	26482.173
##	PSTRICT	LVARs	MEANS			
## chisq	365.203	385.429	717.219			
## df	138.000	142.000	146.000			
## npar	70.000	66.000	62.000			
## cfi	0.917	0.911	0.792			
## rmsea	0.085	0.087	0.131			
## rmsea.ci.lower	0.074	0.076	0.122			
## rmsea.ci.upper	0.096	0.097	0.141			
## srmr	0.070	0.108	0.273			
## bic	26323.399	26319.135	26626.436			

```
summary(WIIIMMV.fit, stand = T)
```

```

## lavaan 0.6-9 ended normally after 151 iterations
##
##      Estimator                      ML
##      Optimization method          NLMINB
##      Number of model parameters      96
##      Number of equality constraints    30
##
##      Number of observations per group:
##      Group 1                        349
##      Group 2                        107
##
## Model Test User Model:
##
##      Test statistic                  385.429
##      Degrees of freedom              142
##      P-value (Chi-square)            0.000
##      Test statistic for each group:

```

```

##      Group 1                      258.342
##      Group 2                      127.087
##
## Parameter Estimates:
##
##      Standard errors              Standard
##      Information                  Expected
##      Information saturated (h1) model Structured
##
##
## Group 1 [Group 1]:
##
## Latent Variables:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## VCI =~
##   I          1.000
##   V      (.p2.) 0.937   0.055  16.993   0.000   2.033   0.804
##   S          1.141   0.070  16.230   0.000   2.476   0.823
##   C      (.p4.) 0.934   0.056  16.631   0.000   2.025   0.787
##   A      (.p5.) 0.476   0.067   7.121   0.000   1.033   0.345
## POI =~
##   MR          1.000
##   PC      (.p7.) 1.068   0.074  14.388   0.000   2.023   0.658
##   BD      (.p8.) 1.043   0.071  14.673   0.000   1.975   0.708
##   PA      (.p9.) 1.038   0.061  16.976   0.000   1.967   0.915
## WMI =~
##   A          1.000
##   DS      (.11.) 1.529   0.190   8.064   0.000   2.010   0.718
##   LNS      (.12.) 1.609   0.201   8.018   0.000   2.115   0.795
## PSI =~
##   DSC          1.000
##   SS      (.14.) 1.063   0.067  15.803   0.000   2.315   0.921
##
## Covariances:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## VCI ~~
##   POI          1.908   0.280   6.813   0.000   0.464   0.464
##   WMI          1.000   0.225   4.448   0.000   0.351   0.351
##   PSI          0.730   0.290   2.519   0.012   0.155   0.155
## POI ~~
##   WMI          1.288   0.227   5.670   0.000   0.517   0.517
##   PSI          2.424   0.305   7.951   0.000   0.587   0.587
## WMI ~~
##   PSI          1.204   0.237   5.090   0.000   0.420   0.420
##
## Intercepts:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
##   .I          11.750   0.152  77.167   0.000  11.750   4.131
##   .V      (.39.)  14.560   0.135 107.582   0.000  14.560   5.759
##   .S          14.220   0.161  88.299   0.000  14.220   4.727
##   .C          12.480   0.138  90.642   0.000  12.480   4.852
##   .A           9.580   0.160  59.702   0.000   9.580   3.196
##   .MR          10.590   0.141  75.149   0.000  10.590   4.023
##   .PC      (.44.)  10.438   0.156  66.916   0.000  10.438   3.397

```

```

##      .BD      (.45.)    10.692    0.145    73.940    0.000    10.692    3.831
##      .PA      (.46.)    10.093    0.115    87.873    0.000    10.093    4.695
##      .DS      (.47.)     9.438    0.144    65.330    0.000     9.438    3.372
##      .LNS     (.48.)     8.920    0.140    63.715    0.000     8.920    3.354
##      .DSC     (.49.)     7.791    0.141    55.147    0.000     7.791    2.908
##      .SS      (.50.)     8.224    0.134    61.267    0.000     8.224    3.272
##      VCI              0.000              0.000    0.000
##      POI              0.000              0.000    0.000
##      WMI              0.000              0.000    0.000
##      PSI              0.000              0.000    0.000
##
## Variances:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
##      .I      (.15.)    3.387    0.280    12.112    0.000    3.387    0.419
##      .V      (.16.)    2.258    0.203    11.135    0.000    2.258    0.353
##      .S              2.923    0.312     9.378    0.000    2.923    0.323
##      .C      (.18.)    2.516    0.217    11.577    0.000    2.516    0.380
##      .A      (.19.)    5.239    0.395    13.267    0.000    5.239    0.583
##      .MR     (.20.)    3.342    0.261    12.798    0.000    3.342    0.482
##      .PC              5.351    0.445    12.032    0.000    5.351    0.567
##      .BD     (.22.)    3.891    0.299    13.009    0.000    3.891    0.499
##      .PA              0.752    0.139     5.407    0.000    0.752    0.163
##      .DS     (.24.)    3.795    0.384     9.888    0.000    3.795    0.484
##      .LNS     (.25.)    2.601    0.360     7.228    0.000    2.601    0.368
##      .DSC     (.26.)    2.432    0.312     7.804    0.000    2.432    0.339
##      .SS              0.957    0.314     3.045    0.002    0.957    0.152
##      VCI     (.28.)    4.704    0.514     9.147    0.000    1.000    1.000
##      POI     (.29.)    3.588    0.425     8.440    0.000    1.000    1.000
##      WMI     (.30.)    1.727    0.394     4.389    0.000    1.000    1.000
##      PSI     (.31.)    4.747    0.504     9.412    0.000    1.000    1.000
##
##
## Group 2 [Group 2]:
##
## Latent Variables:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
##      VCI =~
##      I              1.000              2.169    0.762
##      V      (.p2.)    0.937    0.055    16.993    0.000    2.033    0.804
##      S              0.482    0.056     8.644    0.000    1.046    0.724
##      C      (.p4.)    0.934    0.056    16.631    0.000    2.025    0.787
##      A      (.p5.)    0.476    0.067     7.121    0.000    1.033    0.337
##      POI =~
##      MR              1.000              1.894    0.720
##      PC      (.p7.)    1.068    0.074    14.388    0.000    2.023    0.782
##      BD      (.p8.)    1.043    0.071    14.673    0.000    1.975    0.708
##      PA      (.p9.)    1.038    0.061    16.976    0.000    1.967    0.615
##      WMI =~
##      A              1.000              1.314    0.429
##      DS      (.11.)    1.529    0.190     8.064    0.000    2.010    0.718
##      LNS     (.12.)    1.609    0.201     8.018    0.000    2.115    0.795
##      PSI =~
##      DSC              1.000              2.179    0.813
##      SS      (.14.)    1.063    0.067    15.803    0.000    2.315    0.759

```

```

##
## Covariances:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## VCI ~~
##   POI      2.236   0.412   5.429   0.000   0.544   0.544
##   WMI      1.435   0.337   4.258   0.000   0.503   0.503
##   PSI      1.201   0.530   2.267   0.023   0.254   0.254
## POI ~~
##   WMI      1.578   0.306   5.157   0.000   0.634   0.634
##   PSI      2.442   0.432   5.655   0.000   0.592   0.592
## WMI ~~
##   PSI      1.633   0.351   4.650   0.000   0.570   0.570
##
## Intercepts:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
##   .I          16.207   0.449  36.070   0.000  16.207   5.697
##   .V      (.39.)   14.560   0.135 107.582   0.000  14.560   5.759
##   .S          11.022   0.359  30.680   0.000  11.022   7.630
##   .C          14.607   0.405  36.062   0.000  14.607   5.679
##   .A          11.452   0.486  23.560   0.000  11.452   3.735
##   .MR          12.720   0.263  48.276   0.000  12.720   4.832
##   .PC      (.44.)   10.438   0.156  66.916   0.000  10.438   4.034
##   .BD      (.45.)   10.692   0.145  73.940   0.000  10.692   3.831
##   .PA      (.46.)   10.093   0.115  87.873   0.000  10.093   3.155
##   .DS      (.47.)    9.438   0.144  65.330   0.000    9.438   3.372
##   .LNS      (.48.)    8.920   0.140  63.715   0.000    8.920   3.354
##   .DSC      (.49.)    7.791   0.141  55.147   0.000    7.791   2.908
##   .SS      (.50.)    8.224   0.134  61.267   0.000    8.224   2.695
##   VCI         -6.017   0.463 -13.000   0.000   -2.774   -2.774
##   POI         -2.040   0.268  -7.617   0.000   -1.077   -1.077
##   WMI          0.074   0.170   0.435   0.663    0.056    0.056
##   PSI          1.729   0.277   6.250   0.000    0.794    0.794
##
## Variances:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
##   .I      (.15.)    3.387   0.280  12.112   0.000    3.387    0.419
##   .V      (.16.)    2.258   0.203  11.135   0.000    2.258    0.353
##   .S          0.992   0.163   6.089   0.000    0.992    0.476
##   .C      (.18.)    2.516   0.217  11.577   0.000    2.516    0.380
##   .A      (.19.)    5.239   0.395  13.267   0.000    5.239    0.557
##   .MR      (.20.)    3.342   0.261  12.798   0.000    3.342    0.482
##   .PC          2.603   0.489   5.322   0.000    2.603    0.389
##   .BD      (.22.)    3.891   0.299  13.009   0.000    3.891    0.499
##   .PA          6.364   0.967   6.579   0.000    6.364    0.622
##   .DS      (.24.)    3.795   0.384   9.888   0.000    3.795    0.484
##   .LNS      (.25.)    2.601   0.360   7.228   0.000    2.601    0.368
##   .DSC      (.26.)    2.432   0.312   7.804   0.000    2.432    0.339
##   .SS          3.948   0.788   5.007   0.000    3.948    0.424
##   VCI      (.28.)    4.704   0.514   9.147   0.000    1.000    1.000
##   POI      (.29.)    3.588   0.425   8.440   0.000    1.000    1.000
##   WMI      (.30.)    1.727   0.394   4.389   0.000    1.000    1.000
##   PSI      (.31.)    4.747   0.504   9.412   0.000    1.000    1.000

```

#Mean Homogeneity #1

```
WIIISA1.model <- '  
VCI =~ I + V + S + C + A  
POI =~ MR + PC + BD + PA  
WMI =~ A + DS + LNS  
PSI =~ DSC + SS  
  
VCI ~ c(0,0)*1'
```

#Mean Homogeneity #2

```
WIIISA2.model <- '  
VCI =~ I + V + S + C + A  
POI =~ MR + PC + BD + PA  
WMI =~ A + DS + LNS  
PSI =~ DSC + SS  
  
POI ~ c(0,0)*1'
```

#Mean Homogeneity #3

```
WIIISA3.model <- '  
VCI =~ I + V + S + C + A  
POI =~ MR + PC + BD + PA  
WMI =~ A + DS + LNS  
PSI =~ DSC + SS  
  
WMI ~ c(0,0)*1'
```

#Mean Homogeneity #4

```
WIIISA4.model <- '  
VCI =~ I + V + S + C + A  
POI =~ MR + PC + BD + PA  
WMI =~ A + DS + LNS  
PSI =~ DSC + SS  
  
PSI ~ c(0,0)*1'
```

#Mean Homogeneity #5

```
WIIISA5.model <- '  
VCI =~ I + V + S + C + A  
POI =~ MR + PC + BD + PA  
WMI =~ A + DS + LNS  
PSI =~ DSC + SS  
  
VCI ~ c(0,0)*1  
POI ~ c(0,0)*1'
```

#Mean Homogeneity #6

```

WIIISA6.model <- '
VCI =~ I + V + S + C + A
POI =~ MR + PC + BD + PA
WMI =~ A + DS + LNS
PSI =~ DSC + SS

```

```

VCI ~ c(0,0)*1
WMI ~ c(0,0)*1'

```

#Mean Homogeneity #7

```

WIIISA7.model <- '
VCI =~ I + V + S + C + A
POI =~ MR + PC + BD + PA
WMI =~ A + DS + LNS
PSI =~ DSC + SS

```

```

VCI ~ c(0,0)*1
PSI ~ c(0,0)*1'

```

#Mean Homogeneity #8

```

WIIISA8.model <- '
VCI =~ I + V + S + C + A
POI =~ MR + PC + BD + PA
WMI =~ A + DS + LNS
PSI =~ DSC + SS

```

```

POI ~ c(0,0)*1
WMI ~ c(0,0)*1'

```

#Mean Homogeneity #9

```

WIIISA9.model <- '
VCI =~ I + V + S + C + A
POI =~ MR + PC + BD + PA
WMI =~ A + DS + LNS
PSI =~ DSC + SS

```

```

POI ~ c(0,0)*1
PSI ~ c(0,0)*1'

```

#Mean Homogeneity #10

```

WIIISA10.model <- '
VCI =~ I + V + S + C + A
POI =~ MR + PC + BD + PA
WMI =~ A + DS + LNS
PSI =~ DSC + SS

```

```

WMI ~ c(0,0)*1
PSI ~ c(0,0)*1'

```

#Mean Homogeneity #11

```
WIIISA11.model <- '  
VCI =~ I + V + S + C + A  
POI =~ MR + PC + BD + PA  
WMI =~ A + DS + LNS  
PSI =~ DSC + SS  
  
VCI ~ c(0,0)*1  
POI ~ c(0,0)*1  
WMI ~ c(0,0)*1'
```

#Mean Homogeneity #12

```
WIIISA12.model <- '  
VCI =~ I + V + S + C + A  
POI =~ MR + PC + BD + PA  
WMI =~ A + DS + LNS  
PSI =~ DSC + SS  
  
VCI ~ c(0,0)*1  
POI ~ c(0,0)*1  
PSI ~ c(0,0)*1'
```

#Mean Homogeneity #13

```
WIIISA13.model <- '  
VCI =~ I + V + S + C + A  
POI =~ MR + PC + BD + PA  
WMI =~ A + DS + LNS  
PSI =~ DSC + SS  
  
VCI ~ c(0,0)*1  
WMI ~ c(0,0)*1  
PSI ~ c(0,0)*1'
```

#Mean Homogeneity #14

```
WIIISA14.model <- '  
VCI =~ I + V + S + C + A  
POI =~ MR + PC + BD + PA  
WMI =~ A + DS + LNS  
PSI =~ DSC + SS  
  
POI ~ c(0,0)*1  
WMI ~ c(0,0)*1  
PSI ~ c(0,0)*1'
```

```
WIIIMME1.fit <- cfa(WIIISA1.model, sample.cov = GCovs, sample.mean = GMeans,  
  sample.nobs = GNs, std.lv = F, orthogonal = F,  
  group.equal = c("loadings", "intercepts", "residuals",  
    "residual.covariances", "lv.variances"),  
  group.partial = c("VCI=~S", "I~1", "MR~1", "C~1", "A~1", "S~1",
```



```

"PA~~PA", "SS~~SS", "S~~S", "PC~~PC"))

WIIIMME2.fit <- cfa(WIIISA2.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals",
    "residual.covariances", "lv.variances"),
  group.partial = c("VCI~~S", "I~1", "MR~1", "C~1", "A~1", "S~1",
    "PA~~PA", "SS~~SS", "S~~S", "PC~~PC"))

WIIIMME3.fit <- cfa(WIIISA3.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals",
    "residual.covariances", "lv.variances"),
  group.partial = c("VCI~~S", "I~1", "MR~1", "C~1", "A~1", "S~1",
    "PA~~PA", "SS~~SS", "S~~S", "PC~~PC"))

WIIIMME4.fit <- cfa(WIIISA4.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals",
    "residual.covariances", "lv.variances"),
  group.partial = c("VCI~~S", "I~1", "MR~1", "C~1", "A~1", "S~1",
    "PA~~PA", "SS~~SS", "S~~S", "PC~~PC"))

WIIIMME5.fit <- cfa(WIIISA5.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals",
    "residual.covariances", "lv.variances"),
  group.partial = c("VCI~~S", "I~1", "MR~1", "C~1", "A~1", "S~1",
    "PA~~PA", "SS~~SS", "S~~S", "PC~~PC"))

WIIIMME6.fit <- cfa(WIIISA6.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals",
    "residual.covariances", "lv.variances"),
  group.partial = c("VCI~~S", "I~1", "MR~1", "C~1", "A~1", "S~1",
    "PA~~PA", "SS~~SS", "S~~S", "PC~~PC"))

WIIIMME7.fit <- cfa(WIIISA7.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals",
    "residual.covariances", "lv.variances"),
  group.partial = c("VCI~~S", "I~1", "MR~1", "C~1", "A~1", "S~1",
    "PA~~PA", "SS~~SS", "S~~S", "PC~~PC"))

WIIIMME8.fit <- cfa(WIIISA8.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals",
    "residual.covariances", "lv.variances"),
  group.partial = c("VCI~~S", "I~1", "MR~1", "C~1", "A~1", "S~1",
    "PA~~PA", "SS~~SS", "S~~S", "PC~~PC"))

WIIIMME9.fit <- cfa(WIIISA9.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,

```

```

        group.equal = c("loadings", "intercepts", "residuals",
                        "residual.covariances", "lv.variances"),
        group.partial = c("VCI=~S", "I~1", "MR~1", "C~1", "A~1", "S~1",
                        "PA~~PA", "SS~~SS", "S~~S", "PC~~PC"))

WIIIMME10.fit <- cfa(WIIISA10.model, sample.cov = GCovs, sample.mean = GMeans,
                    sample.nobs = GNs, std.lv = F, orthogonal = F,
                    group.equal = c("loadings", "intercepts", "residuals",
                                    "residual.covariances", "lv.variances"),
                    group.partial = c("VCI=~S", "I~1", "MR~1", "C~1", "A~1", "S~1",
                                    "PA~~PA", "SS~~SS", "S~~S", "PC~~PC"))

WIIIMME11.fit <- cfa(WIIISA11.model, sample.cov = GCovs, sample.mean = GMeans,
                    sample.nobs = GNs, std.lv = F, orthogonal = F,
                    group.equal = c("loadings", "intercepts", "residuals",
                                    "residual.covariances", "lv.variances"),
                    group.partial = c("VCI=~S", "I~1", "MR~1", "C~1", "A~1", "S~1",
                                    "PA~~PA", "SS~~SS", "S~~S", "PC~~PC"))

WIIIMME12.fit <- cfa(WIIISA12.model, sample.cov = GCovs, sample.mean = GMeans,
                    sample.nobs = GNs, std.lv = F, orthogonal = F,
                    group.equal = c("loadings", "intercepts", "residuals",
                                    "residual.covariances", "lv.variances"),
                    group.partial = c("VCI=~S", "I~1", "MR~1", "C~1", "A~1", "S~1",
                                    "PA~~PA", "SS~~SS", "S~~S", "PC~~PC"))

WIIIMME13.fit <- cfa(WIIISA13.model, sample.cov = GCovs, sample.mean = GMeans,
                    sample.nobs = GNs, std.lv = F, orthogonal = F,
                    group.equal = c("loadings", "intercepts", "residuals",
                                    "residual.covariances", "lv.variances"),
                    group.partial = c("VCI=~S", "I~1", "MR~1", "C~1", "A~1", "S~1",
                                    "PA~~PA", "SS~~SS", "S~~S", "PC~~PC"))

WIIIMME14.fit <- cfa(WIIISA14.model, sample.cov = GCovs, sample.mean = GMeans,
                    sample.nobs = GNs, std.lv = F, orthogonal = F,
                    group.equal = c("loadings", "intercepts", "residuals",
                                    "residual.covariances", "lv.variances"),
                    group.partial = c("VCI=~S", "I~1", "MR~1", "C~1", "A~1", "S~1",
                                    "PA~~PA", "SS~~SS", "S~~S", "PC~~PC"))

round(cbind(LVARS = fitMeasures(WIIIMV.fit, FITM),
            MEANS = fitMeasures(WIIIMME.fit, FITM),
            MEAN1 = fitMeasures(WIIIMME1.fit, FITM),
            MEAN2 = fitMeasures(WIIIMME2.fit, FITM),
            MEAN3 = fitMeasures(WIIIMME3.fit, FITM),
            MEAN4 = fitMeasures(WIIIMME4.fit, FITM),
            MEAN5 = fitMeasures(WIIIMME5.fit, FITM),
            MEAN6 = fitMeasures(WIIIMME6.fit, FITM),
            MEAN7 = fitMeasures(WIIIMME7.fit, FITM),
            MEAN8 = fitMeasures(WIIIMME8.fit, FITM),
            MEAN9 = fitMeasures(WIIIMME9.fit, FITM),
            MEAN10 = fitMeasures(WIIIMME10.fit, FITM),
            MEAN11 = fitMeasures(WIIIMME11.fit, FITM),

```

```

MEAN12 = fitMeasures(WIIIMME12.fit, FITM),
MEAN13 = fitMeasures(WIIIMME13.fit, FITM),
MEAN14 = fitMeasures(WIIIMME14.fit, FITM)),3)

```

	LVARs	MEANS	MEAN1	MEAN2	MEAN3	MEAN4
## chisq	385.429	717.219	674.795	451.458	385.617	422.807
## df	142.000	146.000	143.000	143.000	143.000	143.000
## npar	66.000	62.000	65.000	65.000	65.000	65.000
## cfi	0.911	0.792	0.807	0.888	0.912	0.898
## rmsea	0.087	0.131	0.128	0.097	0.086	0.093
## rmsea.ci.lower	0.076	0.122	0.118	0.087	0.076	0.083
## rmsea.ci.upper	0.097	0.141	0.137	0.107	0.097	0.103
## srmr	0.108	0.273	0.285	0.132	0.109	0.117
## bic	26319.135	26626.436	26602.378	26379.041	26313.201	26350.390
	MEAN5	MEAN6	MEAN7	MEAN8	MEAN9	MEAN10
## chisq	675.034	694.896	705.891	466.009	513.564	427.105
## df	144.000	144.000	144.000	144.000	144.000	144.000
## npar	64.000	64.000	64.000	64.000	64.000	64.000
## cfi	0.807	0.800	0.796	0.883	0.866	0.897
## rmsea	0.127	0.130	0.131	0.099	0.106	0.093
## rmsea.ci.lower	0.118	0.120	0.121	0.089	0.096	0.083
## rmsea.ci.upper	0.137	0.139	0.141	0.109	0.116	0.103
## srmr	0.290	0.246	0.275	0.118	0.132	0.113
## bic	26596.496	26616.357	26627.352	26387.470	26435.025	26348.566
	MEAN11	MEAN12	MEAN13	MEAN14		
## chisq	697.979	708.076	711.907	515.690		
## df	145.000	145.000	145.000	145.000		
## npar	63.000	63.000	63.000	63.000		
## cfi	0.799	0.795	0.794	0.865		
## rmsea	0.129	0.131	0.131	0.106		
## rmsea.ci.lower	0.120	0.121	0.121	0.096		
## rmsea.ci.upper	0.139	0.140	0.141	0.116		
## srmr	0.262	0.294	0.245	0.125		
## bic	26613.318	26623.415	26627.245	26431.029		

The best-fitting model with the South African configuration was one in which the means for WMI did not differ.

#Measurement Model - British Configuration, with Partial Models

#Configural

```

WIIIMC.fit <- cfa(WIIUKR.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNS, std.lv = T, orthogonal = F)

```

#Metric

```

WIIIMM.fit <- cfa(WIIUKR.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNS, std.lv = F, orthogonal = F,
  group.equal = "loadings")

```

#Partial Metric Model

```

WIIIMMP.fit <- cfa(WIIIUKR.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = "loadings", group.partial = "VCI=~S")

#Scalar

WIIIMMS.fit <- cfa(WIIIUKR.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts"),
  group.partial = "VCI=~S")

#Partial Scalar Model

WIIIMMSP.fit <- cfa(WIIIUKR.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts"),
  group.partial = c("VCI=~S", "I~1", "MR~1", "DSC~1", "SS~1", "S~1",
    "V~1"))

1 - pchisq(293.571-276.594, 4)

```

```
## [1] 0.00195294
```

```

#Strict

WIIIMMF.fit <- cfa(WIIIUKR.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals"),
  group.partial = c("VCI=~S", "I~1", "MR~1", "DSC~1", "SS~1", "S~1",
    "V~1"))

#Partial Strict

WIIIMMFP.fit <- cfa(WIIIUKR.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals"),
  group.partial = c("VCI=~S", "I~1", "MR~1", "DSC~1", "SS~1", "S~1",
    "V~1", "PA~~PA", "S~~S"))

1 - pchisq(329.011-295.571, 11)

```

```
## [1] 0.0004463542
```

```

#Residual Variance Homogeneity

WIIIMMVR.fit <- cfa(WIIIUKR.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals",
    "residual.covariances"),
  group.partial = c("VCI=~S", "I~1", "MR~1", "DSC~1", "SS~1", "S~1",
    "V~1", "PA~~PA", "S~~S"))

1 - pchisq(359.669-329.011, 1)

```

```
## [1] 3.077551e-08
```

```
#Residual covariance identity is untenable
```

```
#Latent Variance Homogeneity
```

```
WIIIMMV.fit <- cfa(WIIUKR.model, sample.cov = GCovs, sample.mean = GMeans,  
  sample.nobs = GNs, std.lv = F, orthogonal = F,  
  group.equal = c("loadings", "intercepts", "residuals",  
    "lv.variances"),  
  group.partial = c("VCI~~S", "I~1", "MR~1", "DSC~1", "SS~1", "S~1",  
    "V~1", "PA~~PA", "S~~S"))
```

```
1 - pchisq(349.327-329.011, 3)
```

```
## [1] 0.0001459755
```

```
#Latent variance homogeneity is borderline tenable, so it will be marked untenable. Additional tests of
```

```
#Mean Homogeneity
```

```
WIIIMME.fit <- cfa(WIIISA.model, sample.cov = GCovs, sample.mean = GMeans,  
  sample.nobs = GNs, std.lv = F, orthogonal = F,  
  group.equal = c("loadings", "intercepts", "residuals",  
    "means"),  
  group.partial = c("VCI~~S", "I~1", "MR~1", "DSC~1", "SS~1", "S~1",  
    "V~1", "PA~~PA", "S~~S"))
```

```
round(cbind(CONFIGURAL = fitMeasures(WIIIMC.fit, FITM),  
  METRIC = fitMeasures(WIIIMM.fit, FITM),  
  PMETRIC = fitMeasures(WIIIMMP.fit, FITM),  
  SCALAR = fitMeasures(WIIIMMS.fit, FITM),  
  PSCALAR = fitMeasures(WIIIMMSP.fit, FITM),  
  STRICT = fitMeasures(WIIIMMF.fit, FITM),  
  PSTRICT = fitMeasures(WIIIMMFP.fit, FITM),  
  RESCOV = fitMeasures(WIIIMMVR.fit, FITM),  
  LVARs = fitMeasures(WIIIMMV.fit, FITM),  
  MEANS = fitMeasures(WIIIMME.fit, FITM)), 3)
```

##	CONFIGURAL	METRIC	PMETRIC	SCALAR	PSCALAR	STRICT
## chisq	249.398	315.892	276.594	580.428	293.571	470.437
## df	118.000	130.000	129.000	139.000	133.000	146.000
## npar	90.000	78.000	79.000	69.000	75.000	62.000
## cfi	0.952	0.932	0.946	0.839	0.942	0.882
## rmsea	0.070	0.079	0.071	0.118	0.073	0.099
## rmsea.ci.lower	0.058	0.068	0.059	0.108	0.062	0.089
## rmsea.ci.upper	0.082	0.090	0.082	0.128	0.084	0.109
## srmr	0.057	0.070	0.062	0.114	0.065	0.079
## bic	26330.044	26323.068	26289.892	26532.502	26282.379	26379.653
##	PSTRICT	RESCOV	LVARs	MEANS		
## chisq	329.011	359.669	349.327	548.344		
## df	144.000	145.000	147.000	143.000		
## npar	64.000	63.000	61.000	65.000		

```
## cfi                0.933      0.922      0.926      0.853
## rmsea              0.075      0.081      0.078      0.112
## rmsea.ci.lower     0.064      0.070      0.067      0.102
## rmsea.ci.upper     0.086      0.091      0.088      0.121
## srmr               0.073      0.073      0.114      0.187
## bic                26250.472 26275.008 26252.421 26475.928
```

```
summary(WIIIMMFP.fit, stand = T)
```

```
## lavaan 0.6-9 ended normally after 124 iterations
##
##      Estimator                      ML
##      Optimization method          NLMINB
##      Number of model parameters      93
##      Number of equality constraints    29
##
##      Number of observations per group:
##      Group 1                        349
##      Group 2                        107
##
## Model Test User Model:
##
##      Test statistic                 329.011
##      Degrees of freedom              144
##      P-value (Chi-square)            0.000
##      Test statistic for each group:
##      Group 1                        207.973
##      Group 2                        121.037
##
## Parameter Estimates:
##
##      Standard errors                Standard
##      Information                    Expected
##      Information saturated (h1) model Structured
##
##
## Group 1 [Group 1]:
##
## Latent Variables:
##      Estimate  Std.Err  z-value  P(>|z|)  Std.lv  Std.all
##      POI =~
##      PA                1.000                2.075  0.924
##      SS      (.p2.)    0.730    0.061   12.043   0.000   1.514  0.566
##      DSC      (.p3.)    0.626    0.063    9.868   0.000   1.300  0.482
##      PC      (.p4.)    1.038    0.061   17.144   0.000   2.154  0.702
##      MR      (.p5.)    0.971    0.056   17.266   0.000   2.014  0.739
##      BD      (.p6.)    1.016    0.056   18.065   0.000   2.108  0.729
##      VCI =~
##      V                1.000                2.127  0.831
##      S                1.188    0.067   17.729   0.000   2.527  0.832
##      I      (.p9.)    1.035    0.060   17.228   0.000   2.201  0.766
##      C      (.10.)    0.835    0.052   15.935   0.000   1.775  0.675
##      A      (.11.)    0.353    0.050    7.117   0.000   0.751  0.247
##      WMI =~
```

```

##      C      1.000      0.865      0.329
##      DS      (.13.)      2.634      0.344      7.663      0.000      2.278      0.783
##      LNS      (.14.)      2.413      0.316      7.631      0.000      2.087      0.757
##      A      (.15.)      1.897      0.265      7.157      0.000      1.641      0.540
##
## Covariances:
##      Estimate      Std.Err      z-value      P(>|z|)      Std.lv      Std.all
##      .SS ~~
##      .DSC      3.615      0.309      11.710      0.000      3.615      0.694
##      POI ~~
##      VCI      1.877      0.291      6.445      0.000      0.425      0.425
##      WMI      0.972      0.170      5.718      0.000      0.541      0.541
##      VCI ~~
##      WMI      0.513      0.134      3.841      0.000      0.279      0.279
##
## Intercepts:
##      Estimate      Std.Err      z-value      P(>|z|)      Std.lv      Std.all
##      .PA      (.36.)      10.090      0.120      84.120      0.000      10.090      4.496
##      .SS      8.250      0.143      57.650      0.000      8.250      3.086
##      .DSC      7.720      0.144      53.473      0.000      7.720      2.862
##      .PC      (.39.)      10.478      0.159      66.002      0.000      10.478      3.414
##      .MR      10.590      0.146      72.624      0.000      10.590      3.887
##      .BD      (.41.)      10.682      0.150      71.052      0.000      10.682      3.695
##      .V      14.560      0.137      106.333      0.000      14.560      5.692
##      .S      14.220      0.162      87.510      0.000      14.220      4.684
##      .I      11.750      0.154      76.421      0.000      11.750      4.091
##      .C      (.45.)      12.462      0.141      88.592      0.000      12.462      4.737
##      .A      (.46.)      9.684      0.158      61.331      0.000      9.684      3.188
##      .DS      (.47.)      9.430      0.152      61.904      0.000      9.430      3.240
##      .LNS      (.48.)      8.916      0.143      62.210      0.000      8.916      3.235
##      POI      0.000      0.000      0.000      0.000      0.000      0.000
##      VCI      0.000      0.000      0.000      0.000      0.000      0.000
##      WMI      0.000      0.000      0.000      0.000      0.000      0.000
##
## Variances:
##      Estimate      Std.Err      z-value      P(>|z|)      Std.lv      Std.all
##      .PA      0.732      0.135      5.409      0.000      0.732      0.145
##      .SS      (.18.)      4.853      0.336      14.438      0.000      4.853      0.679
##      .DSC      (.19.)      5.585      0.379      14.731      0.000      5.585      0.768
##      .PC      (.20.)      4.783      0.358      13.369      0.000      4.783      0.508
##      .MR      (.21.)      3.364      0.261      12.866      0.000      3.364      0.453
##      .BD      (.22.)      3.914      0.300      13.044      0.000      3.914      0.468
##      .V      (.23.)      2.021      0.197      10.263      0.000      2.021      0.309
##      .S      2.829      0.311      9.093      0.000      2.829      0.307
##      .I      (.25.)      3.406      0.282      12.063      0.000      3.406      0.413
##      .C      (.26.)      2.167      0.190      11.391      0.000      2.167      0.313
##      .A      (.27.)      5.282      0.400      13.193      0.000      5.282      0.572
##      .DS      (.28.)      3.281      0.360      9.110      0.000      3.281      0.387
##      .LNS      (.29.)      3.241      0.325      9.959      0.000      3.241      0.427
##      POI      4.305      0.394      10.924      0.000      1.000      1.000
##      VCI      4.523      0.483      9.357      0.000      1.000      1.000
##      WMI      0.748      0.185      4.036      0.000      1.000      1.000
##
##
##

```

```

## Group 2 [Group 2]:
##
## Latent Variables:
##      Estimate   Std.Err   z-value   P(>|z|)   Std.lv   Std.all
##      POI =~
##      PA          1.000
##      SS      (.p2.)  0.730    0.061   12.043    0.000    1.031    0.424
##      DSC      (.p3.)  0.626    0.063    9.868    0.000    0.885    0.351
##      PC      (.p4.)  1.038    0.061   17.144    0.000    1.467    0.557
##      MR      (.p5.)  0.971    0.056   17.266    0.000    1.372    0.599
##      BD      (.p6.)  1.016    0.056   18.065    0.000    1.436    0.587
##      VCI =~
##      V          1.000
##      S          0.508    0.064    7.890    0.000    0.976    0.700
##      I      (.p9.)  1.035    0.060   17.228    0.000    1.988    0.733
##      C      (.10.)  0.835    0.052   15.935    0.000    1.603    0.663
##      A      (.11.)  0.353    0.050    7.117    0.000    0.679    0.247
##      WMI =~
##      C          1.000
##      DS      (.13.)  2.634    0.344    7.663    0.000    1.505    0.639
##      LNS      (.14.)  2.413    0.316    7.631    0.000    1.378    0.608
##      A      (.15.)  1.897    0.265    7.157    0.000    1.084    0.395
##
## Covariances:
##      Estimate   Std.Err   z-value   P(>|z|)   Std.lv   Std.all
##      .SS ~~
##      .DSC          1.766    0.473    3.733    0.000    1.766    0.339
##      POI ~~
##      VCI          1.081    0.368    2.937    0.003    0.398    0.398
##      WMI          0.476    0.142    3.355    0.001    0.590    0.590
##      VCI ~~
##      WMI          0.464    0.163    2.842    0.004    0.423    0.423
##
## Intercepts:
##      Estimate   Std.Err   z-value   P(>|z|)   Std.lv   Std.all
##      .PA      (.36.)  10.090    0.120   84.120    0.000   10.090    3.495
##      .SS          11.208    0.277   40.506    0.000   11.208    4.607
##      .DSC          11.035    0.284   38.839    0.000   11.035    4.373
##      .PC      (.39.)  10.478    0.159   66.002    0.000   10.478    3.979
##      .MR          12.672    0.269   47.055    0.000   12.672    5.533
##      .BD      (.41.)  10.682    0.150   71.052    0.000   10.682    4.370
##      .V          13.077    0.378   34.626    0.000   13.077    5.472
##      .S          10.231    0.317   32.307    0.000   10.231    7.343
##      .I          14.492    0.419   34.620    0.000   14.492    5.342
##      .C      (.45.)  12.462    0.141   88.592    0.000   12.462    5.157
##      .A      (.46.)   9.684    0.158   61.331    0.000    9.684    3.527
##      .DS      (.47.)   9.430    0.152   61.904    0.000    9.430    4.005
##      .LNS      (.48.)   8.916    0.143   62.210    0.000    8.916    3.933
##      POI          -2.052    0.231   -8.896    0.000   -1.452   -1.452
##      VCI          -4.157    0.398  -10.450    0.000   -2.164   -2.164
##      WMI           0.056    0.091    0.617    0.537    0.098    0.098
##
## Variances:
##      Estimate   Std.Err   z-value   P(>|z|)   Std.lv   Std.all

```


##	.PA		6.341	0.956	6.635	0.000	6.341	0.761
##	.SS	(.18.)	4.853	0.336	14.438	0.000	4.853	0.820
##	.DSC	(.19.)	5.585	0.379	14.731	0.000	5.585	0.877
##	.PC	(.20.)	4.783	0.358	13.369	0.000	4.783	0.690
##	.MR	(.21.)	3.364	0.261	12.866	0.000	3.364	0.641
##	.BD	(.22.)	3.914	0.300	13.044	0.000	3.914	0.655
##	.V	(.23.)	2.021	0.197	10.263	0.000	2.021	0.354
##	.S		0.990	0.165	6.015	0.000	0.990	0.510
##	.I	(.25.)	3.406	0.282	12.063	0.000	3.406	0.463
##	.C	(.26.)	2.167	0.190	11.391	0.000	2.167	0.371
##	.A	(.27.)	5.282	0.400	13.193	0.000	5.282	0.701
##	.DS	(.28.)	3.281	0.360	9.110	0.000	3.281	0.592
##	.LNS	(.29.)	3.241	0.325	9.959	0.000	3.241	0.630
##	POI		1.997	0.424	4.711	0.000	1.000	1.000
##	VCI		3.691	0.662	5.571	0.000	1.000	1.000
##	WMI		0.326	0.105	3.118	0.002	1.000	1.000

#Latent Variance Homogeneity #1

```
WIIIUKR1.model <- '
POI =~ PA + SS + DSC + PC + MR + BD
VCI =~ V + S + I + C + A
WMI =~ C + DS + LNS + A
```

```
DSC ~~ SS
```

```
POI ~~ c(1,1)*POI'
```

#Latent Variance Homogeneity #2

```
WIIIUKR2.model <- '
POI =~ PA + SS + DSC + PC + MR + BD
VCI =~ V + S + I + C + A
WMI =~ C + DS + LNS + A
```

```
DSC ~~ SS
```

```
VCI ~~ c(1,1)*VCI'
```

#Latent Variance Homogeneity #3

```
WIIIUKR3.model <- '
POI =~ PA + SS + DSC + PC + MR + BD
VCI =~ V + S + I + C + A
WMI =~ C + DS + LNS + A
```

```
DSC ~~ SS
```

```
WMI ~~ c(1,1)*WMI'
```

#Latent Variance Homogeneity #4

```
WIIIUKR4.model <- '
POI =~ PA + SS + DSC + PC + MR + BD
```

```

VCI =~ V + S + I + C + A
WMI =~ C + DS + LNS + A

DSC ~~ SS

POI ~~ c(1,1)*POI
VCI ~~ c(1,1)*VCI'

#Latent Variance Homogeneity #5

WIIIIUKR5.model <- '
POI =~ PA + SS + DSC + PC + MR + BD
VCI =~ V + S + I + C + A
WMI =~ C + DS + LNS + A

DSC ~~ SS

POI ~~ c(1,1)*POI
WMI ~~ c(1,1)*WMI'

#Latent Variance Homogeneity #6

WIIIIUKR6.model <- '
POI =~ PA + SS + DSC + PC + MR + BD
VCI =~ V + S + I + C + A
WMI =~ C + DS + LNS + A

DSC ~~ SS

VCI ~~ c(1,1)*VCI
WMI ~~ c(1,1)*WMI'

WIIIMMV1.fit <- cfa(WIIIIUKR1.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals"),
  group.partial = c("VCI=~S", "I~1", "MR~1", "DSC~1", "SS~1", "S~1",
    "V~1", "PA~~PA", "S~~S"))

WIIIMMV2.fit <- cfa(WIIIIUKR2.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals"),
  group.partial = c("VCI=~S", "I~1", "MR~1", "DSC~1", "SS~1", "S~1",
    "V~1", "PA~~PA", "S~~S"))

WIIIMMV3.fit <- cfa(WIIIIUKR3.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals"),
  group.partial = c("VCI=~S", "I~1", "MR~1", "DSC~1", "SS~1", "S~1",
    "V~1", "PA~~PA", "S~~S"))

WIIIMMV4.fit <- cfa(WIIIIUKR4.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals"),

```

```

group.partial =c("VCI=~S", "I~1", "MR~1", "DSC~1", "SS~1", "S~1",
                 "V~1", "PA~~PA", "S~~S"))

WIIIMMV5.fit <- cfa(WIIIUUKR5.model, sample.cov = GCovs, sample.mean = GMeans,
                  sample.nobs = GNs, std.lv = F, orthogonal = F,
                  group.equal = c("loadings", "intercepts", "residuals"),
                  group.partial =c("VCI=~S", "I~1", "MR~1", "DSC~1", "SS~1", "S~1",
                                   "V~1", "PA~~PA", "S~~S"))

WIIIMMV6.fit <- cfa(WIIIUUKR6.model, sample.cov = GCovs, sample.mean = GMeans,
                  sample.nobs = GNs, std.lv = F, orthogonal = F,
                  group.equal = c("loadings", "intercepts", "residuals"),
                  group.partial =c("VCI=~S", "I~1", "MR~1", "DSC~1", "SS~1", "S~1",
                                   "V~1", "PA~~PA", "S~~S"))

round(cbind(PSTRICKT = fitMeasures(WIIIMMFP.fit, FITM),
            LVAR5 = fitMeasures(WIIIMMV5.fit, FITM),
            LVAR1 = fitMeasures(WIIIMMV1.fit, FITM),
            LVAR2 = fitMeasures(WIIIMMV2.fit, FITM),
            LVAR3 = fitMeasures(WIIIMMV3.fit, FITM),
            LVAR4 = fitMeasures(WIIIMMV4.fit, FITM),
            LVAR5 = fitMeasures(WIIIMMV5.fit, FITM),
            LVAR6 = fitMeasures(WIIIMMV6.fit, FITM)),3)

```

##	PSTRICKT	LVAR5	LVAR1	LVAR2	LVAR3	LVAR4
## chisq	329.011	349.327	485.645	455.896	345.557	607.145
## df	144.000	147.000	146.000	146.000	146.000	148.000
## npar	64.000	61.000	62.000	62.000	62.000	60.000
## cfi	0.933	0.926	0.876	0.887	0.927	0.833
## rmsea	0.075	0.078	0.101	0.096	0.077	0.117
## rmsea.ci.lower	0.064	0.067	0.091	0.087	0.067	0.107
## rmsea.ci.upper	0.086	0.088	0.111	0.107	0.088	0.126
## srmr	0.073	0.114	0.137	0.112	0.090	0.162
## bic	26250.472	26252.421	26394.861	26365.113	26254.773	26504.116
##	LVAR5	LVAR6				
## chisq	500.115	475.709				
## df	148.000	148.000				
## npar	60.000	60.000				
## cfi	0.872	0.881				
## rmsea	0.102	0.099				
## rmsea.ci.lower	0.092	0.089				
## rmsea.ci.upper	0.112	0.109				
## srmr	0.148	0.122				
## bic	26397.086	26372.680				

```
1 - pchisq(345.557-329.011, 2)
```

```
## [1] 0.0002553182
```

Latent variance homogeneity was not tenable using the British configuration.

#Mean Homogeneity #1

```
WIIIUKR1.model <- '  
POI =~ PA + SS + DSC + PC + MR + BD  
VCI =~ V + S + I + C + A  
WMI =~ C + DS + LNS + A  
  
DSC ~~ SS  
  
POI ~ c(0,0)*1'
```

#Mean Homogeneity #2

```
WIIIUKR2.model <- '  
POI =~ PA + SS + DSC + PC + MR + BD  
VCI =~ V + S + I + C + A  
WMI =~ C + DS + LNS + A  
  
DSC ~~ SS  
  
VCI ~ c(0,0)*1'
```

#Mean Homogeneity #3

```
WIIIUKR3.model <- '  
POI =~ PA + SS + DSC + PC + MR + BD  
VCI =~ V + S + I + C + A  
WMI =~ C + DS + LNS + A  
  
DSC ~~ SS  
  
WMI ~ c(0,0)*1'
```

#Mean Homogeneity #4

```
WIIIUKR4.model <- '  
POI =~ PA + SS + DSC + PC + MR + BD  
VCI =~ V + S + I + C + A  
WMI =~ C + DS + LNS + A  
  
DSC ~~ SS  
  
POI ~ c(0,0)*1  
VCI ~ C(0,0)*1'
```

#Mean Homogeneity #5

```
WIIIUKR5.model <- '  
POI =~ PA + SS + DSC + PC + MR + BD  
VCI =~ V + S + I + C + A  
WMI =~ C + DS + LNS + A  
  
DSC ~~ SS
```

```

POI ~ c(0,0)*1
WMI ~ C(0,0)*1'

#Mean Homogeneity #6

WIIUKR6.model <- '
POI =~ PA + SS + DSC + PC + MR + BD
VCI =~ V + S + I + C + A
WMI =~ C + DS + LNS + A

DSC ~~ SS

VCI ~ c(0,0)*1
WMI ~ C(0,0)*1'

WIIIMME1.fit <- cfa(WIIISA1.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals"),
  group.partial = c("VCI=~S", "I~1", "MR~1", "DSC~1", "SS~1", "S~1",
    "V~1", "PA~~PA", "S~~S"))

```

```

## Warning in lav_model_vcov(lavmodel = lavmodel, lavsamplestats = lavsamplestats, : lavaan WARNING:
##   The variance-covariance matrix of the estimated parameters (vcov)
##   does not appear to be positive definite! The smallest eigenvalue
##   (= 1.167127e-14) is close to zero. This may be a symptom that the
##   model is not identified.

```

```

WIIIMME2.fit <- cfa(WIIISA2.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals"),
  group.partial = c("VCI=~S", "I~1", "MR~1", "DSC~1", "SS~1", "S~1",
    "V~1", "PA~~PA", "S~~S"))

```

```

## Warning in lav_model_vcov(lavmodel = lavmodel, lavsamplestats = lavsamplestats, : lavaan WARNING:
##   The variance-covariance matrix of the estimated parameters (vcov)
##   does not appear to be positive definite! The smallest eigenvalue
##   (= 6.070837e-15) is close to zero. This may be a symptom that the
##   model is not identified.

```

```

WIIIMME3.fit <- cfa(WIIISA3.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals"),
  group.partial = c("VCI=~S", "I~1", "MR~1", "DSC~1", "SS~1", "S~1",
    "V~1", "PA~~PA", "S~~S"))

```

```

## Warning in lav_model_vcov(lavmodel = lavmodel, lavsamplestats = lavsamplestats, : lavaan WARNING:
##   The variance-covariance matrix of the estimated parameters (vcov)
##   does not appear to be positive definite! The smallest eigenvalue
##   (= 5.728766e-15) is close to zero. This may be a symptom that the
##   model is not identified.

```

```
WIIIMME4.fit <- cfa(WIIISA4.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals"),
  group.partial = c("VCI=~S", "I~1", "MR~1", "DSC~1", "SS~1", "S~1",
    "V~1", "PA~~PA", "S~~S"))
```

```
WIIIMME5.fit <- cfa(WIIISA5.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals"),
  group.partial = c("VCI=~S", "I~1", "MR~1", "DSC~1", "SS~1", "S~1",
    "V~1", "PA~~PA", "S~~S"))
```

```
## Warning in lav_model_vcov(lavmodel = lavmodel, lavsamplestats = lavsamplestats, : lavaan WARNING:
##   The variance-covariance matrix of the estimated parameters (vcov)
##   does not appear to be positive definite! The smallest eigenvalue
##   (= 8.466408e-15) is close to zero. This may be a symptom that the
##   model is not identified.
```

```
WIIIMME6.fit <- cfa(WIIISA6.model, sample.cov = GCovs, sample.mean = GMeans,
  sample.nobs = GNs, std.lv = F, orthogonal = F,
  group.equal = c("loadings", "intercepts", "residuals"),
  group.partial = c("VCI=~S", "I~1", "MR~1", "DSC~1", "SS~1", "S~1",
    "V~1", "PA~~PA", "S~~S")); "\n"
```

```
## Warning in lav_model_vcov(lavmodel = lavmodel, lavsamplestats = lavsamplestats, : lavaan WARNING:
##   The variance-covariance matrix of the estimated parameters (vcov)
##   does not appear to be positive definite! The smallest eigenvalue
##   (= 9.634988e-15) is close to zero. This may be a symptom that the
##   model is not identified.
```

```
## [1] "\n"
```

```
round(cbind(PSTRICKT = fitMeasures(WIIIMMFP.fit, FITM),
  MEANS = fitMeasures(WIIIMME.fit, FITM),
  MEAN1 = fitMeasures(WIIIMME1.fit, FITM),
  MEAN2 = fitMeasures(WIIIMME2.fit, FITM),
  MEAN3 = fitMeasures(WIIIMME3.fit, FITM),
  MEAN4 = fitMeasures(WIIIMME4.fit, FITM),
  MEAN5 = fitMeasures(WIIIMME5.fit, FITM),
  MEAN6 = fitMeasures(WIIIMME6.fit, FITM)),3)
```

	PSTRICKT	MEANS	MEAN1	MEAN2	MEAN3	MEAN4
## chisq	329.011	548.344	514.537	474.059	400.940	400.316
## df	144.000	143.000	140.000	140.000	140.000	140.000
## npar	64.000	65.000	68.000	68.000	68.000	68.000
## cfi	0.933	0.853	0.864	0.879	0.905	0.905
## rmsea	0.075	0.112	0.108	0.102	0.090	0.090
## rmsea.ci.lower	0.064	0.102	0.098	0.092	0.080	0.080
## rmsea.ci.upper	0.086	0.121	0.118	0.113	0.101	0.101
## srmr	0.073	0.187	0.152	0.115	0.073	0.073
## bic	26250.472	26475.928	26460.488	26420.010	26346.892	26346.267

##	MEAN5	MEAN6
## chisq	531.673	523.655
## df	141.000	141.000
## npar	67.000	67.000
## cfi	0.858	0.861
## rmsea	0.110	0.109
## rmsea.ci.lower	0.100	0.099
## rmsea.ci.upper	0.120	0.119
## srmr	0.199	0.142
## bic	26471.502	26463.483

No level of mean constraint was possible using the British configuration.

Part III: Bias Estimation

This section utilized Gunn, Grimm & Edwards' (2019) SDI_2 measure of bias, which could be roughly understood in terms of Cohen's d . Details can be found in their paper that introduced the measure. Briefly, the measure yielded expectations such as if a group showed a higher (unstandardized) intercept than another, they would receive higher scores at the same levels of ability. The values derived through this measure could be added to the differences in Cohen's d provided by Cockcroft et al. (2015, Table 1) to arrive at bias-corrected mean differences but that would not add anything to this study beyond the SDI_2 values themselves. The secondary group for computing SDI_2 was the British one.

When the residual variances of items are biased, they indicate that different influences are operating on the scores measured in each group (DeShon, 2004); despite the importance of this part of testing for measurement invariance, the effect of a violation cannot yet be directly adjusted but future methodological innovations may. In lieu of estimating new variances, I adopted Millsap & Olivera-Aguilar's (2012) approach of estimating the proportion of the difference in manifest variances due to differences in the residual variances.

```
#South African Inputs. Values are derived from the most restrictive non-mean model
#summary results for this model are printed above.

#Parameters for POI. Order is MR, PC, BD, PA.

Pp = 4; P11 = c(0, 0, 0, 0); P12 = c(0, 0, 0, 0)
Pi1 = c(12.720, 0, 0, 0); Pi2 = c(10.590, 0, 0, 0); Psd = c(2.71, 3.13, 2.87, 2.26)
Pfm = 1.077; Pfsd = 1

#Parameters for VCI. Order is V, S, I, C, A.

Vp = 5; V11 = c(0, 0.724, 0, 0, 0); V12 = c(0, 0.823, 0, 0, 0)
Vi1 = c(0, 11.022, 16.207, 14.607, 11.452); Vi2 = c(0, 14.220, 11.750, 12.480, 9.580)
Vsd = c(2.66, 3.04, 2.72, 2.59, 3.14); Vfm = 2.774; Vfsd = 1

#Parameter inputs for POI and VCI

PES <- SDI2.UDI2(Pp, P11, P12, Pi1, Pi2, Psd, Pfm, Pfsd)
VES <- SDI2.UDI2(Vp, V11, V12, Vi1, Vi2, Vsd, Vfm, Vfsd)

PES$SDI2

##          SDI2
## [1,] 0.7859
```

```
## [2,] 0.0000
## [3,] 0.0000
## [4,] 0.0000
```

```
VES$SDI2
```

```
##          SDI2
## [1,] 0.0000
## [2,] -1.1266
## [3,] 1.6176
## [4,] 0.8107
## [5,] 0.5885
```

```
#For calculation of the proportion of the difference in variances due to bias
```

```
#PA
```

```
MOES(0.752, 6.364, 2.26, 2.82)
```

```
## [1] 1.972722
```

```
#SS
```

```
MOES(0.957, 3.948, 2.62, 2.67)
```

```
## [1] 11.30813
```

```
#S
```

```
MOES(2.923, 0.992, 3.04, 1.40)
```

```
## [1] 0.265189
```

```
#PC
```

```
MOES(5.351, 2.603, 3.13, 2.11)
```

```
## [1] 0.5141446
```

```
#British Inputs
```

```
#POI. Order is MR, PC, BD, PA, DSC, SS.
```

```
Pp = 6; P11 = c(0, 0, 0, 0, 0, 0); P12 = c(0, 0, 0, 0, 0, 0)
Pi1 = c(12.672, 0, 0, 0, 11.035, 11.208); Pi2 = c(10.590, 0, 0, 0, 7.720, 8.250)
Psd = c(2.71, 3.13, 2.87, 2.26, 2.68, 2.62); Pfm = 1.452; Pfsd = 2.156 #SD Ratio
```

```
#VCI
```

```
Vp = 5; V11 = c(0, 0.700, 0, 0, 0); V12 = c(0, 0.832, 0, 0, 0)
```



```
Vi1 = c(13.077, 10.231, 14.492, 0, 0); Vi2 = c(14.560, 14.220, 11.750, 0, 0)
Vsd = c(2.66, 3.04, 2.72, 2.59, 3.14); Vfm = 2.164; Vfsd = 1.225
```

```
#Parameter inputs for POI and VCI
```

```
PES <- SDI2.UDI2(Pp, Pl1, Pl2, Pi1, Pi2, Psd, Pfm, Pfsd)
VES <- SDI2.UDI2(Vp, Vl1, Vl2, Vi1, Vi2, Vsd, Vfm, Vfsd)
```

```
PES$SDI2
```

```
##          SDI2
## [1,] 0.7290
## [2,] 0.0000
## [3,] 0.0000
## [4,] 0.0000
## [5,] 1.1738
## [6,] 1.0714
```

```
VES$SDI2
```

```
##          SDI2
## [1,] -0.5518
## [2,] -1.3904
## [3,] 0.9978
## [4,] 0.0000
## [5,] 0.0000
```

```
#PA
```

```
MOES(0.732, 6.341, 2.26, 2.82)
```

```
## [1] 1.971668
```

```
#S
```

```
MOES(2.829, 0.990, 3.04, 1.40)
```

```
## [1] 0.2525544
```

Section Conclusion

Conclusions were similar regardless of model: bias was present but it generally favored the South African group. If the South African configuration was accepted, Matrix Reasoning, Information, Comprehension, and Arithmetic were biased in favor of the South African group while Similarities was biased in favor of the British one (i.e., 80% were biased in favor of the South African group versus 20% in favor of the British one). If the British configuration was used, Mental Rotations, Digit-Symbol Coding, Symbol Search, and Information were biased in favor of the South African group while Vocabulary and Similarities were biased in favor of the British group (i.e., 67% were biased in favor of the South African group versus 33% in favor of the British one). 26.52 and 51.41% of the difference in the SDs for Similarities and Picture Completion were due to bias and 25.25% of the difference in the SD for Similarities was due to bias if the South African or British configurations, respectively, were used. Three of the five estimates for the proportions of the SD differences due to bias were not interpretable (for more information, see Millsap & Olivera-Aguilar, 2012).

Conclusion

Section-by-section, the PCA results from Cockcroft et al. (2015) were accurate but the evidence for different configural factor structures was poor and it should not have been concluded that those differed based on exploratory results. Cross-loadings - which are problematic for assessing bias - could be eliminated and replaced with residual covariances while maintaining theoretical coherence for the factors; because of this, it may have been better to work towards models without cross-loadings due to the aforementioned difficulty of assessing bias when they are in the model. Luckily, the variant loading in Cockcroft et al.'s model(s) did not occur for a cross-loaded variable. Results regarding the degrees and directions of bias were largely consistent whether the South African or the British factor configurations were used.

Discussion and Limitations

Cockcroft et al. (2015) found evidence for measurement bias in a comparison of South African and British university students. Their interpretation was that the Wechsler Adult Intelligence Scale Third Edition showcased a “Eurocentric” bias. However, when the degree and direction of bias in variable means was computed, bias consistently favored the South African group. Though the test (and most contemporary tests) was (were) developed among Europeans, that clearly did not make it a test that was biased in their favor. As a general rule, without computing the direction of bias, the direction should not be interpreted in favor of or against a group regardless of how they scored relative to another one. As these results show, theoretical cases for which direction bias should point are not always correct even though they are often intuitively plausible, as in Cockcroft et al.'s speculation about which tests were more or less biased, which were based on interpreting the group differences as a measure of bias. Though my results were about a specific, perhaps counterintuitive case, it should be stated that they may not apply to others. In any case, it should never be assumed that bias goes in any particular direction and the presence of a mean difference cannot be thought of as *prima facie* evidence for bias. With intelligence tests, bias is usually absent, although it may be reasonable to assume that is not true for international comparisons even though it has been clearly true for (the far more common) intranational ones. The ultimate lesson here should be obvious: bias is always an empirical matter and it should not be interpreted naively. With the advent of effect sizes for measurement bias, there is no excuse not to use them when aiming to interpret bias.

Model choices, though theoretically appropriate, may have affected the results. This inference is supported by the slight differences in the degrees of bias when using models derived from the exploratory results from the South African or British groups. Whatever the ‘true’ model was, it may have yielded a different amount of bias, but the degree of difference is likely not much and the direction of bias is almost certainly consistent. Furthermore, although I advised computing effect sizes for bias, it is worth noting that even if bias points one or another direction, that cannot alone vindicate a particular reason for bias. Effect sizes can only inform about the direction of bias, not why the bias emerged. They should not be interpreted as evidence for a particular cause without other forms of evidence that are more than just musing about what that might be. A final limitation is that even if bias points mostly in one or another direction, it may be the case that bias from a particular source is present and operating in whatever proposed direction even if it is outweighed by other sources of bias. It is simply not possible to identify why bias happens from finding bias and computing an effect size alone, it is only sufficient to say that there is bias and the degree to which it affects tests. This means that, though Cockcroft et al.'s (2015) data was not clearly consistent at a first glance with a Eurocentric bias for the WAIS-III, that sort of bias may have been present, albeit outweighed by other types of bias. The conclusion that unevicenced forms of bias are present should be treated as dubious because they are practically pseudoscientific (i.e., unfalsifiable) without coherent and plausible theories that pave the way for other tests to evidence them. There is definitely room for more discussion of bias - complex and simple - but it does not belong here; the main conclusion to take from this work is to be cautious in interpreting bias with psychological instruments.

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